Original Research Article

Realization and Implementation of Polynomial Chaotic Sun System

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ABSTRACT

In this study, the circuit realization and its corresponding implementation by means of analog components is presented for a new chaotic system. In particular, the polynomial chaotic Sun system, which has 12 terms, twelve parameters and six nonlinearities, is considered. A relation for converting the chaotic ODE system parameters into circuit parameters is provided. The circuit realization of such system is simulated by PSPice-A/D. Next, the circuit is implemented by means of analog electronic components such as operational amplifiers and multipliers. The signals measured from experiments agreed with numerical simulations.

Keywords: Chaotic systems, simulation, circuit model, analog circuit implementation.

1. INTRODUCTION

Chaos is a phenomenon that could be modeled with nonlinear systems of equations [1]. A nonlinear, aperiodic and continuous-time system is said to be chaotic if it exhibits sensitive dependence on initial conditions; this behavior makes it practically impossible to predict a future state of the system given that we cannot with pinpoint accuracy ascertain the initial states [2]. A positive Lyapunov exponent is also an indication that a nonlinear continuous system is chaotic. Resurgence of interests in chaotic systems started after an MIT professor, E. Lorenz, in 1963, applied it to weather forecasting [3]. With chaos stabilization and synchronization [4], engineering applications have soared. More recently, chaotic systems have been used in private communications [5, 6]. Chaotic systems have also found applications in many other areas such as ecology [7, 8], robotics [9], lasers [10, 11], neural networks [12, 13], chemical reaction [14], cryptosystem [15], finance and economy [16, 17], medicine and biology [18, 19], and so on. Polynomial chaotic Sun system [20] is one example of very many such systems. The present system on discourse can find applications in chaotic scenarios where many quadratic interactions including self-interactions are present.

Several electronic circuits with chaotic responses have been designed and realized [21, 22, 23, 24, 5], however the polynomial chaotic Sun system is of interest for a couple of reasons including the fact that it has twelve parameters and six nonlinearities. These many parameters and nonlinearities compared to contemporary chaotic systems imply the system holds potential

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to accommodate real or more complex quadratic interactions. For instance, in applying chaos to communication, more parameter degrees of freedom can be exploited as extra layers of security although that would fundamentally require more resources; more so, in economics or in chemical reactions for instance, many nonlinearities would allow for representing more interactions amongst competing variables. Hence, the polynomial chaotic Sun system deserves investigation at different levels.

Engineering applications of chaotic systems often involve design and hardware implementation, hence the motivation for the present study. In this report, we present an electronic circuit model and implementation of the novel polynomial chaotic Sun system. To the best of our knowledge, circuit realization and or implementation of this system has never been reported.

2. METHODOLOGY

The polynomial chaotic Sun system is stated as:

$$\begin{aligned} \dot{x}_1 &= p_1 x_1 + p_2 x_2 + p_3 x_2 x_3 + p_4 {x_2}^2 \\ \dot{x}_2 &= p_5 x_1 + p_6 x_2 + p_7 x_3 + p_8 x_1 x_2 + p_9 {x_2}^2 \\ \dot{x}_3 &= p_{10} x_3 + p_{11} x_1 x_2 + p_{12} {x_2}^2, \end{aligned} \tag{1}$$

where the parameter values are $p_1 = -2, p_2 = 10, p_3 = -1, p_4 = 2, p_5 = 18, p_6 = -8, p_7 = 8, p_8 = -1, p_9 = -2, p_{10} = -2, p_{11} = 2, p_{12} = 1$. x_k $(k = 1,2,3)$, are time dependent state variables. A dot on a state variable implies derivative with respect to time, for example $\dot{x}_k = {dx_k}/{dt}$.

The amplitudes and scales of the state variables differ markedly and are also too large for a DC supply that integrated circuits (IC) can handle [22]; to take care of these issues, we make the transformations given in eq.(2). The transformation was necessary because *PSPice-A/D* simulation would not give a response that agreed with *Matlab* for a voltage supply within safe range for electronic components like the integrated circuits. Also, running the *Matlab* *max* and *min* functions on the numerical data stored in x_1, x_2 and x_3 reveals the wide differences between these variables.

$$x_1 = 6x; \quad x_2 = 4y; \quad x_3 = 11z\,. \tag{2}$$

Now, we obtain a more physically viable polynomial chaotic Sun system which reads:

$$\begin{aligned} \dot{x} &= p_1 x + \frac{2}{3} p_2 y + \frac{22}{3} p_3 yz + \frac{8}{3} p_4 y^2 \\ \dot{y} &= \frac{3}{2} p_5 x + p_6 y + \frac{11}{4} p_7 z + 6 p_8 xy + 4 p_9 y^2 \\ \dot{z} &= p_{10} z + \frac{24}{11} p_{11} xy + \frac{16}{11} p_{12} y^2\,. \end{aligned} \tag{3}$$

Numerical data from the solutions of eq.(3) is plotted in Fig 1 and the dynamics agree with the chaotic Sun system as originally reported in Ref.[20]

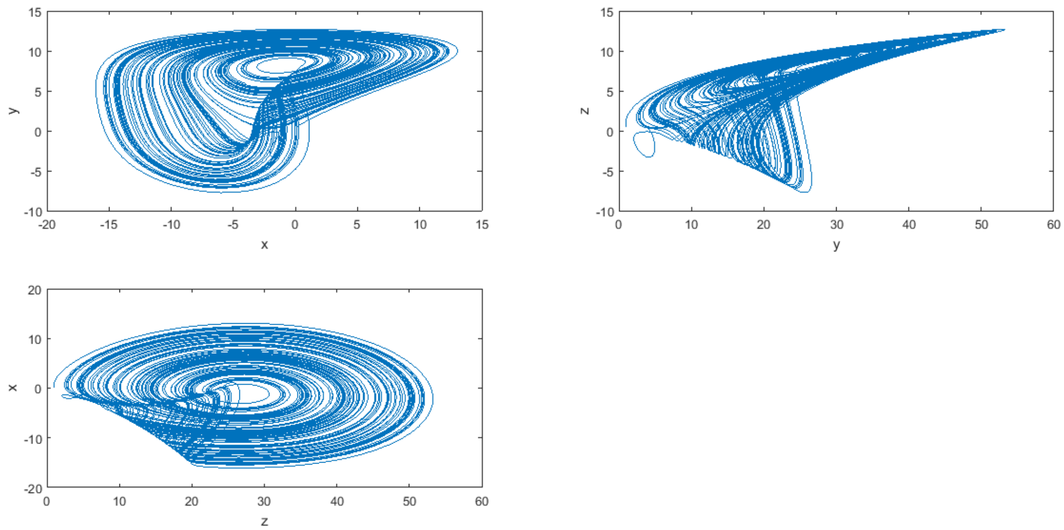


Fig 1. Phase portraits from numerical solutions of the scaled polynomial chaotic Sun system. *Matlab ode45* with automatic time step and initial conditions $x(0) = 0.1, y(0) = 0.5, z(0) = 1.0$ were used.

Now, let v_x, v_y and v_z be physical signals (e.g. electric voltages) corresponding to the mathematical objects x, y and z respectively. Then the electronic circuit model corresponding to eq.(3) is given as follows.

$$
\begin{aligned}
\dot{v}_x &= -\frac{1}{R_1C_1}v_x + \frac{1}{R_2C_1}\frac{r_1}{r_2}v_y - \frac{1}{R_3C_1}v_yv_z + \frac{1}{R_4C_1}{v_y}^2 \\
\dot{v}_y &= \frac{1}{R_5C_2}\frac{r_3}{r_4}v_x - \frac{1}{R_6C_2}v_y + \frac{1}{R_7C_2}\frac{r_5}{r_6}v_z - \frac{1}{R_8C_2}v_xv_y - \frac{1}{R_9C_2}{v_y}^2 \\
\dot{v}_z &= -\frac{1}{R_{10}C_3}v_z + \frac{1}{R_{11}C_3}v_xv_y + \frac{1}{R_{12}C_3}{v_y}^2,
\end{aligned} \tag{4}
$$

where R_i, $(i = 1,2, \ldots ,12)$ and r_j, $(j = 1,2, \ldots 6)$ are electronic resistances to be computed and selected respectively and C_k, $(k = 1,2,3)$ are capacitances of capacitors. A circuit realization for the model is displayed in Fig. 2 and choosing a scale of 2500, the formula given in eq.(5) provides the relationship between parameters of the chaotic Sun system and the electronic circuit parameters. Indeed the formula applies to any such system of ordinary differential equations for which one wishes to convert the equation parameters to electronic parameters for any desired scale.

$$R_i = \frac{1/C_k}{s\times|p_i|\times2500} \times \frac{1}{10^{n-1}}, \tag{5}$$

where for a reasonably chosen circuit capacitance, $C_k (= 1nF$ in our case), R_i is a circuit resistance corresponding to system parameter p_i, s is a scalar from the transformation and n is the (polynomial) power of the i^{th} term; for example, looking at eq.(1) and eq.(3), it is clear that for $p_4 = 2$, and $s = 8/3$, the power of the corresponding variable y, is $n = 2$ and the corresponding circuit resistance R_4 is thus calculated as:

$$R_4 = \frac{1/_{10^{-9}}}{8/3 \times 2 \times 2500} \times \frac{1}{10} = 7.5k\,;$$

other R_i are computed in a similar fashion using eq.(5). Each r_i is chosen to be $100k\Omega$ as indicated in the circuit schematics; they only serve to provide the appropriate gains after signal inversion[1].

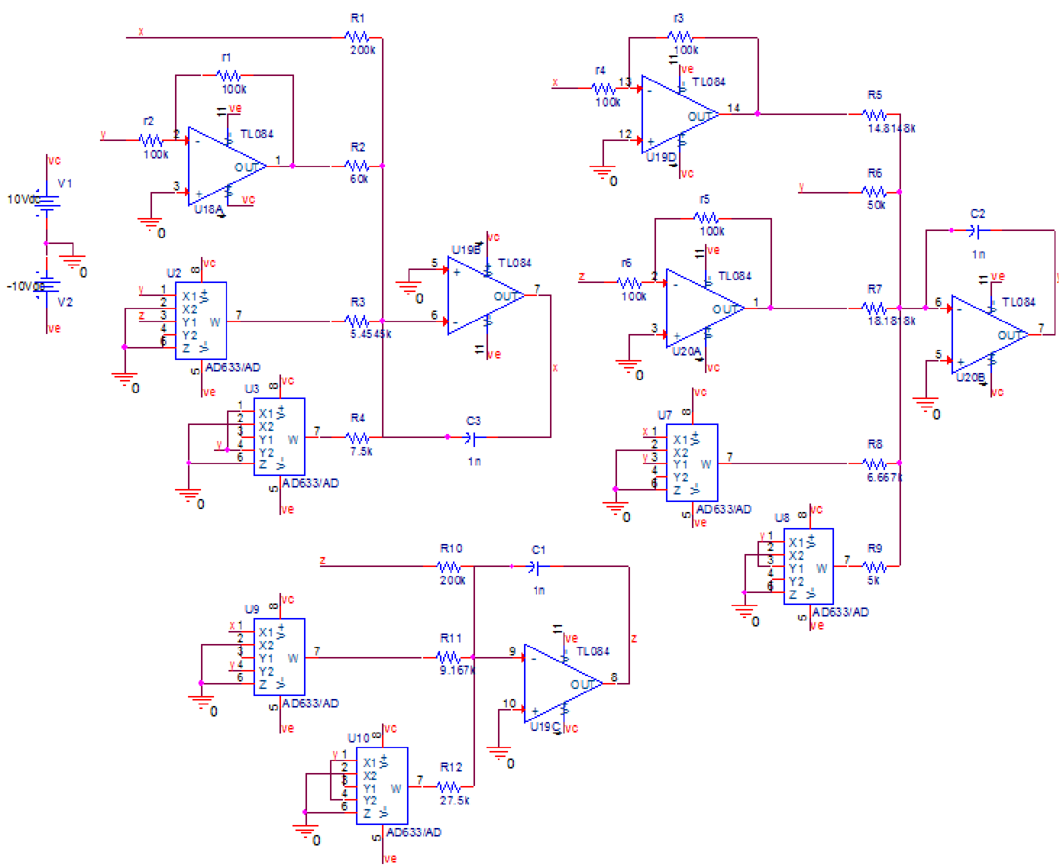


Fig 2. Circuit realization of the polynomial chaotic Sun system. *OrCAD-Capture* was used to layout the schematic.

[1] *Netlist* by *PSpice-A/D* makes no distinction between R_i and r_i. We have adopted the labels on the schematics after simulation for convenience because r_i are not derivable from eq.(5) but chosen to give the appropriate gain.

3. SCHEMATIC MODEL SIMULATION

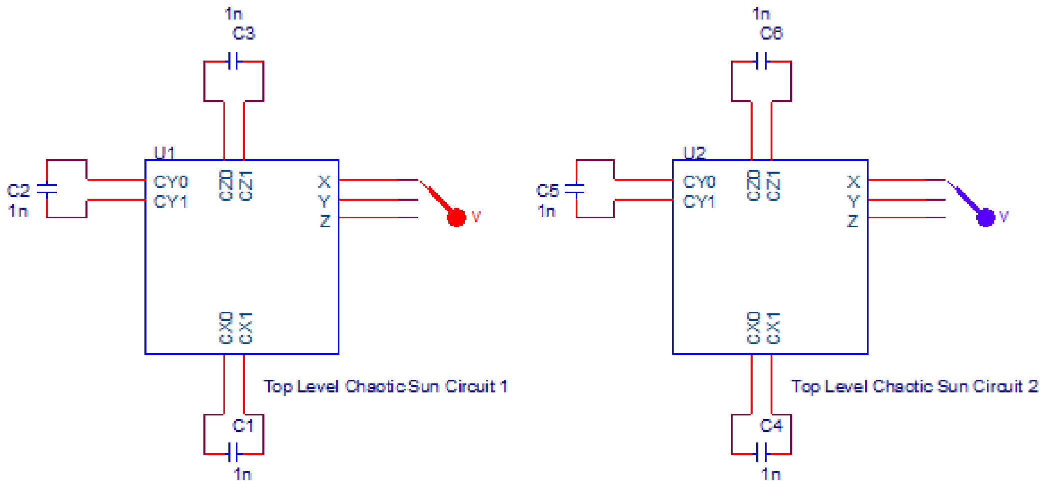
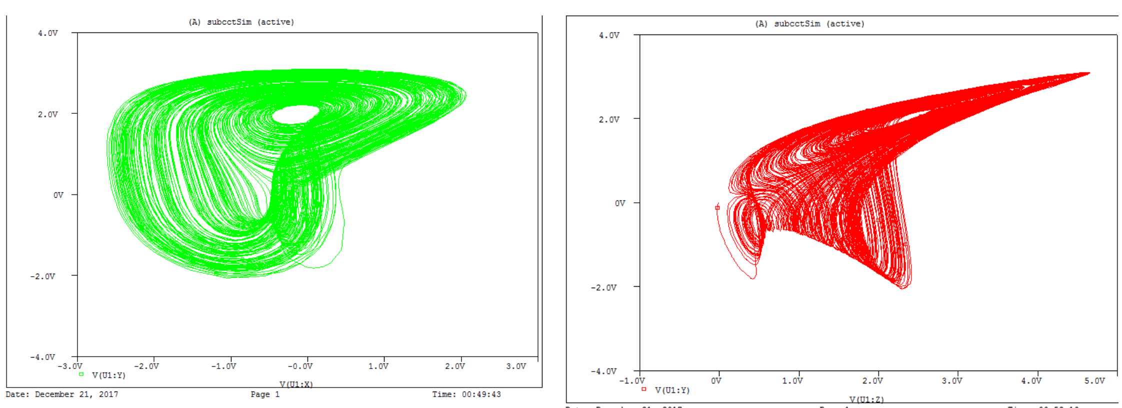


Fig 3. Each of the toplevel parts contain the chaotic Sun system as given in Fig. 2, with the capacitors ported to the outside. The initial stored voltage in each can be varied slightly in order to observe for the circuit, *sensitive dependence*.

We have designed *PSPice* top-level part as shown in Fig. 3; each contains the circuit design of Fig. 2 except that terminals for the capacitors are mapped out to the top-level; initial stored voltages of the capacitors of a part were set at $(0.01, 0.00, 0.01)$ *volts* and in the other part were set at $(0.02, 0.01, 0.02)$ *volts*. The simulation results are given in Fig. 4 (*Middle-Right*, *Bottom-Left*, *Bottom-Right*) and they show sensitive dependence on initial stored voltages. Also in Fig. 4 are phase portraits from *PSPice-A/D* simulations, which are in concordance with phase portraits from *Matlab* solutions as shown in Fig. 1.

We present in the following section hardware implementation of the circuit realization.



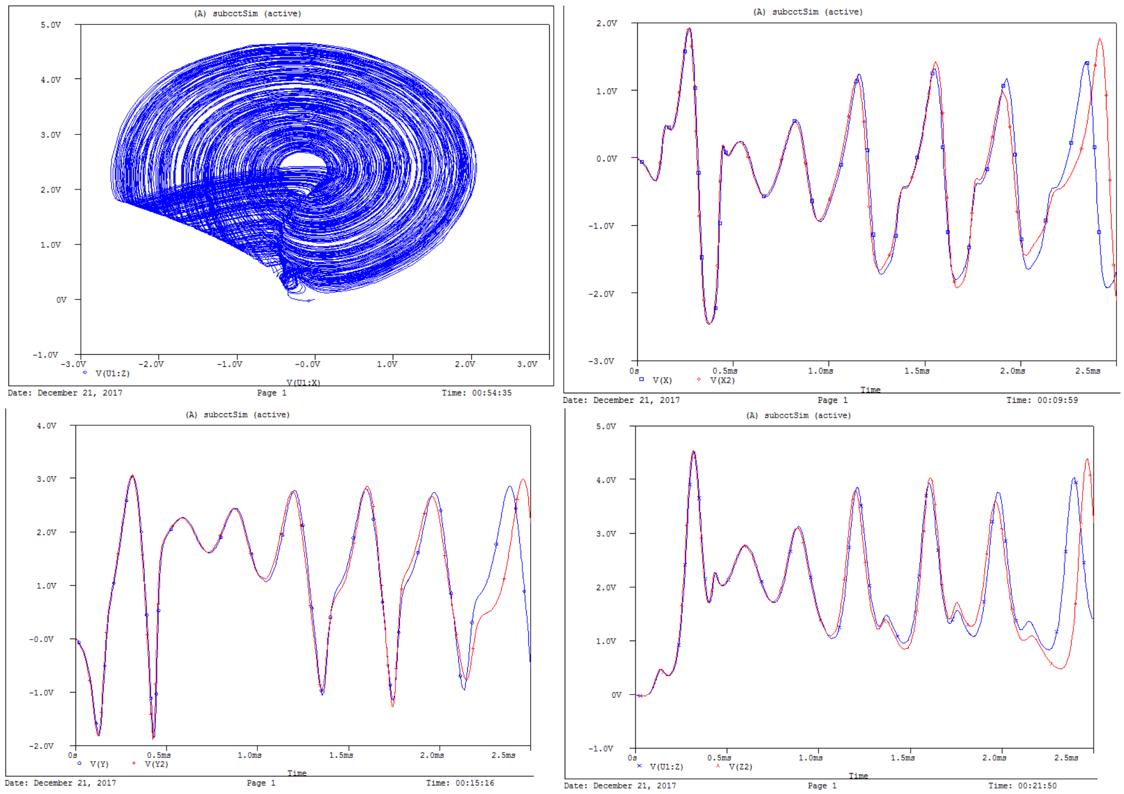


Fig 4. *PSPice-A/D* simulation reuslts. ***Top-Left***: yx-portrait. Top-Right: yz-portrait. ***Middle-Left***: zx–portrait. ***Middle-Right***: Time series plots of x & x2, ***Bottom-Left::*** Time series plots of y & y2, ***Bottom-Right:*** Time series plots of z & z2, with initial stored voltages of capacitors at (0.01, 0.00, 0.01) volts and (0.02, 0.01, 0.01) volts for the two chaotic systems shown in Fig. 3.

4. IMPLEMENTATION

The chaotic Sun system was realized in electronic circuit using AD633JN for multiplication, TL084CN, containing four operational amplifiers (op-amps), for signal inversion, multi-turn trimpots were used to meticulously tune the circuit parameters and three capacitors each of $1nF$ together with op-amps implemented the mathematical integration and the outputs on the oscilloscope are displayed in Fig. 5.

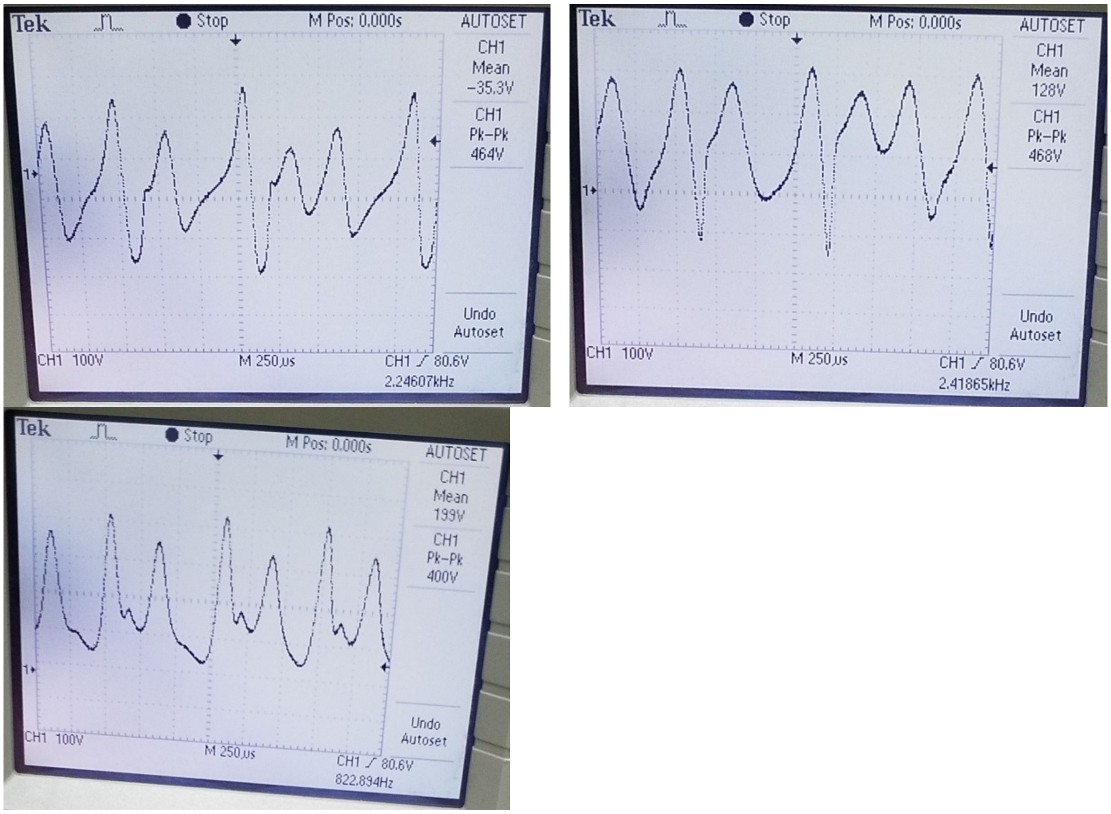


Fig 5. Circuit implementation outputs as seen on the oscilloscope. ***Top-Left***: v_x signal. ***Top-Right***: v_y signal. ***Bottom-Left***: v_z signal.

5. CONCLUSION

The output signals of Fig. 4 agree completely with numerical results from the abstract mathematical model plotted in Fig.1 and with Ref. [20] Also, the electronic circuit implementation gives results that are consistent with the schematic model simulations shown in Fig. 4. Hence the circuit realization effectively represents the polynomial chaotic Sun system. Also, the circuit operation is near room temperature. Furthermore, the circuit implementation of the polynomial chaotic Sun system can find applications in situations where many variables interact in pairs and with themselves; more parameters compared to contemporary chaotic systems give more access points to experiment with the system's properties. Finally, in a forthcoming paper, we are investigating an FPGA[2] realization of the analog circuit presented in this report.

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[2] FPGA – Field Programmable Gate Arrays.

COMPETING INTERESTS

There are no competing interests associated with this research.

AUTHORS' CONTRIBUTIONS

The authors of this report have worked together on every aspect during the research.

REFERENCES

1. Lilian H, Mao W, Rupeng F. Parameters identification and adaptive synchronization of chaotic systems with unknown parameters. *Physics Letters A*. 2005;342(4):299-304.
2. Pérez-Cruz JH, Portilla-Flores EA, Niño-Suárez PA, Aguilar ROV. Design of a nonlinear controller and its intelligent optimization for exponential synchronization of a new chaotic system. *Optik - International Journal for Light and Electron Optics*. 2017;130:201-212.
3. Edward L. Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*. 1963;20:130-141.
4. Pecora LM, Carroll TL. Synchronization in chaotic systems. *Physical Review Letters*. 1990;64(8):821-825.
5. Kevin C, Alan O, Strogats S. Synchronization of Lorenz-based chaotic circuits with applications to communications. *IEEE*. 1993;40(10):626-633.
6. Smaoui N, Karouma A, Zribi M. Secure communications based on the synchronization of the hyperchaotic chen and the unified chaotic systems. *Commun Nonlinear Sci Numer Simulat*. 2011;16:3279–3293.
7. Sprott JC, Vano J. C. Wildenberg JA, Anderson MB, Noel JK. Coexistence and chaos in complex ecologies. *Physics Letters A*. 2005;335:207-212.
8. Sahoo B, Poria S. The chaos and control of a food chain model supplying additional food to top-predator. *Chaos, Solitons & Fractals*. 2104;58:52-64.
9. Volos CK, Kyprianidis IM, Stouboulos IN. A chaotic path planning generator for autonomous mobile robots. *Robotics and Autonomous Systems*. 2012;60:651-656.
10. Yuan G, Zhang X, Z W. Generation and synchronization of feedback-induced chaos in semiconductor ring lasers by injection-locking. *Optik*. 2014;125:1950– 1953.
11. Hu J, Jia K, Ma J. Chaos synchronization and encoding in coupled semiconductor lasers of multiple modulated time delays. *Optik*. 2011;122:2071– 2074.
12. Kaslik E, Sivasundaram S. Nonlinear dynamics and chaos in fractional-order neural networks. *Neural Networks*. 2012`;32:245–256.
13. He G, Shrimali MD, Aihara K. Threshold control of chaotic neural network. *Neural Networks*. 2008;21:114–121.
14. Villegas M, Augustin F, Gilg A, Hmaidi A, Wever U. Application of the polynomial chaos expansion to the simulation of chemical reactors with uncertainties. *Mathematics and Computers in Simulation*. 2012;82:805–817.
15. Rhouma R, Belghith S. Cryptanalysis of a chaos-based cryptosystem on DSP. *Commun Nonlinear Sci Numer Simulat*. 2011;16:876–884.
16. Faggini M, Parziale A. The failure of economic theory. Lessons from chaos theory. *Modern Economy*. 2102;3:1-10.
17. Hayward T, Preston J. Chaos theory, economics and information: the implications for

strategic decision-making. *Journal of Information Science*. 1999;25(3):173-182.

18. Fu C, Zhang G, Bian O, Lei W, Ma H. A novel medical image protection scheme using a 3-dimensional chaotic system. *PLoS ONE 9(12)*. 2014;9(12):1-25.
19. Kyriazis M. Applications of chaos theory to the molecular biology of aging. *Experimental Gerontology*. 1991;26:569-572.
20. Sun C, Zhu B, Xu Q, Ai Y. Stabilization of a polynomial chaotic system based on T-S fuzzy model. *IEEE*. 2016:1071-1075.
21. Yilmaz PIaU. A new 3D chaotic system with golden proportion equilibria: Analysis and electronic circuit realization. *Computers & Electrical Engineering*. 2012:1777 - 1784.
22. Zhi-Yu XLaLYJaZQMaZ. Circuit implementation and antisynchronization of an improved Lorenz chaotic system. *Shock and Vibration*. 2016;2016:1-12.
23. Guanrong YSaLJaC. Theoretical design and circuit implementation of multidirectional multi-torus chaotic attractors. *IEEE Transactions on Circuits and Systems I*. 2007;54(9):2087-2098.
24. Zhou WjaWZpaWMwaZWhaWJf. Dynamics analysis and circuit implementation of a new three-dimensional chaotic system. *Optik-International Journal for Light and Electron Optics*. 2015;126(7):765-768.