Ad Hoc Error Correction vs. Removal of Error's root

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Abstract

An ad hoc correction of a theoretical error does not change other elements of the relevant theory. The mathematical structure of theoretical physics indicates that a removal of the root of an error has a much more profound meaning than that of just correcting an error. Herein, this general principle is applied to four physical issues: the standard derivation of the electromagnetic energy-momentum tensor; gauge transformations of the 4-potential of electromagnetic fields; the idea called *the hadronic structure of the photon;* the $(1 \pm \gamma^5)$ factor that aims to project a quantum function into a parity violating form. The erroneous root of each of these issues is discussed and an appropriate line of research is suggested.

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1 Introduction

A typical attribute of an ad hoc error correction of a problematic physical expression is that this operation is performed *after an apparently legitimate calculation has been finished.* In this case, the result is modified because its form is physically unacceptable. In particular, an ad hoc correction does not modify other elements of the theory. This kind of activity resembles attempts of an accountant who adds an artificial line to his report in order to balance it. By contrast, a correction of the root of an error aims to yield a better theory from which physically meaningful expressions are derived. This kind of activity resembles the work of an accountant who corrects erroneous lines that are included in his report. In this work the term *ad hoc error correction* means an application of an ad hoc mathematical manipulation aiming to correct an erroneous result whereas other elements of the theory are unaltered.

The amazing success of mathematics in attempts to describe the physical world is recognized by scientists [1] as well as by engineers who use mathematics in their work. Evidently, cases where a legitimate mathematical procedure yields an incorrect physical result can be considered as a counter-example that weakens the above mentioned status of mathematics. This issue is an unfavorable attribute of an ad hoc error correction.

In this work an ad hoc correction is regarded as a hint that the mathematical structure of the relevant theory probably contains an inherent problem. This issue stimulates an examination of the relevant theory. Obviously, a further analysis of any physical theory is always a good idea, because results of this assignment may either substantiate the correctness of the examined theory or remove erroneous elements from it. Four different cases are examined below and the discussion indicates that a further theoretical work is needed.

In this work units where $\hbar = c = 1$ are used. Therefore, just one dimension is required and the dimension of length $[L]$ is used. The Minkowski metric is diagonal and its entries are (1,-1,-1,-1). Relativistic expressions are written in the standard notation. Sections 2-5 discuss ad hoc corrections that are used in the following topics: the electromagnetic energy-momentum tensor; gauge transformations of the electromagnetic 4-potential; the hadronic structure of the photon; the factor $(1 \pm \gamma^5)$ of the electroweak theory. The last section summarized this work.

2 The Construction of the Electromagnetic Energy-Momentum Tensor

Textbooks on classical electrodynamics derive the fields energy-momentum tensor $T^{\mu\nu}$ from the following Lagrangian density

$$\mathcal{L}_{Fields} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}. \tag{1}$$

(see [2], pp. 86-89, [3] pp. 601-608). The actual calculation yields an unacceptable expression. For example, the derived tensor is non-symmetric and depends explicitly on a partial derivative of the 4-potential A_μ. This problem is recognized and corrected by an ad hoc addition of this quantity

$$\frac{1}{4\pi}(A^\mu F^{\nu\lambda})_{,\lambda}. \tag{2}$$

The addition of (2) yields the following symmetric traceless tensor which is independent of the 4-potential (see [2], p. 87, [3], p. 605)

$$T^{\mu\nu} = \frac{1}{4\pi}(g^{\mu\alpha} F_{\alpha\beta} F^{\beta\nu} + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}). \tag{3}$$

This is the required energy-momentum tensor of the electromagnetic fields.

Although the foregoing procedure yields the required energy-momentum tensor, the correction process leaves open following crucial question:

> Why a legitimate application of important theories like the variational principle and Maxwellian electrodynamics yields an unacceptable result?

The inconsistent energy-momentum tensor which is obtained from the electromagnetic Lagrangian density (1) provides a good reason for carrying out an examination of the present form of electrodynamics in general and of Quantum Electrodynamics (QED) in particular.

An analysis of the structure of Maxwellian electrodynamics has recently been published [4]. An important result which is obtained from this analysis proves that *radiation fields and bound fields represent different physical objects.* Therefore the electromagnetic Lagrangian density (1) should treat these fields separately. Furthermore, bound fields do not represent an independent physical object. As a matter of fact, bound field can be removed from the Lagrangian density. In the classical case, the Darwin Lagrangian (see [2], pp. 179-182, [3], pp. 593-595) plays the role of bound fields. In the quantum case, the Breit interaction replaces the role of the Darwin Lagrangian (see [5], pp. 170-178).

The need for a distinction between radiation fields and bound fields relies on well-established experimental data [4]. The same is true for the removal of bound fields from the Lagrangian density [4]. If the Lagrangian density (1) contains only radiation fields then *the correct symmetric traceless energy-momentum tensor* (3) *is directly obtained without any further manipulation* [6]. This result shows that the mathematical structure of the electromagnetic Lagrangian density can take a consistent form.

Evidently, the above mentioned modification of the electrodynamics Lagrangian density may affect many other electrodynamic calculations.

3 The Problem of Gauge Transformation of the Electromagnetic 4-potential

A classical electrodynamics theory can be constructed on the basis of Maxwell equations and the Lorentz law of force. These differential equations depend on the electromagnetic fields $F^{\mu\nu}$ and are independent of the 4-potential. These equations can be used for a construction of the entire theory. This approached is called below MLE. The acronym MLE stands for Maxwell-Lorentz Electrodynamics, namely the theory which relies on the above mentioned differential equations. Here the 4-potential is an auxiliary quantity that may be used for finding solution to a given problem. Let $\Lambda(x)$ be an arbitrary gauge function, where $x \equiv (t, \mathbf{x})$ denotes a set of four space-time coordinates. The associated gauge transformation of the 4-potential is

$$A_\mu(x) \to A_\mu(x) + \Lambda(x)_{,\mu}. \tag{4}$$

This gauge transformation does not change the electromagnetic fields which are the 4-curl of the 4-potential

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}. \tag{5}$$

This evidence means that gauge transformations are acceptable in MLE, because the fields (5) are invariant under this transformation.

The variational principle provides an alternative to MLE. It can be used for a construction of classical electrodynamics [2] and quantum electrodynamics (see [7], p. 349, [8], p. 78). This theory is derived from a Lagrangian density. In the quantum case of a Dirac particle, the Lagrangian density takes the following form

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} - e\bar{\psi}\gamma^\mu A_\mu \psi. \tag{6}$$

Below, electrodynamics that relies on the variational principle is called VE. Evidently, the last term of (6) shows that the 4-potential A_μ is used explicitly in the Lagrangian density of VE.

It should be pointed out that the gauge function $\Lambda(x)$ is an *arbitrary* function of the four space-time coordinates (see [7], p. 342, [9], [10]). Hence, in general, the gauge transformation (4) proves that its derivative alters the 4-potential. Indeed, a direct calculation shows that the Lagrangian density (6) is *not* invariant under the gauge transformation (4). An approach aiming to correct this error modifies the gauge transformation (4) and casts it into the following form (see [7], p. 345, [8], p. 78,)

$$A_\mu(x) \rightarrow A_\mu(x) + \Lambda(x)_{,\mu}, \quad \psi(x) \rightarrow \exp(-ie\Lambda(x))\psi(x), \tag{7}$$

where the symbol e in the exponent denotes the electronic charge. Here the quantity obtained from the second transformation on the right hand side of (7) cancels the additional term which is obtained from (4).

Apart from canceling the erroneous quantity that is obtained from the transformation (4), the second transformation of (7) has no other theoretical justification. In the terminology used herein, its introduction is an example of an ad hoc error correction. This is another reason for carrying out a theoretical analysis of its structure.

The following arguments show that the gauge transformation (7) is unacceptable. The root of the problem is that the dimension of the 4-potential A_μ is $[L^{-1}]$ whereas an arbitrary function of the four space-time coordinate may have a different dimension. Indeed, the power series expansion of the exponential function of (7) is

$$\exp(-ie\Lambda(x)) = 1 - ie\Lambda(x) + ... \tag{8}$$

A basic law of physics says that all terms of a physically valid expression must have the same dimension. Furthermore, in the case of a relativistic expression, these terms must also undergo the same Lorentz transformation. The first term of (8) is the pure number 1, which is a dimensionless Lorentz scalar. The same is true for the imaginary number i, and in the units used herein also the electric charge e is a dimensionless Lorentz scalar. This argument proves that the gauge function $\Lambda(x)$ must be a dimensionless Lorentz scalar. This outcome is inconsistent with the arbitrariness of the gauge function that is used in the literature (see [7], p. 342, [9], [10]).

Another erroneous element of the gauge transformation (7) is revealed in cases where the Dirac Hamiltonian is required. Applying the Legendre transformation to the Dirac Lagrangian density (6), one obtains the Dirac Hamiltonian (see [11], p. 48). This Hamiltonian stands on the right hand side of the Dirac equation

$$i\frac{\partial \psi}{\partial t} = [\boldsymbol{\alpha} \cdot (\mathbf{P} - e\mathbf{A}) + \beta m + e\phi]\psi. \tag{9}$$

Let us see what happens in the simple case of a motionless Dirac particle which is in a field-free region where $A_\mu = 0$. Here (9) reduces to

$$i\frac{\partial \psi}{\partial t} = \boldsymbol{\alpha} \cdot \mathbf{P} + \beta m. \tag{10}$$

It follows that the energy of the system is $E = m$. At this point, let us introduce the following gauge transformation

$$\Lambda(t, \mathbf{x}) = t^2, \tag{11}$$

which is a legitimate example of an arbitrary function of the four space-time coordinates. Substituting this gauge transformation into the Dirac equation (9) and using the gauge transformation (7), one finds that in this case the energy of the system is

$$E = m + 2t. \tag{12}$$

This is a contradiction. It violates the law of energy conservation as well as the uniform dimension of all terms of a physical expression.

Another problem stems from the additional phase of the gauge transformation (7). Evidently, this change of the phase of a Dirac particle may affect the calculation of an interference pattern of two electronic sub-beams. And indeed, it can be proved that if a gauge transformation is permissible then the interference calculation of these sub-beams is destroyed [12].

These examples show how errors arise if the root of the problem is not properly corrected. In particular, the present form of the QED gauge transformation (7) is unacceptable. It means that QED should be reconstructed without an application of gauge transformations.

4 The Hadronic Structure of the Photon

Experiments that have been carried out about 60 years ago prove that the cross section of a hard photon scattered on a proton is similar to that of a neutron [13]. Furthermore, the electric charge of the nucleon quark constituents is too small and it cannot explain this finding. Historically, the photon has been regarded as a pure electromagnetic particle which is associated with Maxwellian radiation fields. Therefore, the new data about the hard photon-nucleon interaction is inconsistent with the prevailing theory of that time.

An idea called Vector Meson Dominance (VMD) has been suggested in order to explain the new data. Similar ideas are called Modified Vector Meson Dominance (MVMD), Vector Dominance Model (VDM) and Hadronic Structure of the Photon. The primary element of these ideas states that the wave function of an energetic photon takes the form (see [13], p. 271)

$$\mid \gamma > = c_0 \mid \gamma_0 > + c_h \mid h >, \tag{13}$$

where $\mid \gamma >$ denotes the wave function of a physical photon, $\mid \gamma_0 >$ denotes the pure electromagnetic component of a physical photon and $\mid h >$ denotes its hypothetical hadronic component. c_0 and c_h are appropriate numerical coefficients. c_h tends to zero in the case of a soft photon. On the other hand, a hard photon is described by a non-negligible value of c_h. Evidently, the second term on the right hand side of (13) is an example of an ad hoc error correction. Indeed, it has been suggested *after* the new photon-nucleon data have been measured and it aims to explain them by changing an established theoretical expression. The following arguments prove that (13) is inconsistent with fundamental physical principles.

Expression (13) is inconsistent with Wigner's analysis of the unitary representations of the inhomogeneous Lorentz group (see [7, 14, 15]). This analysis proves that a massive quantum particle has a well-defined mass and spin whereas a massless

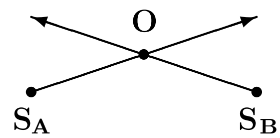

Figure 1: *Two beams of optical photons intersect at point* **O**. *(See text.)*

particle has two components of helicity. It means that the first term of (13) has only two values of the z-component of its spin whereas the second term of (13) has three values of the z-component of its spin. This is a contradiction because a free quantum particle has a well-defined spin (see [7, 14, 15]).

The following simple experiment provides an illustration of the validity of the above mentioned Wigner's work. It demonstrates that VMD is inconsistent with special relativity. Let S_A and S_B denote two sources of optical beams whose position is at $\mathbf{r} = (\pm 1, 0, 0)$, respectively (see fig. 1). The figure is embedded in the (x, y) plane. The beams intersect at point O and each of them continues in its original direction. Namely, in the case of optical photons no photon-photon scattering event takes place. Now let us examine this system from an inertial frame Σ' that moves nearly at the speed of light in the negative direction of the y-axis. In Σ', the photons of the beams are very energetic. Therefore, if VMD is correct then these energetic photons contain hadrons, and hadron-hadron scattering events should take place. It means that contradictory results are obtained from two observations of one and the same process. This contradiction proves that VMD is inconsistent with special relativity.

This discussion shows that the ad hoc modification (13) of the standard form of electrodynamics is unacceptable. The Standard model of particle physics contains the electromagnetic and the strong interactions sectors. It follows that the Standard Model provides no satisfactory explanation for hard photon-nucleon scattering that belongs to its domain of validity.

It is interesting to point out that a regular charge-monopole Lagrangian density provides a straightforward explanation for the hard photon-hadron interaction [16]. This is an example that shows how a consistent mathematical expression yields explanation of a physical effect.

5 The $(1 \pm \gamma^5)$ Factor of the Electroweak Theory

Lee and Yang analyzed unexplained experimental data obtained in the 1950s (see e.g. [17], pp. 136-137). Their conclusion was that the data could be explained if weak interactions do not conserve parity. Their conjecture stimulated Wu to organize an appropriate experiment. The results of this experiment substantiated the concept of parity violation in β decay. This result was at odds with field theories of that time.

The parity of the Dirac γ^5 matrix is odd (see [11], p. 26). Therefore, the operators

$$\hat{O} = (1 \pm \gamma^5) \tag{14}$$

contain equal amount of even and odd terms, which means that each of the operators (14) represents a maximal parity violation. These operators are used in the electroweak theory as ad hoc factors that account for the experimentally valid effect of parity violation in weak interactions (see e.g. [18], p. 308). The factor (14) has been introduced before the formulation of the electroweak theory [19]. This projection operator (see [19], p. 194) applies to any field theory. Its application means that *a special factor is introduced in order to account for parity violation of any Lagrangian density.*

The following argument proves that a contradiction arises from the parity violating factor (14). This analysis relies on the following experimental evidence: "neutrinos can no longer be considered as massless particles" [20].

Let us use the notation of of [11] (see pp. 17, 25, 28) for the γ matrices and for the 4-spinor. Here the product of each of the projection operators (14) and a motionless spin-up 4-spinor is

$$\begin{pmatrix} 1 & 0 & \pm 1 & 0 \\ 0 & 1 & 0 & \pm 1 \\ \pm 1 & 0 & 1 & 0 \\ 0 & \pm 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \pm 1 \\ 0 \end{pmatrix}. \tag{15}$$

The right hand side of (15) describes a massive spinor that moves at the speed of light in the z-direction (see [11], p. 30). This contradiction undermines the validity of the parity violating factor (14). Section 2 of [21] describes other erroneous elements of the electroweak theory.

It is interesting to note that a consistent Lagrangian density describes parity violating weak interaction processes. Here the associated *Hamiltonian* is a parity violating sum of a vector and an axial vector [21, 22]. This is certainly a better theory because it *proves* parity violation in weak interaction processes whereas the electroweak theory introduces the projection operator (14) in order to account for the parity violation effect.

6 Conclusions

Four examples of an ad hoc correction of unacceptable physical expressions are discussed. Drawbacks of this approach are explained. In particular, this approach does not help people to pay attention to a problematic element which may be hidden in the theory's structure. Another unfavorable aspect of this issue is that an ad hoc correction casts doubt on the essential principle that says that consistent mathematics provides a solid basis for physical theories.

The distinction between electromagnetic radiation fields and bound fields is based on experimental evidence which has no apparent connection to the derivation of the fields' energy-momentum tensor. The fact that a Lagrangian density that is based on this distinction yields the correct energy-momentum tensor is an indication that this is the right form of the Lagrangian density of electromagnetic fields.

The standard form of gauge transformation of the electromagnetic 4-potential (7) is unacceptable because an arbitrary gauge function yields crucial expressions (like the Hamiltonian) whose terms do not have the same dimension. Evidently, this is an unacceptable error. Other errors of the gauge transformation (7) are mentioned in section 3.

The photon has been regarded as a pure electromagnetic particle for many decades. The experimental evidence of hard photon-nucleon scattering was the reason for the idea called hadronic structure of the photon which takes the form (13). Section 4 proves that this is an erroneous idea and mentions that a regular charge-monopole theory solves the problem.

The last example is the introduction of the parity violating factors $(1 \pm \gamma^5)$ to the electroweak Lagrangian density. Section 5 proves that this is an erroneous idea. As a matter of fact, an appropriate weak interaction Lagrangian density yields a Hamiltonian that is a sum of a vector and an axial vector [21, 22]. Obviously, this Hamiltonian provides an explanation for the violation of parity conservation in weak interaction processes.

It can be concluded that this work shows the significance of a critical examination of ad hoc corrections as a tool for the removal of erroneous elements from the mathematical structure of physical theories.

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