NATURAL CONVECTIVEHEAT TRANSFER IN A LAMINAR FLOW OVER AN IMMERSED CURVED SURFACE

3

4 ABSTRACT

5 Numerical solutions of unsteady laminar free flow of a viscous fluid past an immersed curved surface were presented in this research study. The two-dimensional fluid flow in consideration 6 was unsteady and incompressible. Flows of this nature are commonly encountered in engineering 7 studies such as Aerodynamics and Hydrodynamics. In our study, the continuity, the momentum 8 and thermal energy equations were non-dimensionalized and the solutions of the dimensionless 9 governing equations approximated using finite-difference method. The velocity and temperature 10 fields were studied by varying various parameters in the equations governing the fluid flow. The 11 results obtained were presented graphically for comprehensive and easier interpretation. From 12 the results, it was found out that the dissipation of heat within the boundary layer increases with 13 increase in the length of the curvature i.e when the length of the curvature was increased, a 14 consequence increase in the amount of heat dissipated within the boundary layer was noted. 15 Also at large Reynolds number, minimal amount of heat dissipated within the boundary layer 16 was recorded. These findings would assist Engineers in making appropriate designs and estimate 17 improvements in equipment that require minimal resistance to the fluid in motion. 18

19 1 BACKGROUND INFORMATION

Natural Convective heat transfer over an immersed curved surface is receiving research attention
 due to its wide applications in designing of devices such as flying planes, submarines, pumps,
 cooling fans among many others.

23

24 In the study of laminar flows, Gupta et al (2003) investigated heat transfer along the surface

25 with a longitudinal curvature and concluded that as the curvature changes from concave to

26 convex, the Nusselt number decreases for Eckert number being small and increases if the

27 Eckert number is increased to unity.

Bradshaw et al (2006) extended the study on the use of the algebraic analogy to the curved shear
layers and the effects of the curvature on the mixing length if the shear layer thickness exceeds
1/300 of the radius of the curvature. In their study they concluded that large effects occurred in
compressible fluid flows.

From the investigations conducted by Khoshevis<mark>et al</mark> (2007) on effects of the concave curvature on turbulent fluid flows, it was found that turbulent intensities as well as shear stresses are high on concave surfaces compared to a flat surface under similar conditions. In their study, they concluded that the de-stabilizing effects on the boundary layer of the concave surface leads to increase in turbulence between the fluid particles similar to the way concave curvature would cause the flow to be destabilized.

Mugambi et al (2008) in their investigation on the forces produced by the fluid motion on a sub-merged finite curved plates established the relationship between geometrical shape of the curvature and the variation of drag force of specific velocities of the viscous fluid.

George et al (2009) in their study on the convective heat transfer over curved surface established that as fluid flows over an immersed curved surface, some work is done against viscous effect and energy spent is convertedinto heat. The vortices formed in the boundary layer due to high velocity gradient is swept outwards from the boundary layer. They established that the rate of heat transfer is considerably high at points close to the convex surface within the boundary layer thickness. This, as a result leads to a decrease in fluid viscosity.

Kioiet al (2011) in their study noted that when the Reynolds number is high, the heat dissipation in the boundary layer also goes high. Their study concluded that when the Reynolds number is increased, the consequence is decrease in drag. When the Reynolds number decreased, the effect of drag goes high. At high Reynolds number the lift is increased and vise versa, hence a direct proportionality of the two quantities. 52 Mawiraet al(2014) investigated the convective transfer of heat in a laminar boundary layer over 53 an immersed curved surface. In their study, concluded that when the surface area of the curvature 54 was increased, the velocity and temperature of the fluid increased and vise versa.

From the above discussed research investigations and findings, it is clear that limited or little attention has been paid on the extent to which varying the length of the curvature would affect the velocity and temperature profiles along the unsteady laminar fluid flow. This was the

- 58 motivation of this research work.
- 59

60 **3 DESCRIPTION OF THE FLOW MODEL**

In this research work, a two dimensional laminar unsteady flow of a fluid over an immersed
 curved surface is studied. Since the body had both convex and concave surfaces there existed
 two non- zero pressure gradients as shown in the schematic diagram below.





72 4 EQUATIONS GOVERNING THE FLUID FLOW

73 **4.1 Equation of continuity**:

74 The general continuity equation is given as:

75
$$\frac{\partial \rho}{\partial t} + \nabla (\rho u) = 0$$
 (1.0)

For two-dimensional fluid flow with constant density, equation (1.0) reduces to: 76

77
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
 (1.1)

4.2 Momentum equation 78

Along the x-axis; 79

80
$$\rho \frac{\partial u}{\partial t} + \rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left\{ \mu \left[2 \frac{\partial u}{\partial x} - \frac{2}{3} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] \right\} +$$

81
$$\frac{\partial}{\partial y} \left[\mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right\} + \rho F_{x}$$

Along the y- axis; 82

83
$$\rho \frac{\partial v}{\partial t} + \rho \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial y} \left\{ \mu \left[2 \frac{\partial v}{\partial y} - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \right\} + \frac{\partial}{\partial x} \left\{ \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right\} + \rho F_{y}$$

Since $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, equations the above equations reduces to: 85

86
$$\rho \frac{\partial u}{\partial t} + \rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial P}{\partial x} + 2\mu \frac{\partial^2 u}{\partial x^2} + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} \right) + \rho F_x \qquad (1.2a)$$

87 and

84

88
$$\rho \frac{\partial v}{\partial t} + \rho \left[u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial P}{\partial y} + 2\mu \frac{\partial^2 v}{\partial x^2} + \mu \left(\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 v}{\partial y^2} \right) + \rho F_y$$
(1.2b)

- respectively. 89
- From the boundary layer approximations, equation (1.2a) reduces to: 90

91
$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + F_x$$
92 But $\frac{\mu}{\rho} = \nu$ and thus the above equation further reduces to:
93 $\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + F_x$ (1.3a)
94 Also , equation (1.2b) reduces to:
95 $0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} + F_y$ (1.3b)
96 From Bernoulli's equations, we have
97 $P + \frac{1}{2}\rho u^2 = \text{constant}$ (1.4)
98 The curved surfaces provides both adverse and favorable pressure gradients whose tangential
99 components of the velocity of the outer flow reveals a power law dependence on the stream wise
100 x measured along the curved surface boundary as;
101 $\frac{u}{c} = x^m$ (1.5)
102 Differentiating partially equation (1.4) with respect to x, we obtain
103 $\frac{\partial p}{\partial x} + \rho u \frac{\partial u}{\partial x} = 0$ (1.6)
104 Which implied that:
105 $-\frac{1}{\rho} \frac{\partial p}{\partial x} = u \frac{\partial u}{\partial x}$ (1.7)
106 But from the power law dependence,
107 $u \frac{\partial u}{\partial x} = \text{me}^2 x^{2m + 1}$ (1.8)
108 Hence equation (1.3a) reduces to;

109
$$\frac{\partial u}{\partial t} = P_t + v \frac{\partial^2 u}{\partial y^2} + F_x \text{ where } P_t = m c^2 x^{2m-1}$$
(1.9)

- 111 Prandtl proposed to account for curvature effect by multiplying the length of the curvature by
- 112 factor f given by: $f = -\frac{1}{4} \frac{k_r u}{\left(\frac{\partial u}{\partial y}\right)} + 1$ (2.0)
- 113 He also deduced that the boundary layer equation on the curved surface is written as ;
- 114 $\rho k_r u^2 = h_1 \frac{\partial P}{\partial y}$, which is re-written as
- 115 $\frac{1}{\rho}\frac{\partial P}{\partial y} = \frac{k_r u^2}{h_1}$ (2.1)
- 116 Where k_r and h_l are curvature parameters which are defined as

117 Kr =
$$-\frac{1}{c(x)}$$

- 118 $h_1 = 1 + k_r y$
- 119 Where c(x) is the radius of the curvature.
- 120 Body forces, F_x and F_y due to the gravitational pull are assumed to be a constant in both cases
- 121 and thus the assumption:
- $F_x = F_y \tag{2.2}$
- 123 Hence the generalized equation of conservation of momentum for fluid flow over an immersed
- 124 curved surface is derived as;
- 125 $\frac{\partial u}{\partial t} = P_t + v \frac{\partial^2 u}{\partial y^2} + \frac{k_r u^2}{h_1}$
- 126 Since $h_1 = 1 + k_r y$, the term $\frac{k_r u^2}{h_1}$ is written in Taylor series as
- 127 $k_r u^2 (1 + k_r y)^{-1} = k_r u^2 (1 k_r y + k_r^2 y +)$
- 128 And therefore, equation (2.3) yields

(2.3)

129
$$\frac{\partial u}{\partial t} = \mathbf{P}_{t} + v \frac{\partial^{2} u}{\partial y^{2}} + k_{r} u^{2} \left(1 - k_{r} y + k_{r}^{2} y + \dots\right)$$
(2.4)

130 The flow is along the x- axis. This implies that $y \approx 0$ and for every small value of k_r we have (1-131 $k_r y + k_r^2 y +) = 1$. Consequently, equation (2.4) reduces to

132
$$\frac{\partial u}{\partial t} = P_t + v \frac{\partial^2 u}{\partial y^2} + k_r u^2$$
 (2.5)

133 This is our momentum equation in consinderation

134 **4.3 The Energy equation**

135 The general equation is given as

136
$$\rho C_p \frac{Dh}{Dt} = K \nabla^2 T + \mu \emptyset, \qquad (2.6)$$

137 Where

138
$$\varphi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2$$
 (2.7)

139 By considering unsteady incompressible flow in a control volume, the standard thermal energy

140 equation for the thermal boundary layer is given by

141
$$\rho v \frac{\partial h}{\partial y} + \rho \frac{\partial h}{\partial t} + \rho u \frac{\partial h}{\partial x} = (\mu \phi + q) + \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \left(u \frac{\partial P}{\partial x} + v \frac{\partial P}{\partial y} \right)$$
(2.8)

- 142 where h was the enthalpy and q was the rate of heat dissipation.
- 143 Now the enthalpy h is given by:

144
$$h = E + P\left(\frac{1}{\rho}\right) \tag{2.9}$$

- 145 then, the first order derivative of enthalpy becomes
- 146 $dh = dE + \left(\frac{1}{\rho}\right) dP + pd\left(\frac{1}{\rho}\right)$ (3.0)
- 147 But $dQ = dE + dW = dE + pd\left(\frac{1}{\rho}\right)$ and for a unit mass and a single species fluid,

148
$$dQ = Tds$$
. Therefore we have
149 $dE = Tds - pd(\frac{1}{p})$ (3.1)
150 In view of (3.1), equation (3.0) yields:
151 $dh = Tds + (\frac{1}{p}) dP + pd(\frac{1}{p}) - pd(\frac{1}{p})$ (3.2)
152 hence
153 $dh = Tds + (\frac{1}{p}) dP$ (3.3)
154 Assuming that $u\frac{\partial P}{\partial x}$ and $v\frac{\partial P}{\partial y}$ were negligible and $dh = CpdT$, equation (2.8) reduces to
155 $C_p \rho \frac{\partial T}{\partial t} + C_p \rho (u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}) = k \frac{\partial^2 T}{\partial y^2} + \mu (\frac{\partial u}{\partial y})^2 + q$ (3.4)
156 Now, the convection equation is expressed as:
157 $q = KAdT$ (3.5)

where $dT = (T_{\infty} - T_s)$ is the difference in temperature between the body surface and the bulk fluid. A is the area of the surface.

160 In this case, the area of the surface was the length of the curved surface and for this concave 161 surface which had a destabilizing effect, the effect of the curved surface was taken into account 162 by multiplying the area, A by a dimensionless factor earlier defined. This resulted to:

$$163 q = AfK dT (3.6)$$

- 164 Where q is the heat transferred per unit time.
- 165 On replacing f, equation (3.6) reduces to

166
$$q = k \left(1 - \frac{1}{4} \frac{k_r u}{\left(\frac{\partial u}{\partial y}\right)} \right) A(T_{\infty} - T_s)$$
(3.7)

167 From Newton's law of cooling, the local heat flux is given by

168
$$q_s^{"} = h(T_{\infty} - T_s)$$
 (3.8)

169 Where h is the local convection coefficient.

170 Since the flow conditions varied from one point to another on the curved surface, both $q_s^{"}$ and h 171 also varied along the curved surface.

For any particular distance x from the edge of the curved surface, $q_s^{"}$ was found by applying the Fourier's Law to the fluid. This was done at y = 0 and was given as:

174
$$-q_s'' = k \frac{\partial T}{\partial y}$$
, which can be re-written as:

175
$$q_{s}^{\prime\prime} = -k \frac{\partial T}{\partial y}$$
(3.9)

176 The local convection heat transfer is expressed as

178 At the thermal boundary layer, the rate of heat conduction along the y- direction was larger than 179 that along the x- axis i.e $\frac{\partial T}{\partial y} >> \frac{\partial T}{\partial x}$

180 Then thwe have:

181
$$C_p \rho \left(u \; \frac{\partial T}{\partial x} + v \; \frac{\partial T}{\partial y} \right) + C_p \rho \; \frac{\partial T}{\partial t} = q + k \; \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \tag{4.1}$$

182 From the above approximations, equation (4.1) reduces to

183
$$C_p \rho \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2 + q$$
(4.2)

But the value of q is replaced with equation (3.7) in order to take care of the curvature effects and hence on substituting equation (3.7) in equation (4.2) we have:

186
$$C_p \rho \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2 + k \left(1 - \frac{1}{4} \frac{k_r u}{\left(\frac{\partial u}{\partial y}\right)}\right) A(T_{\infty} - T_s)$$
(4.3)

Equation (4.3) gives the equation of energy for convective heat transfer over an immersed curved
surface.

189 **5 NON-DIMENSIONALIZING THE EQUATIONS GOVERNING THE FLOW**

In our research work, we let L, V, P and T to be the characteristic length, velocity, pressure and
temperature respectively. The following transformations were used to reduce our equations in a
dimensionless form;

193
$$\frac{x}{x^*} = L, \quad \frac{y}{y^*} = L, \quad \frac{u}{u^*} = V, \quad \frac{v}{v^*} = V, \quad \frac{p}{p^*} = P, \quad T^*(T_{\infty} - T_S)^{-1} = T - T_S$$

194 $t^*L = tV \text{ or } t = \frac{t^*V}{L}$

195 **5.1 Equation of Continuity**

196 For this particular fluid flow, the equation of continuity is given by

197
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$
 (4.4)

198 On non-dimensionalizing, the equation of continuity becomes

199
$$\frac{\partial(u^*V)}{\partial(x^*L)} + \frac{\partial(v^*V)}{\partial(y^*L)} = 0$$
(4.5)

200

201 Or
$$\frac{V}{L}\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*}\right) = 0$$
 (4.6)

202

203 Or
$$\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*}\right) = 0$$
 (4.7)

204 5.2 TheMomentum Equation

205 The equation of conservation of momentum for this flow problem is given by

206
$$\frac{\partial u}{\partial t} = \mathbf{P}_{t} + v \frac{\partial^{2} u}{\partial y^{2}} + k_{r} u^{2}$$
 (4.8)

207 On non-dimensionalizing, the equation becomes:

208
$$\frac{\partial (u^*V)}{\partial (\frac{t^*L}{V})} = \mathbf{P}^* \mathbf{P}_t + v \frac{\partial^2 (u^*V)}{\partial (y^*L)^2} + k_r (u^*V)^2$$
(4.9)

209 On re-arrangement, the equation becomes

210
$$\frac{V^2}{L}\frac{\partial u^*}{\partial t^*} = \operatorname{PP}_t^* + \frac{vV}{L^2}\frac{\partial^2 u^*}{\partial y^{*2}} + k_r V^2 u^{*2}$$
(5.0)

211 Multiplying both sides by $\frac{L}{V^2}$ we have

212
$$\frac{\partial u^*}{\partial t^*} = \frac{PL}{V^2} P_t^* + \frac{v}{LV} \frac{\partial^2 u^*}{\partial {y^*}^2} + k_r L u^{*2}$$
 (5.1)

213 This is the equation of momentum in non-dimensional form

214 **5.3 TheEnergy Equation**

The equation of conservation of energy is given by

216
$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{c_p \rho} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{c_p \rho} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{kA}{c_p \rho} (T_{\infty} - T_s) \left(1 - \frac{1}{4} \frac{k_r u}{(\frac{\partial u}{\partial y})}\right)$$
(5.2)

217 From the boundary approximations the above equation reduces to

218
$$\frac{\partial T}{\partial t} = \frac{k}{c_p \rho} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{c_p \rho} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{kA}{c_p \rho} (T_{\infty} - T_s) \left(1 - \frac{1}{4} \frac{k_r u}{(\frac{\partial u}{\partial y})}\right)$$
(5.3)

- 219 From the non-dimensional form of T, we have:
- 220 $T^* = \frac{T T_s}{(T_{\infty} T_s)}$, which on making T the subject of the formulae yields
- 221 $T = T^*(T_{\infty} T_s) + T_s$ and thus the equation of energy becomes

$$\frac{\partial \left[T^*(T_{\infty} - T_S) + T_S\right]}{\partial \left(\frac{t^*L}{V}\right)} = \frac{k}{c_p \rho} \frac{\partial^2 \left[T^*(T_{\infty} - T_S) + T_S\right]}{\partial (y^*L)^2} + \frac{\mu}{c_p \rho} \left(\frac{\partial (u^*V)}{\partial (y^*L)}\right)^2 + \frac{kA}{c_p \rho} \left(T_{\infty} - T_S\right) \left(1 - \frac{1}{4} \frac{k_r (u^*V)}{\left(\frac{\partial (u^*V)}{\partial (y^*L)}\right)}\right)$$

$$223 \tag{5.4}$$

224 On further simplification, the above equation yields

$$225 \qquad \frac{V(T_{\infty} - T_{S})}{L} \frac{\partial T^{*}}{\partial t^{*}} = \frac{k}{c_{p}\rho} \frac{(T_{\infty} - T_{S})}{L^{2}} \frac{\partial^{2} T^{*}}{\partial y^{*2}} + \frac{\mu V^{2}}{c_{p}\rho L^{2}} \left(\frac{\partial u^{*}}{\partial y^{*}}\right)^{2} + \frac{kA}{c_{p}\rho} (T_{\infty} - T_{S}) \left(1 - \frac{1}{4} \frac{k_{r} u^{*}L}{\left(\frac{\partial u^{*}}{\partial y^{*}}\right)}\right)$$
(5.5)

226 Diving all through by the term $\frac{V(T_{\infty} - T_s)}{L}$, we obtain

$$227 \qquad \frac{\partial T^*}{\partial t^*} = \frac{k}{c_p \rho L V} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu V}{c_p \rho L (T_{\infty} - T_s)} \left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{k L A}{c_p \rho V} \left(1 - \frac{1}{4} \frac{k_r u^* L}{\left(\frac{\partial u^*}{\partial y^*}\right)}\right)$$
(5.6)

228 Multiplying the term $\frac{\mu V}{C_p \rho L(T_{\infty} - T_s)} \left(\frac{\partial u^*}{\partial y^*}\right)^2$ by V in the numerator and the denominator, we obtain

229
$$\frac{\partial T^*}{\partial t^*} = \frac{k}{c_p \rho L V} \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\mu V V}{c_p \rho L V (T_\infty - T_S)} \left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{k L A}{c_p \rho V} \left(1 - \frac{1}{4} \frac{k_r u^* L}{\left(\frac{\partial u^*}{\partial y^*}\right)}\right)$$
(5.7)

230 The equation (5.7) represent the equation of conservation of energy in non-dimensional form

231 **6 VALIDATION OF THE FLOW MODEL**

- In order to validate our flow model, we assume the initial boundary conditions as per each of the
- 233 equations as in below;
- 234 **6.1 Equation of momentum**
- 235 Our momentum equation is solved subject to the following boundary and initial conditions in
- 236 non-dimensional form
- 237 $u^*(t^*, 0) = 0$
- 238 $u^{*}(t^{*},\infty) = 1$
- 239 $u^*(0, y^*) = y$
- 240 **6.2 Energy equation**
- 241 The energy equation is solved subject to the following boundary and initial conditions in the
- 242 non-dimensional form



248 **6 REPRESENTATION OF THE RESULTS**

- 249 We solved our governing equations and obtained the results which were presented graphically
- 250 using MATLAB software as below



From figure 2, when the length of the curvature is increased form L= 0.5 to L= 1.0, the free

stream velocity is accompanied by a considerable increase from 0.275501 to 0.360971 as shown



This is because as the length of the curvature increases, the velocity gradient also increases. Increase in velocity gradient increases the velocity of the fluid flow in considerationi.e. when the length of the curvature is increased, the velocity gradient also increases and a consequence increase in free stream velocity is recorded.

260 More so, when the velocity gradient is increased, the kinetic energy of the fluid particles in

261 motion increases at the boundary layer which implies that the fluid particles are at high velocities

262



264

Figure 3: velocity profiles for Re=1.3, Pe=1, V=1, Kr = 1, Ec = 2, A = 2, Pt = 1

266 **DISCUSSION**

From Figure 3, we note that when the length of the curvature is increased from L = 0.5 to L = 0.5

1.0, the heat dissipation in the boundary layer increases from 0.392678 to 0.572599.

This is because increase in the length of the curvature increases the velocity gradient which leads to increase in shear stresses. The friction between the fluid particles and the surface in consideration is brought about by these shear stresses. In return, this friction force causes the dissipation of heat in the boundary layer. This is due to the fact that the shear stress is directly

- 273 proportional to velocity gradient. i.e $\tau = \mu \frac{\partial u}{\partial y}$ when the velocity gradient is increased, the shear 274 stress increases which brings about friction between the fluid particles leading to increase in heat 275 dissipation.
- 276
- 277





280 **DISCUSSION**

278

From Figure 4, we note that as the Reynolds number increases from 0.7 to 1.3, a direct consequence of the increase in inertia forces occurred leading to increase in velocity from 0.297405 to 0.367155. When the Reynolds number is large, the inertia forces tend to dominated over the viscous force and consequently, the friction of the fluid particles and the surface in consideration is very minimal resulting to increase in velocity of the fluid flow. At large inertia

- forces, the velocity of the fluid is high since low viscous forces implies that little or minimal
- 287 friction exists between the fluid particles and the surface in consideration.
- 288
- 289





292 **DISCUSSION**

From Figure 5, we note that when the Reynolds number is increased from 0.8 to 1.3, the heat dissipation in the boundary layer reduces from 0.613144 to 0.508381.

295 This is because when the value of the Reynolds number is low, the inertia forces are minimal.

296 The viscosity of the fluid thus dominate over the inertia forces and consequently, the friction of

the fluid particles with surface increases resulting to increase in heat dissipation within the

- 298 boundary layer. When Reynolds number is large, the viscous forces are very minimal since
- 299 inertia forces dominate in the fluid flow. Consequently, the friction of the fluid particles with the
- 300 surface is minimal and this results to minimal dissipation of heat within the boundary.

2	n	1
Э	υ	L

- 303
- 304

305 6 CONCLUSION AND RECOMMENDATIONS

Numerical investigations of the convective heat transfer in a laminar boundary layerover an immersed curved surface has been carried out. The variations of the length of the curvature as well as the Reynolds number affect the velocity and temperature profiles in the laminar boundary layer.

310 When the length of the curvature was increased, this led to velocity and temperature rise. This

311 matched the theoretical explanation since increase in velocity gradient increases the velocity of

the fluid flow. Also at high velocity gradients, the shear stresses are high which brings about the

313 friction between the fluid particles and the surface. Consequently, heat is dissipated. It thus

follows that the length of the curvature is directly proportional to the velocity and temperature

315 distribution.

It is also observed that at large Reynolds number, the inertia forces are large compared to the viscous effect of the fluid and consequently, the fluid velocity increases. This is in line with theoretical explanation, since at low viscosity, minimal shear stresses exist between the fluid particles and the surface and thus the velocity of the fluid is favored. At low Reynolds number, the viscosity of the fluid is high since there are minimal inertia forces. Consequently, the fluid velocity goes down. At large Reynolds number, the amount of heat dissipated at the boundary

322 layer is minimal due to minimal friction between the fluid particles and the surface.

323 It therefore follows that Reynolds number is directly proportional to the velocity distribution and

inversely proportional to the temperature distribution in the boundary layer.

325 It is recommended that further investigations be done in the following areas:

326 1. Compressible fluid over immersed surface

- 327 2. Convective heat transfers on turbulent fluid flows over immersed curved surface
- 328 3. Study of the same orientation but in three-dimension

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