Intermittency of regular and chaotic motion in the dynamic system with multiple Lorenz attractors

ABSTRACT: A new type of intermittency observed in an autostochastic dynamic system with a multicomponent chaotic attractor consisting of several Lorenz attractors is considered. It is shown that it is caused by the coexistence of two types of intermittency: "chaos – chaos" and "quasiperiodic motion – chaos". The main statistical characteristics of this movement are also given.

Keywords: *multiattractor, composite multiattractor, multi-component chaotic attractor, intermittency, chaotic motion, quasi-periodic motion, Lorenz attractor.*

1. INTRODUCION

The study of the unpredictable alternation of different types of motion observed in many physical systems is one of the important problems of nonlinear dynamics. This phenomenon is known as intermittency. It is associated with the coexistence of different types of interacting attractors in the dynamic system phase space and manifests itself, in particular, in the form of the intermittency "quasi-regular motion-chaos" [1-4] and "chaos-chaos" [5-8].

The intermittency of "quasi-regular motion-chaos" is relatively well studied concerning discrete maps, in particular, in the contest of scenarios for the origin of stochastic motion processes, where strictly justified results were obtained [9-10]. This phenomenon is not fully investigated in continuous time systems. The dynamic systems with comparatively simple arranged areas of attraction, consisting of two attractors namely one chaotic and one regular [00], were mainly investigated. Outside the attention of researchers remained, in particular, the intermittency "quasiregular movement - chaos" at chaotic multiattractors described, for example, in [11-17] "scroll grid attractors" [11] and on composite (compound) chaotic multiattractors [8,18-25]. The motion on composite chaotic multiattractors, which is one of the most striking examples of the intermittency of “chaos-chaos”, however, may contain regular motion intervals – during transitions of phase trajectories between local chaotic attractors [8, 18-20]. Thus, they may have a new type of intermittency characterised by the coexistence of both types of intermittency: “chaos-chaos” and “quasi-regular movement-chaos”.

As a rule, because of the short duration of the episodes of transition movements from one local attractor to another, the observation of the proper motion in such systems is difficult, resulting in their dynamics appears as a collection of chaotic fluctuations on a local attractors and chaotic hopping of movement from one of them to another. However, in some cases, a significant increase in the transition time is possible, resulting in a new type of intermittency is quite clear.

2. INTERMITTENCY "QUASIREGULAR MOTION - CHAOS" IN THE DYNAMIC SYSTEM WITH MULTIPLE LORENZ ATTRACTORS

For example, consider the following dynamic system with the amounts of the composite chaotic multiattractor consisting of attractors of Lorenz [8]:

$$\begin{cases} \dfrac{dx}{d\tau} = A\left[H\left(\mu x + y\right) - \mu x - x\right]; \\ \dfrac{dy}{d\tau} = x\left(B - z\right) - H\left(\mu x + y\right) + \mu x; \\ \dfrac{dz}{d\tau} = \left[H\left(\mu x + y\right) - \mu x\right]x - Cz\ . \end{cases} \quad (1)$$

Where,

$$H(\xi) = \xi + (d+1)\left\{P\left(\xi + s + h + \frac{h}{d}\right) + P\left(\xi + s - h - \frac{h}{d}\right) - \right. \\ \left. - \sum_{m=0}^{M}\left[P\left(\xi + s - (2m-1)\left(h + \frac{h}{d}\right)\right) + \frac{h}{d}\right] - \sum_{n=0}^{N}\left[P\left(\xi + s + (2n-1)\left(h + \frac{h}{d}\right)\right) - \frac{h}{d}\right]\right\}, \quad (2)$$

$$P(\xi) = \frac{1}{2}\left(\left|\xi + \frac{h}{d}\right| - \left|\xi - \frac{h}{d}\right|\right),$$

– replicates (reduplicate) operator creates copies of the attractor of the original dynamical system, ordered by coordinate $\xi=\mu x+y$, where μ is a real constant, and their merger into a single multiattractor. It represents a nonlinear function consisting of $1+M+N$ line segments of unit slope, connected by more steep intermediate segments with slope $-d$.

The number of local attractors in the multiattractor of system (1), (2) is equal to the number of line sections with a single slope. Each of them is inside its region of phase space (phase cell), with a length of $2h$ in the coordinate ξ. The constant s accounts for the asymmetry of the local attractors relative to the centre of your cell. The coefficient d determines the width of the transition layer the phase space between adjacent cells (equal to $2h/d$) [8].

Let $A=10.5$, $B=33.2189$, $C=3/8$, $M=1$, $N=0$, $h=22$, $d=10$, $s=0$. In this case, the replicate operator is a nonlinear function of the variable ξ containing two line segments with unit slope, connected by an intermediate segment with a slope $-d$ (Fig.1), and the system (1) has the simplest composition multiattractor containing two local chaotic attractors (Fig.2).

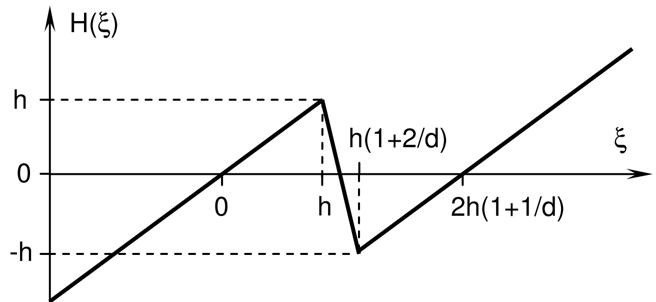

Fig.1

Let us consider the evolution of such multiattractor when we change the value of constants μ. When $\mu < -0.2$, transitions of the phase point between the local attractors occur along short smooth segments (Fig.2, a). In the result, the phases of regular movement look like a fast direct transition of the phase point from one of the local chaotic attractor on the other.

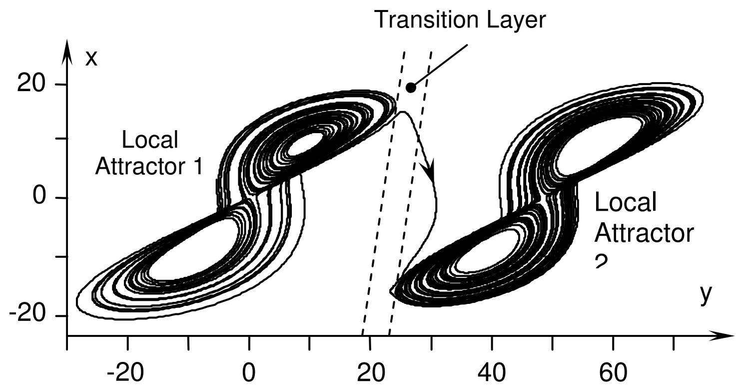

Fig.2,a. Example of the transition movement in the system (1), (2) from local chaotic attractor 1 on the local chaotic attractor 2 when $\mu = -0.2.$

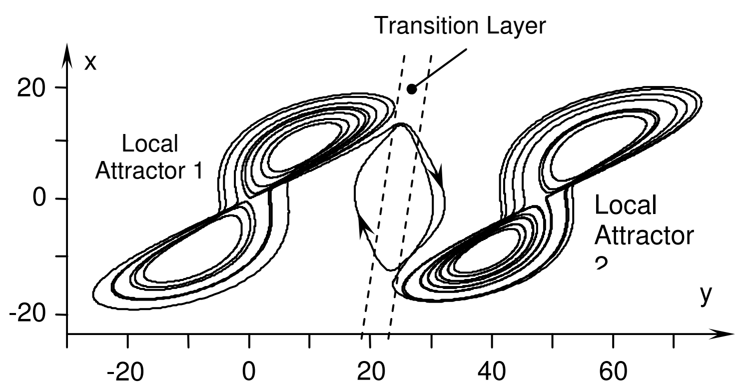

Fig.2,b. Example of the transition movement in the system (1), (2) from local chaotic attractor 1 on the local chaotic attractor 2 when $\mu = -0.15.$

However, if the value of this parameter is increased to -0.15 phase trajectories begin to twist around the unstable cycle, which owes its existence to nonlinearity of the replicate function. First, when $\mu \approx -0.15$, trajectories manage to do a maximum of one turn before it gets into the region of attraction of one of the local attractors and is attracted to it (Fig.2, b).

With the increase of this coefficient the maximum number of turns of the trajectories increases, accordingly, increases the average time of regular motion in the neighbourhood of this cycle. In the timing diagram long sections of quasiperiodic oscillations appear (Fig.3). When $\mu \approx -0.1$ cycle becomes stable. Now the phase trajectory, once finding itself in the region of its attraction cannot leave. That is, the case $\mu \geq -0.1$ corresponds to the global metastability of the system (1), (2). A movement, which begun on any of the local chaotic attractors, through the end time, will always reach a stable cycle corresponding to regular oscillations.

Thus, in the interval of values of the coefficient μ from about -0.15 to -0.1 for the chosen values of the other constants, the system (1) and (2) show a typical example of intermittent dynamics. If the value of μ is close to -0.1 long laminar phases of motion is observed, during which the number of revolutions of the phase trajectory around the unstable cycle can be very large (Fig.3).

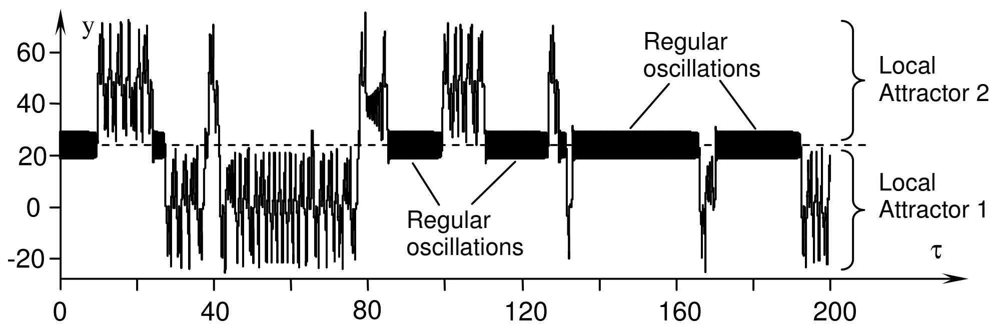

Fig.3, a. An example of time dependencies of dynamic variable y of system (1), (2) when $\mu = -0.109$.

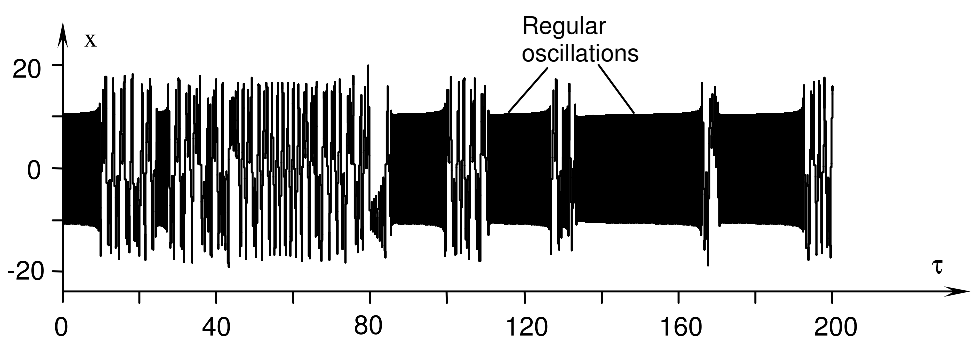

Fig.3, b. An example of time dependencies of dynamic variable x of system (1), (2) when $\mu = -0.109$.

The same behavior of the system (1), (2) is observed in the General case of an arbitrary number of local attractors in the composition of multiattractor [8].

3. STATISTICAL CHARACTERISTICS

Random variables that can be investigated by statistical methods to description of the phenomenon of intermittency in dynamical systems that have multiple chaotic multiattractor are the duration of individual episodes of motion on the chaotic attractors and in the vicinity of the regular attractors, part of multiattractor.

In the present case, the most important are the dependence of the relative total time of the regular movements of the value of the constant μ and frequency distribution of durations of regular and chaotic motions.

The relative total duration of regular motion is equal to $to_{reg} = \lim_{T_\Sigma \to \infty} \frac{\sum_i T_{reg\ i}}{T_\Sigma}$, where T_Σ – total time of observation, $T_{reg\ i}$ – duration of the i-th episode of a regular movement.

The frequency distribution, in this case, represents the relationship "the number of episodes of movement on the selected attractor – the duration of these episodes" for the observation time T_Σ at $T_\Sigma \to \infty$.

The dependence $to_{reg}(\mu)$ for three values of the slope of the intermediate segment of the replicate function ($d=10$, $d=100$, $d=\infty$) is shown in Fig.4. A characteristic feature of this dependence is the existence of the limit of the maximum value of to_{reg} when $d<\infty$. For example, for $d=10$ and $d=100$ the percentage of time consumed on a regular traffic may not exceed approximately *0.55*. In the case of discontinuous replicate function, the upper limit of to_{reg} is equal to *1*.

Note that these dependences are satisfactorily approximated by functions of the form

$$to_{reg} = \frac{\alpha}{(|\mu| - \beta)^{\delta} |\mu|^{\lambda}}, \quad (3)$$

where – $\alpha, \beta, \delta, \lambda$ – are positive constants

For the dependence corresponding to d=10 (Fig.4), these constants have the following values: $\alpha=1.6 \cdot 10^{-8}$, $\beta=0.1005$, $\delta=0.45$, $\lambda=6$. For the dependence corresponding to d=100, these constants have the following values: $\alpha=3 \cdot 10^{-6}$, $\beta=0.0993$, $\delta=0.35$, $\lambda=4$. For the dependence corresponding to $d=\infty$, they are equal $\alpha=1.5 \cdot 10^{-4}$, $\beta=0.09975$, $\delta=0.6$, $\lambda=1.8$.

Frequency distribution of durations of episodes of motion on the chaotic attractors is shown in Fig.5. They show that the duration of motion on the chaotic attractors are concentrated within a limited interval within which appreciable secondary concentration ravnodushie with each other the highs. The values of the maximums are approximately uniformly distributed throughout the interval. The equality of intervals between the peaks is due to the fact that the visit of the phase point of the intersection area of the chaotic attractor with the boundary of its phase cell is mostly quasi-periodic character. Any pronounced dependence of these distributions from μ not observed.

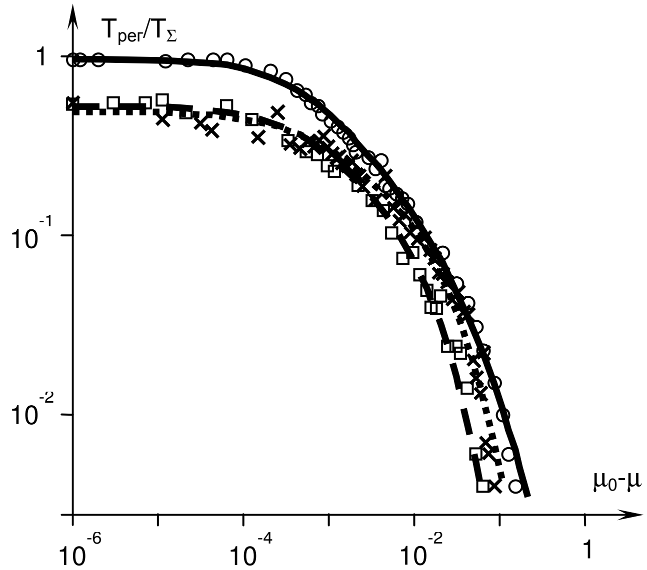

Fig.4. The dependence of the relative total time to the regular movements of the value of the constants μ at *d=10* (- numerical data, dashed line – approximation by function (3)), *d=100* (x - numerical data, small dashed line – approximation by function (3)), *d=∞* (o - numerical data, solid line – approximation by function (3)). μ_0 – limit constant value μ, above which the regular oscillations become stable (for *d=10* $\mu_0 \approx -0.10088$, for *d=100* $\mu_0 \approx -0.09966$, for *d=∞* $\mu_0 \approx -0.1002$).

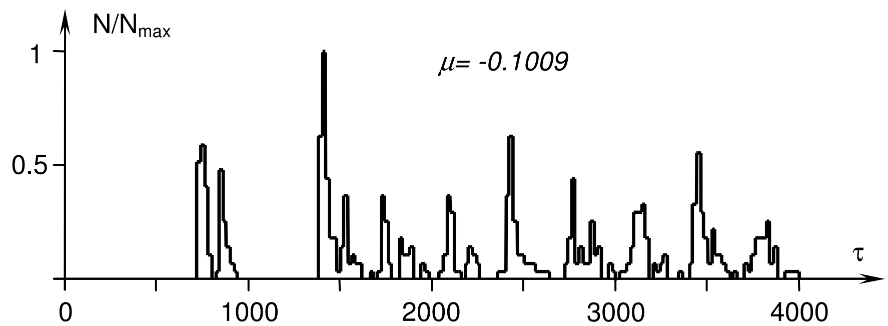

Fig.5, a. Frequency distribution of durations of episodes of movement on the local chaotic attractor 1.

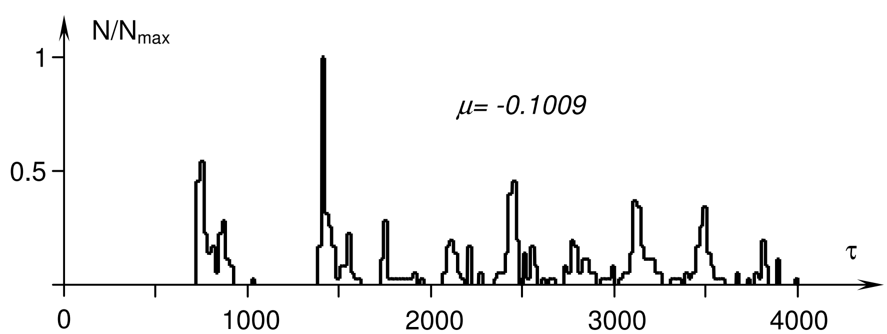

Fig.5, b. Frequency distribution of durations of episodes of movement on the local chaotic attractor 2.

Fig.6 shows the frequency distribution of durations of episodes of regular motion, including at least one rotation of the trajectory around the unstable cycle, with *μ=0.1009, 0.109* and *0.125*, which, according to Fig.4, corresponding to values of relative total duration of regular movement $to_{рег}$ approximately equal to *0.55, 0.1* and *0.03*. It is seen that these distributions have an exponential character. That is, the duration of episodes of regular movement, in general, are concentrated near the minimum value, which is equal to time of one rotation of the phase trajectory around the unstable cycle ($\tau_{turn}\approx 90$). Also, it is seen that the distributions consist of significantly more highly expressed, compared to the distributions in Fig.6, the individual concentrations, separated by equal intervals of τ_{turn} /2, which is a direct consequence of the quasi-periodic nature of the regular movement. (The fact that neighbouring maxima separated by intervals of length exactly τ_{turn} /2, because for every revolution, the trajectory passes through the vicinity of two areas of contact of regular manifolds with chaotic attractors).

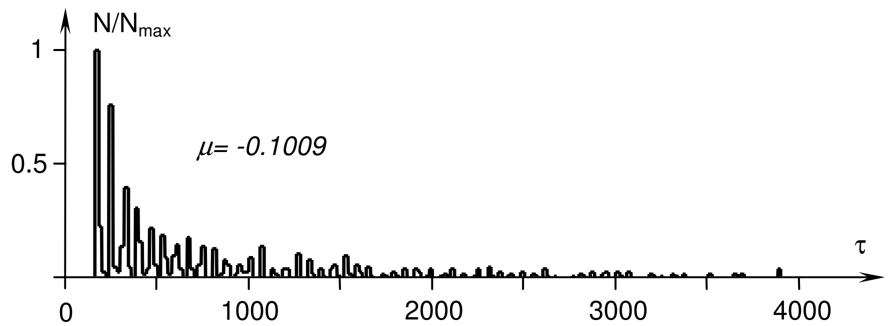

Fig.6, a.

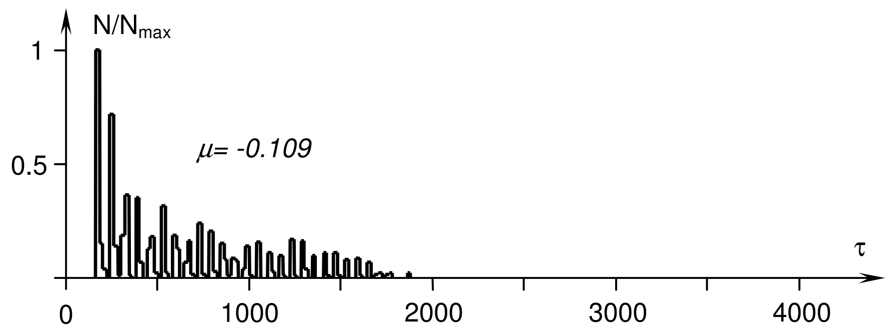

Fig.6, b.

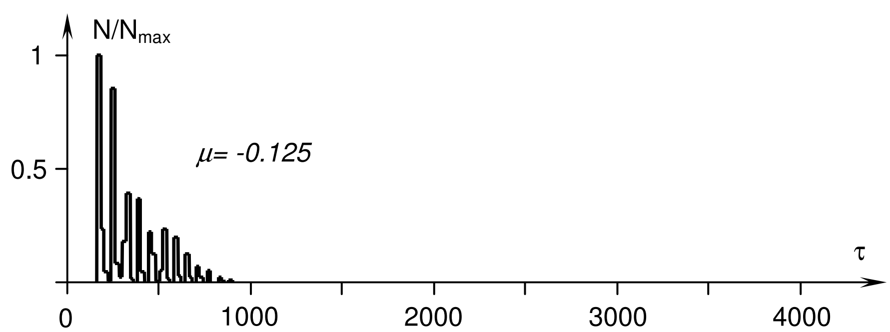

Fig.6, c.

A comparison of these distributions corresponding to different values of the constant μ, shows their strong dependence on to_{reg}. With the reduction in relative overall duration of regular motion, the distribution of the lengths of its intervals is substantially compressed by the ordinate. From Fig.6 it can be seen that when μ changes from *-0.1009* to *-0.125* (in this case to_{reg} is reduced from *0.55* to *0.03* – see Fig.5) maximum observed length of intervals of regular motion is reduced four times – i.e. from *4000* to *1000*.

3. THE MECHANISM OF INTERMITTENCY

The reason for the alternation between chaotic and laminar phases of the movement in the system (1), (2) is the coexistence of interacting attractors of two types (i.e. chaotic and regular) that are in a metastable state, and having such a mutual position that the phase trajectory, leaving the attractor of the same type always appears in the region of attraction of the attractor of another type.

Metastability of regular motion due to instability of the corresponding limit cycle: Metastability of the local chaotic attractors induced by the choice of size of the containing cell of the phase space, so that each of them had crossed the boundaries of its cell, causing the phase trajectory gets the opportunity to leave a local attractor through the area of its intersection with the border of the cell [8,18-20].

Therefore, the mechanism for intermittent oscillations in dynamic systems that have composite chaotic multiattractors, can be described as follows.

For example, the initial conditions are chosen in the domain of attraction of one of the local chaotic attractors. Then, the phase point coming on this attractor will have some time to make chaotic motion on it, until it leaves it through the intersection with the boundary of the phase cell. Getting off a chaotic attractor it gets into the region of attraction of the unstable limit cycle and starts a quasi-periodic motion in its surroundings. Because of the instability cycle, the magnitude of the momentum of the phase trajectory around it over time begins to grow (Fig.3,b) with simultaneous displacement of the region of rotation of the phase trajectories at the unstable manifold – until the phase trajectory crosses the border of the region of attraction of one of the local attractors and be attracted to it. Further, the movement continues on a chaotic attractor, while the phase trajectory will go beyond the boundaries of the containing its cell of the phase space and does into the region of attraction of the cycle, and again started to make momentum around it. The result is a typical pattern of intermittency "quasi-periodic motion – chaos" (Fig.3).

4. CONCLUSION

Thus, in a homogeneous multiattractor system based on the Lorenz attractors, it is possible to observe a new type of intermittency, characterized by the coexistence of two types of intermittency – intermittency of “chaos – chaos” and intermittency “quasiregular movement – chaos.” The manifestation and nature of intermittency “quasiregular traffic – chaos” are controlled by way of the introduction of the replicate operator in the Lorenz equations. That is, a set of those variables (replication variables [19]) relative to which it is set.

From the conducted consideration it is seen that depending on the choice of the replication variable (in the case under consideration, the modification of this variable is carried out by changing the coefficient μ), the alternation of chaotic and quasi-regular behaviour of the system can be very clearly manifested. Therefore, the dynamic systems of the considered type can serve as a very convenient model for demonstrating and more detailed study of such, in many ways still mysterious phenomenon of dynamics as intermittency.

In the context of the material of this article, it is advisable to further investigate, for example, the dependence of the properties of the phenomenon under consideration on the regime of chaotic oscillations on local chaotic attractors, on the parameters of the replicating function, as well as on the modification of the replication variable within a wider range.

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