Energy Spectrum of the k-State Solutions of the Dirac Equation for Modified Eckart Plus Inverse Square Potential Model in the Presence of Spin and Pseudo-Spin Symmetry Within the Framework of Nikifarov-Uvarovmethod

ABSTRACT

The exact analytical bound state solutions of wave equations are still very interesting problems in fundamental quantummechanics.However, there are only a few potentials for which these wave equations can be exactly solved. In this paper, Spin and pseudospin symmetries of the Dirac equation for Modified Eckart plus Inverse square potential within a zero tensor interaction are investigated using the parametricNikiforov-Uvarov method which is based on the solutions of general second-order linear differential equations with special functions. The bound state eigen value was obtained with some few cases of potential considerations.

Keywords: Modified Eckart plus Inverse Square Potential; Dirac Equation; Spin and Pseudospin Symmetry; Nikiforov-Uvarov Method.

1. INTRODUCTION

The exact solutions of wave equations still remain an interesting problem in fundamental quantummechanics.Unfortunately, there are only few known potentials for which the Schrodinger,Dirac, Klein-Gordon, and Duffin-Kemmer-Petiau (DKP) equations can be exactly solved. Several potential models have been introduced to explore the relativistic and nonrelativistic energy spectra and the corresponding wave functions [1–5]. Jia*et al*. [6] have derived the bound-state solution of the Klein-Gordon equation under unequal scalar and vector kink-like potentials.The solutions of the Dirac equation under pseudospin and spin symmetries with a number of potential models have been investigated by many researchers. These potentials include the Manning-Rosen [7], Eckart [8], Hylleraas [9], Deng-Fang [10], Méobious square [11], Tietz [12], hyperbolical [13], Yukawa and inversely quadratic Yukawa [14, 15] potentials. The spin and pseudospin symmetries under various phenomenological potentials have been investigated using various methods, such as the Nikiforov-Uvarov (NU) method [16], supersymmetric quantum mechanics (SUSYQM) [17], and others [18]. On the other hand, we are now almost sure that the spin and pseudospin symmetries of the Dirac equation play a significant role in nuclear and hadronic spectroscopy[19, 20]. The tensor interaction has attracted a great attention as it removes the degeneracy between the doublets [20]. In most studies, due to the mathematical structure of the problem, the tensor interaction is considered as the Coulomb-like [19, 20] or Cornell interaction. Hassanabadi*et al.* were the first to introduce the Yukawa tensor interaction [21]. The investigation has shown that tensor interaction removes the degeneracy between two states in the pseudospin and spin doublets. The effect of tensor coupling under spin and pseudospin symmetries has been studied only for the Coulomb-like interaction until recently that Hassanabadi et al. [21] introduced the Yukawa tensor interaction. Our research group has recently solved the eigenfunctions of Dirac, Klein-Gordon and Schrodinger using combined or superposed potentials. These includeManning-Rosen plus shifted Deng-fang potential [22],

Manning-Rosen plus Yukawa Potential [23], Generalized Woods-Saxon plus Mie-Type Nuclei Potential [24], with Kratzer plus Reduced Pseudoharmonic Oscillator potential [25] and so on.
In the present study, we obtain the approximate analytical solutions of the Dirac equation for the vector Modified Eckart plus Inverse square potentials under zero tensor interaction within the framework of spin and pseudospin symmetry limits.
This paper therefore, is organized as follows. Section 1 covers the introduction, in section 2, we review the NU method, Section 3 is devoted to the Dirac equation for spin and pseudospin symmetries, Special case of the potential is discussed in Section 4, andfinally, we give a brief conclusion.

2. REVIEW ON NIKIFAROV-UVAROV METHOD

The main equation which is closely associated with the method is given in the following form as proposed by Nikiforov and Uvarov 1988 [16].

$$\psi''(s) + \frac{\tilde{\tau}(s)}{\sigma(s)}\tau'(s) + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\psi(s) = 0 \qquad (1)$$

Where $\sigma(s)$ and $\tilde{\sigma}(s)$ are polynomials at most second-degree, $\tilde{\tau}(s)$ is a first-degree polynomial and $\psi(s)$ is a function of the hypergeometric-type.

In order to find the exact solution to Eq. (2), we set the wave function as

$$\psi(x) = \emptyset(s)\mathcal{X}(s) \qquad (2)$$

and on substituting Eq. (3) into Eq. (2), then Eq. (3) reduces to hypergeometric-type,

$$\sigma(s)\mathcal{X}''(s) + \tau(s)\mathcal{X}'(s) + \lambda\mathcal{X}(s) = 0 \qquad (5)$$

where the wave function $\emptyset(s)$ is defined as the logarithmic derivative

$$\frac{\emptyset'(s)}{\emptyset(s)} = \frac{\pi(s)}{\sigma(s)'} \qquad (6)$$

Where $\pi(s)$ is at most first-order polynomial?

The hypergeometric-type function $\emptyset(s)$ whose polynomial solutions are given by the Rodrigues relation

$$\emptyset(s) = \frac{B_n}{\rho(s)}\frac{d^n}{ds^n}[\sigma^n(s)\rho(s)] \qquad (7)$$

Where B_n is the Normalization constant and the weight function $\rho(s)$ most satisfy the condition

$$\frac{d}{ds}[\sigma^n(s)\rho(s)] = \tau(s)\rho(s) \qquad (8)$$

Where

$$\tau(s) = \check{\tau}(s) + 2\pi(s) \tag{9}$$

In order to accomplish the condition imposed on the weight function $\rho(s)$, it is necessary that the classical or polynomials $\tau(s)$ be equal to zero to some point of an interval (a, b) and its derivative at this interval at $\sigma(s) > 0$ will be negative, that is

$$\frac{d\tau(s)}{ds} < 0 \tag{10}$$

Therefore, the function $\pi(s)$ and the parameter λrequired for the NU method are defined as follows:

$$\pi(s) = \frac{\sigma' - \check{\tau}}{2} \pm \sqrt{\left(\frac{\sigma' - \check{\tau}}{2}\right)^2 - \tilde{\sigma} + k\sigma}$$
(10)

Where $\lambda = k + \pi'(s)$

The parametric generalization of the NU method is given by the generalized hypergeometric-type equation as

$$\psi''(s) + \left(\frac{(c_1 - c_2 s)}{s(1 - c_3 s)}\right)\psi'(s) + \left(\frac{-\xi_1 s^2 + \xi_2 s - \xi_3}{s^2(1 - c_3 s)^2}\right)\psi(s) = 0 \tag{11}$$

Equation (11) is solved by comparing it with Eq. (2) and the following polynomials are obtained:

$$\check{\tau}(s) = (c_1 - c_2 s),\ \sigma(s) = s(1 - c_3 s),\ \tilde{\sigma}(s) = -\xi_1 s^2 + \xi_2 s - \xi_3 \tag{12}$$

Now substituting Eq. (12) into Eq. (11), we find

$$\bar{\sigma}(s) = c_4 + c_5 s \pm \sqrt{[(c_6 - c_3 k_{\pm})s^2 + (c_7 + k_{\pm})s + c_8]} \tag{13}$$

Where $c_4 = \frac{1}{2}(1 - c_1)$, $c_5 = \frac{1}{2}(c_2 - 2c_3)$, $c_6 = c_5^2 + \xi_1$, $c_7 = 2c_4 c_5 - \xi_2$, $c_8 = c_4^2 + \xi_3$, $c_9 = c_3 c_7 + c_3^2 c_8 + c_6$, $c_{10} = c_1 + 2c_4 + 2\sqrt{c_8}$, $c_{11} = c_2 - 2c_5 + 2\left(\sqrt{c_9} + c_3\sqrt{c_8}\right)$, $c_{12} = c_4 + \sqrt{c_8}$, $c_{13} = c_5 - \left(\sqrt{c_9} + c_3\sqrt{c_8}\right)$ (14)

The resulting value of *k* in Eq. (13) is obtained from the condition that the function under the square root be square of a polynomials and it yields,

$$k_{\pm} = -(c_7 + 2c_3 c_8) \pm 2\sqrt{c_9 c_8} \tag{15}$$

Where $c_9 = c_3 c_7 + c_3^2 c_8 + c_6$

The new $\pi(s)$ for k becomes

$$\pi(s) = c_4 + c_5 s - \left[\left(\sqrt{c_9} + c_3\sqrt{c_8}\right)s - \sqrt{c_8}\right] \quad (16)$$

Using Eq. (8), we obtain

$$\tau(s) = c_1 + 2c_4 - (c_2 - 2c_5)s - 2\left[\left(\sqrt{c_9} + c_3\sqrt{c_8}\right)s - \sqrt{c_8}\right] \quad (17)$$

We obtain the energy equation as

$$(c_2 - c_3)n + c_3 n^2 - (2n+1)c_5 + (2n+1)\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) + c_7 + 2c_3 c_8 + 2\sqrt{c_8 c_9} = 0 \quad (18)$$

While the wave function is given as

$$\Psi_n(s) = N_{n,l} S^{c_{12}} (1 - c_3 s)^{-c_{12} - \frac{c_{13}}{c_3}} P_n^{\left(c_{10}-1, \frac{c_{11}}{c_3} - c_{10} - 1\right)}(1 - 2c_3 s) \quad (19)$$

Where P_n is the orthogonal polynomials.

Given that $P_n^{(\alpha,\beta)} = \sum_{r=0}^{n} \frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{\Gamma(\alpha+r+1)\Gamma(n+\beta-r+1)(n-r)!r!} \left(\frac{x-1}{2}\right)^r \left(\frac{x+1}{2}\right)^{n-r}$ (20)

This can also be expressed in terms of the Rodriguez's formula

$$P_n^{(\alpha,\beta)}(x) = \frac{1}{2^n n!}(x-1)^{-\alpha}(x+1)^{-\beta}\left(\frac{d}{dx}\right)^n \left((x-1)^{n+\alpha}(x+1)^{n+\beta}\right) \quad (21)$$

3. BOUND STATE SOLUTION OF THE DIRAC EQUATION

The Schrodinger like differential equation [25] for the upper radial spinor component of the Dirac equation is given as

$$\left\{-\frac{d^2}{dr^2} + \frac{k(k+1)}{r^2} + \frac{1}{\hbar^2 c^2}[MC^2 + E_{nk} - \Delta(r)][MC^2 - E_{nk} + \textstyle\sum(r)]\right\} F_{nk}(r) = \frac{\frac{d\Delta r}{dr}\left(\frac{d}{dr} + \frac{k}{r}\right)}{[MC^2 + E_{nk} - \Delta(r)]} F_{nk}(r) \quad (22)$$

Where $\Delta(r) = V(r) - S(r)$ and $\sum(r) = V(r) + S(r)$ are the differences and the sum of the potentials V(r) ans S(r), respectively.

In the presence of the SS, that is, the difference potential $\Delta(r) = V(r) - S(r) = C_s = costant$or $\frac{d\Delta r}{dr} = 0$. Then the above equation becomes

$$\left\{-\frac{d^2}{dr^2} + \frac{k(k+1)}{r^2} + \frac{1}{\hbar^2 c^2}[MC^2 + E_{nk} - C_s]\sum(r)\right\} F_{nk}(r) = [E_{nk}^2 - M^2 C^4 + C_s(MC^2 - E_{nk}]F_{nk}(r)[23]$$

Similarly, under PSS conditions, $\sum(r) = V(r) + S(r) = C_{ps} = constant$ or $\frac{d\sum(r)}{dr} = 0$

$$\left\{-\frac{d^2}{dr^2}+\frac{k(k+1)}{r^2}+\frac{1}{\hbar^2c^2}\left[MC^2-E_{nk}+C_{ps}\right]\Delta(r)\right\}G_{nk}(r)$$
$$=\left[E_{nk}^2-M^2C^4+C_{ps}(MC^2-E_{nk}\right]G_{nk}(r)[24]$$

The Modified Eckart Potential[18]is given as

$$V(r)=-\left(\frac{V_oe^{-\alpha r}}{(1-e^{-\alpha r})^2}\right) \qquad (25)$$

The Inverse Square Potential[15],$V(r)=\frac{A}{r^2}$ (26)

Applying the transformation $S=e^{-\alpha r}$ and pekeris-type approximation.The superposed potential can be represented as MEISP

$$V(s)=-\left(\frac{V_os}{(1-s)^2}\right)+\frac{4A\alpha^2}{(1-s)^2} \qquad (27)$$

By applying the pekeris-type approximation given as [23] and , we obtained the following second order differential equation for Spin Symmetry in the presence of Spin-Orbit Coupling term

$$\frac{d^2R(s)}{ds^2}+\frac{(1-s)}{(1-s)s}\frac{dR(s)}{ds}$$
$$+\frac{1}{(1-s)^2s^2}[(\beta^2+P)s^2+(-2\beta^2-2P-Q)s+(\beta^2-H-P-\lambda)]R(s)=0$$

(28)

Where

$-\beta^2=\left(\frac{E^2-M^2}{4\alpha^2}\right),\ \lambda=(k(k+1)),\ P=\left(\frac{E-M}{4\alpha^2}\right)C_s,\ Q=\left(\frac{E+M-C_s}{4\alpha^2}\right)V_o,\ H=\left(\frac{E+M-C_s}{4\alpha^2}\right)A,$

$$c_1=c_2=c_3=1,c_4=0,c_5=-\frac{1}{2},c_6=\frac{1}{4}+\beta^2+P,c_7=-2\beta^2-2P-Q,$$

$$c_8 = 2\beta^2 - H - \lambda + P, c_9 = \frac{1}{4} - \lambda - H - Q, c_{10} = 1 + 2\sqrt{2\beta^2 - H - \lambda + P}, c_{11} = 2 + 2\left(\sqrt{\frac{1}{4} - \lambda - H - Q} + \sqrt{2\beta^2 - H - \lambda + P}\right), c_{12} = \sqrt{2\beta^2 - H - \lambda + P}\,, c_{13} = -\frac{1}{2} - \left(\sqrt{\frac{1}{4} - \lambda - H - Q} + \sqrt{2\beta^2 - H - \lambda + P}\right), \varepsilon_1 = 2\beta^2 + B + P + K, \varepsilon_2 = 4\beta^2 - \emptyset + B + H, \varepsilon_3 = 2\beta^2 - 2J - K + H$$

Using the eigenvalue equation, the energy eigen spectrum of MEISP is found to be

$$\beta^2 = \left[\frac{(Q+P+2H+2\lambda) - \left(n^2+n-\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4}-\lambda-H-Q}}{\left(n+\frac{1}{2}\right)+2\sqrt{\frac{1}{4}-\lambda-H-Q}}\right]^2 - (H+P+\lambda) \tag{29}$$

3.1.SPIN SYMMETRY

The above equation can be solved explicitly and the energy eigen spectrum can be obtained under the Spin Symmetry$\boldsymbol{k(k+1)}$**,**MEISP

$$E^2 - M^2 = 4\alpha^2 \left\{\left[\frac{\left(\left(\frac{E+M-C_S}{4\alpha^2}\right)V_O + \left(\frac{E-M}{4\alpha^2}\right)C_S + 2\left(\frac{E+M-C_S}{4\alpha^2}\right)A + k(k+1)\right) - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right)V_O - \left(\frac{E+M-C_S}{4\alpha^2}\right)A - k(k+1)}}{\left(n+\frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right)V_O - \left(\frac{E+M-C_S}{4\alpha^2}\right)A - k(k+1)}}\right]^2\right\} - \left(\left(\frac{E-M}{4\alpha^2}\right)C_s + \left(\frac{E+M-C_S}{4\alpha^2}\right)A + k(k+1)\right) \tag{30}$$

3.2.PSEUDO-SPIN SYMMETRY

For Pseudo-Spin consideration$\boldsymbol{k(k-1)}$**,** the explicit energy of the MEISP becomes

$$E^2 - M^2 = 4\alpha^2 \left\{ \left[\frac{\left(\left(\frac{E+M-C_s}{4\alpha^2}\right) V_o + \left(\frac{E-M}{4\alpha^2}\right) C_s + 2\left(\frac{E+M-C_s}{4\alpha^2}\right) A + k(k-1) \right) - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \left(\frac{E+M-C_s}{4\alpha^2}\right) V_o - \left(\frac{E+M-C_s}{4\alpha^2}\right) A - k(k-1)}}{\left(n+\frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - \left(\frac{E+M-C_s}{4\alpha^2}\right) V_o - \left(\frac{E+M-C_s}{4\alpha^2}\right) A - k(k+1)}} \right]^2 \right\} - \left(\left(\frac{E-M}{4\alpha^2}\right) C_s + \left(\frac{E+M-C_s}{4\alpha^2}\right) A + k(k-1) \right) \quad (31)$$

4. DISCUSSION

We consider the following cases of potential from equations $(30) and\ (31)$

(I) When $V_o = 0$, Dirac equation for Inverse square potential for Spin and Pseudo-spin symmetry is obtained as follows

SPIN SYMMETRY

$$E^2 - M^2 = 4\alpha^2 \left\{ \left[\frac{\left(\left(\frac{E-M}{4\alpha^2}\right) C_s + 2\left(\frac{E+M-C_s}{4\alpha^2}\right) A + k(k+1) \right) - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \left(\frac{E+M-C_s}{4\alpha^2}\right) A - k(k+1)}}{\left(n+\frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - \left(\frac{E+M-C_s}{4\alpha^2}\right) A - k(k+1)}} \right]^2 \right\} - \left(\left(\frac{E-M}{4\alpha^2}\right) C_s + \left(\frac{E+M-C_s}{4\alpha^2}\right) A + k(k+1) \right) \quad (32)$$

PSEUDO-SPIN SYMMETRY

$$E^2 - M^2 = 4\alpha^2 \left\{ \left[\frac{\left(\left(\frac{E-M}{4\alpha^2}\right) C_s + 2\left(\frac{E+M-C_s}{4\alpha^2}\right) A + k(k-1) \right) - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \left(\frac{E+M-C_s}{4\alpha^2}\right) A - k(k-1)}}{\left(n+\frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - \left(\frac{E+M-C_s}{4\alpha^2}\right) A - k(k+1)}} \right]^2 \right\} - \left(\left(\frac{E-M}{4\alpha^2}\right) C_s + \left(\frac{E+M-C_s}{4\alpha^2}\right) A + k(k-1) \right) \quad (33)$$

(II) When $A = 0$,Dirac equation for Modified Eckart potential for Spin and Pseudo-spin symmetry is obtained as follows

SPIN SYMMETRY

$$E^2 - M^2 = 4\alpha^2 \left\{ \left[\frac{\left(\left(\frac{E+M-C_S}{4\alpha^2}\right) V_O + \left(\frac{E-M}{4\alpha^2}\right) C_S + k(k+1) \right) - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right) V_O - k(k+1)}}{\left(n+\frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right) V_O - k(k+1)}} \right]^2 \right\} - \left(\left(\frac{E-M}{4\alpha^2}\right) C_s + k(k+1) \right) \quad (34)$$

PSEUDO-SPIN SYMMETRY

$$E^2 - M^2 = 4\alpha^2 \left\{ \left[\frac{\left(\left(\frac{E+M-C_S}{4\alpha^2}\right) V_O + \left(\frac{E-M}{4\alpha^2}\right) C_S + k(k-1) \right) - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right) V_O - k(k-1)}}{\left(n+\frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right) V_O - k(k+1)}} \right]^2 \right\} - \left(\left(\frac{E-M}{4\alpha^2}\right) C_s + k(k-1) \right) \quad (35)$$

5. CONCLUSION

In this paper, we obtained the approximate analytical solutions of the Dirac equation for the Modified Eckart plus Inverse Square potential for zero tensor interaction within the frameworkof pseudospin and spin symmetry limits using the NU technique. We have obtained the energy levels in a closed form and some special case of the potential has been discussed.

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