Review Paper

EFFECTS OF VARIABLE ELECTRICAL CONDUCTIVITY ON THERMAL BOUNDARY LAYER OVER A VERTICAL PLATE WITH BUOYANCY FORCE AND CONVECTIVE SURFACE BOUNDARY CONDITIONS

Abstract

This paper investigates the effects of variable electrical conductivity on thermal boundary layer over a vertical plate with buoyancy force and convective surface boundary conditions. The governing nonlinear partial differential equations are transformed into a set of coupled non-linear ordinary differential equations by using the usual similarity transformation method. The resulting nonlinear ordinary differential equations are then solved numerically by Runge- Kutta fourth order method with Shooting technique to study the effects of variable electrical conductivity on the thermal boundary layer over a vertical plate with buoyancy force and convective surface boundary conditions. The results show that the fluid temperature increases due to increase in magnetic field intensity and decreases due to increase or decrease in electrical conductivity parameter but it is maximum at the plate surface and decreases exponentially to zero far away from the plate thereby satisfying the boundary conditions. The fluid velocity increases with increase or decrease in electrical conductivity parameter and decreases due to magnetic field intensity. The boundary layer thickness increases with an increase in Biot numbers and decreases with increase in Grashof and Prandtl numbers. Convective surface heat transfer enhances thermal diffusion while an increase in Prandtl number which is an intensity of buoyancy force slows down the rate of thermal diffusion within the boundary layer. The skin friction and the rate of heat transfer at the surface increases with an increase in local Grashof number, electrical conductivity parameter and convective surface heat transfer parameter.

Keywords: Electrical conductivity, Thermal boundary layer, Vertical plate, Convective surface boundary conditions.

1. Introduction

The study of heat transfer is an integral part of natural convection flow and a class of boundary layer theory. The quantity of heat transferred is highly dependent upon the fluid motion within the boundary layer.

Convective heat transfer studies are very important in processes involving high temperatures such as gas turbines, nuclear plants, thermal energy storage, etc. The fluid flow along a horizontal, stationary surface located in a uniform free stream was solved for the first time by Blasius (1908) and since then it has been a subject of current research. Cortell [3] in his work presenteda numerical Solutions of the Classical Blasius Flat-Plate Problem using a Runge-Kutta algorithm for high-order initial value problem. He [5] worked on a simple perturbation approach to blasius equation. In his paper, he coupled the iteration method with the perturbation method to solve the well-known Blasius equation.

Also, Bataller [2] presented a numerical solution for the combined effects of thermal radiation and convective surface heat transfer on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow) and about a moving plate in a quiescent ambient fluid (Sakiadis flow).

The study of an incompressible viscous and electrically conducting fluid in the presence of a uniform transverse magnetic field was investigated by Watunade and pop [10]. Shrama and Gurminder[9] investigated the effect of temperature dependent electrical conductivity on steady natural convection flow of a viscous incompressible low Prandtl(Pr<<1) electrically conducting fluid along an isothermal vertical non-conducting plate in the presence of transverse magnetic field and exponentially decayingheat generation. Aziz [1] investigated a similarity solution for laminar thermal boundary layer over a flat-plate with a convective surface boundary condition. Makinde and Sibanda [8] conducted a study on magneto hydrodynamic mixed convective flow and heat and mass transfer past a vertical plate in a porous medium with constant wall suction.

Makinde [7] studied analysis of non-newtonian reactive flow in a cylindrical pipe. Cortell [4] investigated a similarity solutions for flow and heat transfer of a quiescent fluid over a nonlinearly stretching surface.

Makinde and Olanrewaju [6] conducted a study on the effects of buoyancy force on thermal boundary layer over a vertical plate with convective surface boundary conditions. This paper extends the work of Makinde and Olanrewaju (2010) to include the electrical conductivity parameter. The numerical solutions of the resulting momentum and the thermal similarity equations are reported for representative values of thermo physical parameters characterizing the fluid convective process.

2. Mathematical Analysis

Consider a two-dimensional steady incompressible fluid flow coupled with heat transfer by convection over a vertical plate. A stream of cold fluid at temperature T_∞moving over the right surface of the plate with a uniform velocity U_∞ while the left surface of the plate is heated by convection from a hot fluid at temperature T_f,which provides a heat transfer coefficient h_f see Fig.3.1.The *x*-axis is taken along the plate and *y*-axis is normal to the plate. Magnetic field of intensity B_o is applied in the *y*- direction. It is assumed that the external field is zero, also electrical field due to polarization of charges and Hall effect are neglected. Incorporating the Boussinesq's approximation within the boundary layer, the governing equations of continuity, momentum and energy equations are respectively given as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y2} + g\beta(T - T_\infty) - \frac{\sigma^* B_0^2}{\rho} u \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \quad (3)$$

where u and v are the x(along the plate) and the y(normal to the plate) components of the velocity respectively; g is the acceleration due to gravity; x, y are the Cartesian coordinates, B_0 is the Magnetic field intensity, β is the coefficient of thermal expansion, ρ is the density of the fluid, ν is the Kinematic viscosity, α is the coefficient of thermal conductivity, T is the temperature of the fluid, σ^* is the electrical conductivity and it is variable with temperature as given below

$$\sigma^* = \frac{\sigma}{1+\varepsilon\theta} \quad (4)$$

ε is the electrical conductivity parameter. All prime symbols denotes differentiation with respect to η

The velocity boundary conditions can be expressed as:

$$u(x,0) = v(x,0) = 0 \quad (5)$$

$$u(x,\infty) = U_\infty \quad (6)$$

The boundary conditions at the plate surface and far into the cold fluid may be written as:

$$-k\frac{\partial T}{\partial y}(x,0) = h_f[T_f - T(x,0)] \quad (7)$$

$$T(x,\infty) = T_\infty \quad (8)$$

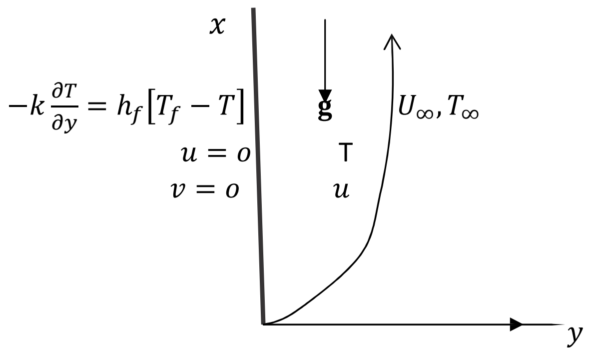

Fig 3.1 Flow Configuration and coordinate system

Introducing the stream function ψ(x,y) such that

$$\left.\begin{aligned} u &= \frac{\partial \psi}{\partial y} \\ v &= -\frac{\partial \psi}{\partial x} \end{aligned}\right\} \quad (9)$$

where $\Psi = (x, y) = x\sqrt{\frac{U_\infty V}{x}}\, f(\eta), \quad U_\infty = ax$ (10)

The similarity variable η, a dimensionless stream function $f(\eta)$ and temperature $\theta(\eta)$ are given as

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}}, \quad u = axf'(\eta), \; v = -\sqrt{a\nu} f(\eta), \quad \theta = \frac{T - T_\infty}{T_f - T_\infty} \quad (11)$$

Thus, the continuity equation (1) is satisfied with u and v of equations (11). Using (11), equations (2) and (3) are transformed into a set of coupled non-linear ordinary differential equation as

$$f'''(\eta) - f'(\eta)^2 + f(\eta) f''(\eta) - \frac{M}{1+\varepsilon\theta} f'(\eta) + Gr\,\theta(\eta) = 0 \quad (12)$$

$$\theta''(\eta) + Pr\, f(\eta)\theta'(\eta) = 0 \quad (13)$$

The boundary conditions (5), (6), (7) and (8) reduced to

$$f(0) = f'(0) = 0 \;, \quad f'(\eta) = 1 \quad as\; \eta \to \infty \quad (14)$$

$$\theta'(0) = -Bi\left[1 - \theta(x, 0)\right] \;, \quad \theta(\infty) = 0 \quad (15)$$

where $Gr = \frac{g\beta(T_f - T_\infty)}{ax^2}$ is the dimensionless Grashof number ,$M = \frac{\sigma\beta_o{}^2}{\rho a}$ is the magnetic parameter , $Pr = \frac{\nu}{\alpha}$ is the prandtl number and $Bi = -\frac{h}{k}\sqrt{\frac{\nu}{a}}$ is the Biot number.

Assuming that equations (12) and (13) have a similarity solution with the parameters Gr and Bi defined as constants.

3. NUMERICAL SOLUTIONS

Solving the governing boundary layer equations (12) and (13) with the boundary conditions (14) and (15) above numerically using Runge- Kutta fourth order method along with shooting technique and implemented on maple 17. The step size of 0.001 is used to obtain the numerical solution correct to eight decimal places as the criterion of the convergence.

4. RESULTS AND DISCUSSION

Numerical calculations have been carried out for different values of the thermo physical parameters controlling the fluid dynamics in the flow region.

Table 4.1: Computations Showing Comparison of the Makinde (2010) and the Present Result

M = Gr = 0 and Pr =0.72				
	Makinde 2010		**Present Work**	
Bi	**-θ'(0)**	**θ(0)**	**-θ'(0)**	**θ(0)**
0.05	0.0428	0.1447	0.04276694	0.14466115
0.10	0.0747	0.2528	0.07472420	0.25275803
0.20	0.1139	0.4035	0.11929550	0.40352251
0.40	0.1700	0.5750	0.16999442	0.57501394
0.60	0.1981	0.6699	0.19805068	0.66991553
0.80	0.2159	0.7302	0.21586402	0.73016997
1.00	0.2282	0.7718	0.22817787	0.77182213
5.00	0.2791	0.9442	0.27913110	0.94417378
10.00	0.2871	0.9713	0.28714625	0.97128538
20.00	0.2913	0.9854	0.29132895	0.98543355

Source: Maple 17 Output

Table 4.1 shows the comparison of Makinde's work (2010) with the present work for Prandtl number Pr=0.72 and it is noteworthy that there is a perfect agreement in the absence of Grashof number. Table 4.2, illustrates the values of the skin-friction coefficient and the local Nusselt number in terms of $f'(0)$and $-\theta'(0)$ respectively, for various values of embedded parameters.

Table4.2: Computational table showing f''(0), -θ'(0) and θ(0)

Pr	M	Gr	ε	Bi	f''(0)	-θ'(0)	θ(0)
0.72	0.1	0.1	0.1	0.1	0.07803359	0.06842756	0.31572443
1	0.1	0.1	0.1	0.1	0.07387282	0.07041418	0.29585824
3	0.1	0.1	0.1	0.1	0.06130059	0.07665015	0.23349847
0.72	**0.1**	0.1	0.1	0.1	0.07803359	0.06842756	0.31572443
0.72	**1**	0.1	0.1	0.1	0.03481939	0.06023466	0.39765338
0.72	**3**	0.1	0.1	0.1	0.02305143	0.05740404	0.42595962
0.72	0.1	**0.1**	0.1	0.1	0.07803359	0.06842756	0.31572443
0.72	0.1	**1**	0.1	0.1	0.30527486	0.07435471	0.25645289
0.72	0.1	**3**	0.1	0.1	0.61823565	0.07792806	0.22071942
0.72	0.1	0.1	**0.1**	0.1	0.07803359	0.06842756	0.31572443
0.72	0.1	0.1	**1**	0.1	0.07983050	0.06856275	0.31437250
0.72	0.1	0.1	**3**	0.1	0.08254355	0.06877520	0.31224801
0.72	0.1	0.1	0.1	**0.1**	0.07803359	0.06842756	0.31572443
0.72	0.1	0.1	0.1	**1**	0.13602308	0.19344578	0.80655422
0.72	0.1	0.1	0.1	**10**	0.15452103	0.24045670	0.97595433

Source: Maple 17 Output

From Table 4.2 and Fig. 4.5, 4.7 and 4.9, it is understood that the skin-friction and the rate of heat transfer at the plate surface increases with an increase in local Grashof number Gr, electrical conductivity parameter ε and convective surface heat transfer parameter Bi . It is also observed that for values of Gr > 0 as in Fig.4.6 there is decrease in the temperature profile which corresponds to the cooling problem. The cooling problem is often encountered in engineering applications; for example, in the cooling of electronic components and nuclear reactors.

However, in Fig. 4.1 and Fig.4.3, an increase in the fluid Prandtl number Pr and magnetic field parameter M decreases the skin-friction but increases the rate of heat transfer at the plate surface. This is attributed to the fact that as the prandtl number decreases, the thermal boundary layer thickness increases, causing reduction in the temperature gradient . $\theta'(0)$ at the surface of the plate.

In Fig. 4.2, the temperature gradient reduces at the surface because low prandtl fluid has high thermal conductivity, causing the fluid to attain higher temperature thereby reducing the heat flux at the surface. Moreover, for such low prandtl number, the velocity boundary layer is inside the thermal boundary layer and its thickness reduces as Prandtl number decreases so the fluid motion is confined in more and more thinner layer near the surface and experiences increase drag (skin-friction) by the fluid. In other words there is more straining motion inside velocity boundary layer resulting in the increase of skin-friction coefficient. It is also observed from the table that increase in magnetic field intensity, the skin-friction coefficient and the rate of heat transfer decreases near the surface; hence the surface experiences reduction in drag.

Generally, Figs. 4.1, 4.3, 4.5, 4.7 and 4.9 show that the fluid velocity is zero at the plate surface and increases gradually away from the plate toward the free stream value satisfying the boundary conditions. Similarly, Figs 4.2, 4.4, 4.6, 4.8 and 4.10 show that the fluid temperature is maximum at the plate surface and decreases exponentially to zero value far away from the plate satisfying the boundary conditions.

From these figures, it is important to note that:

1. thermal boundary layer thickness increases with an increase with biot numbers B_i and decreases with an increase in Grashof number Gr and Prandtl Pr number. Thus, convective surface heat transfer enhances thermal diffusion while an increase in the prandtl number which is an intensity of buoyancy force slows down the rate of thermal diffusion within the boundary layer.
2. fluid velocity increases due to increase or decrease in electrical conductivity parameter while it decreases due to magnetic field intensity
3. fluid temperature increases due to increase in magnetic field intensity while it decreases due increase or decrease in electrical conductivity parameter.

The Graphs below Show the Velocity Profiles and the Temperature Profiles at Various Parameters Values

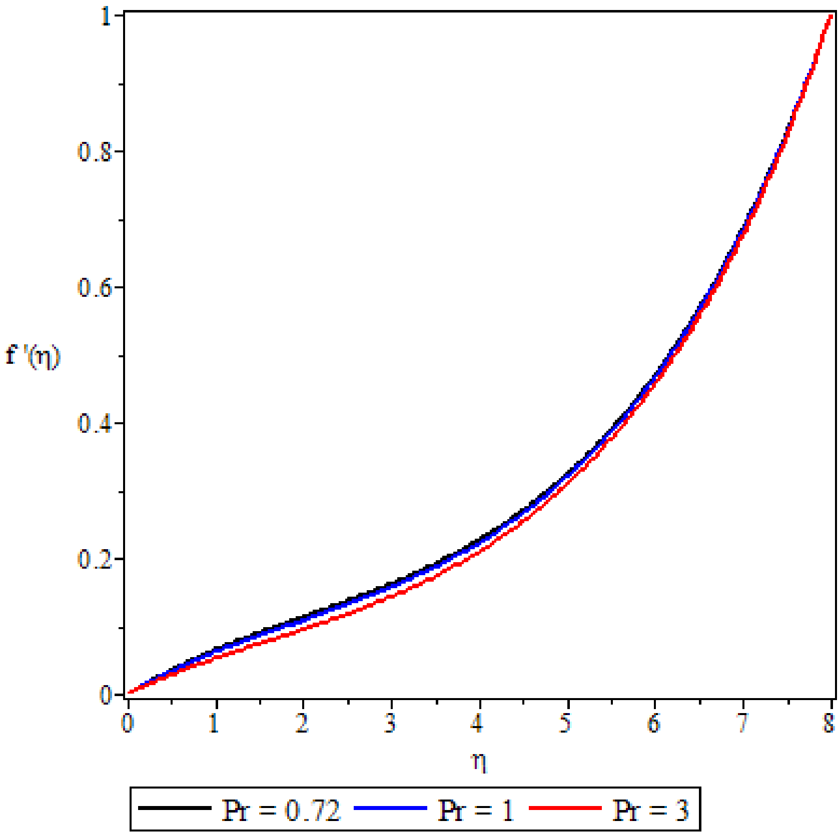

Fig. 4.1 velocity profile for Pr

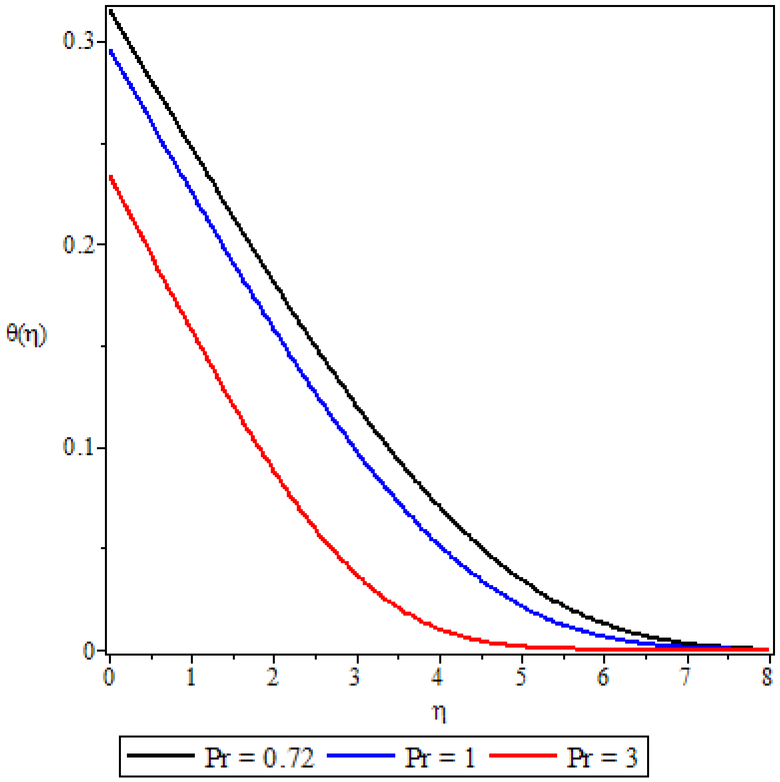

Fig. 4.2 Temperature profile for Pr

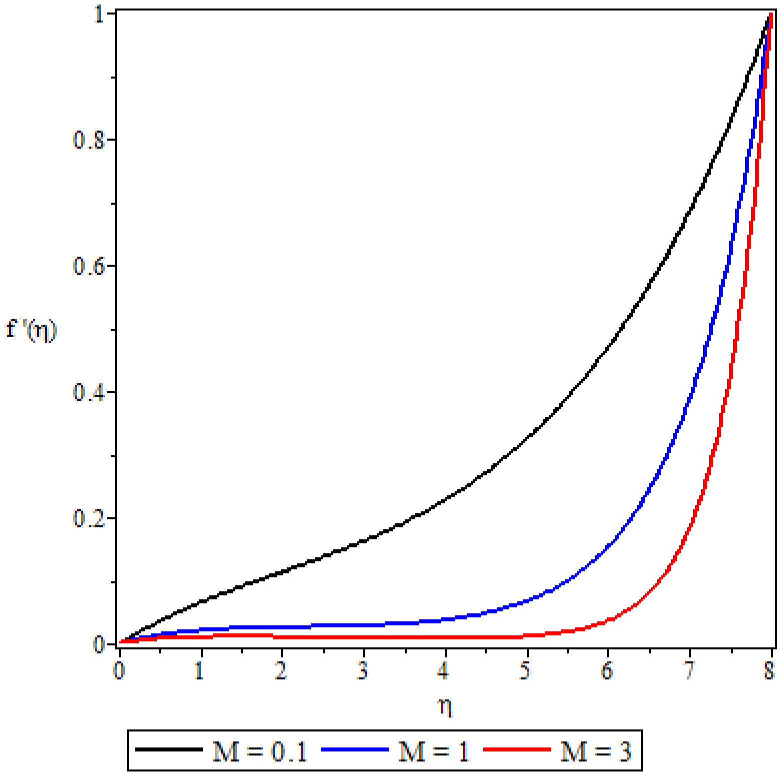

Fig. 4.3 velocity profile for M

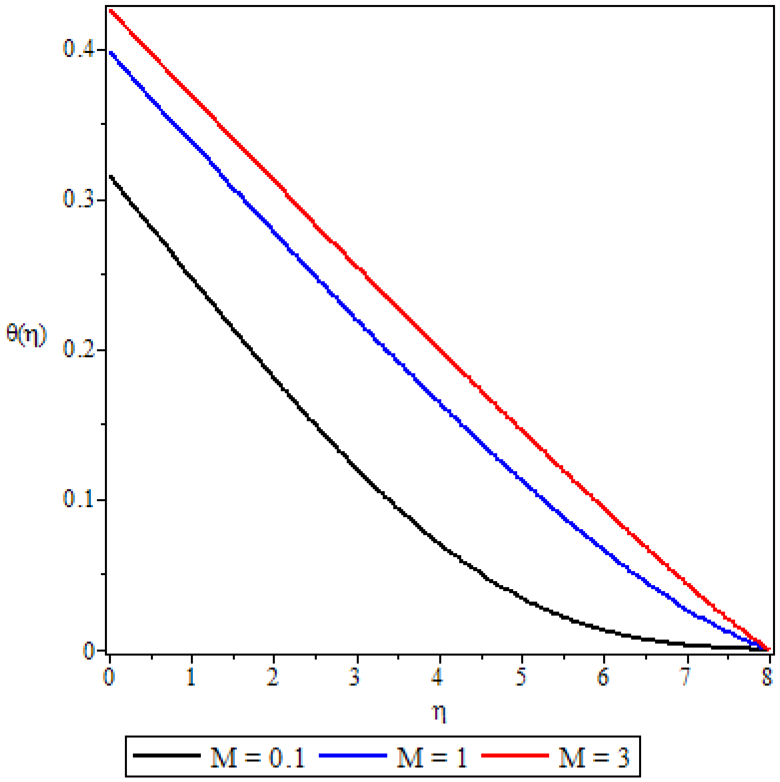

Fig. 4.4 Temperature profile for M

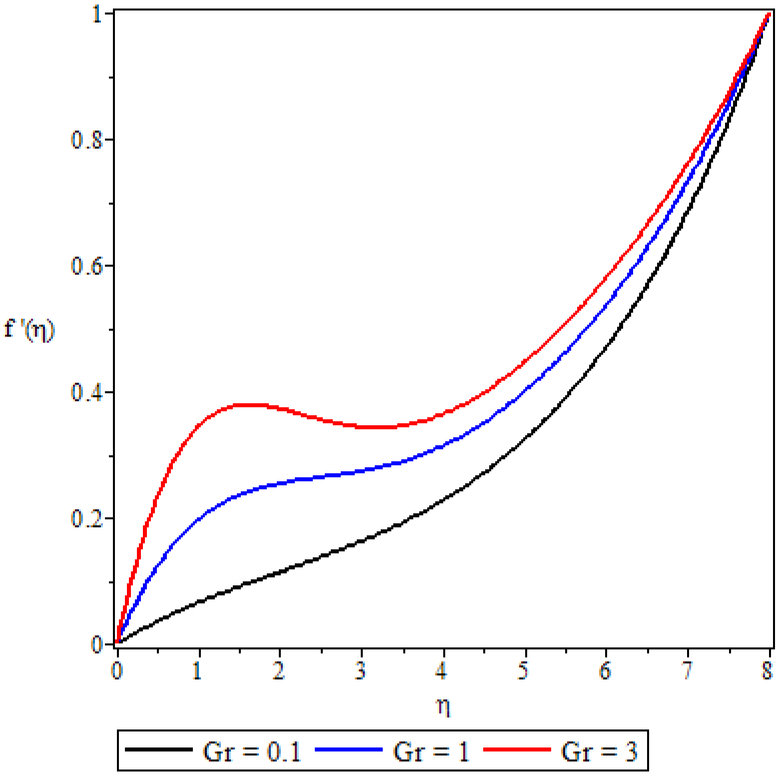

Fig. 4.5 velocity profile for Gr

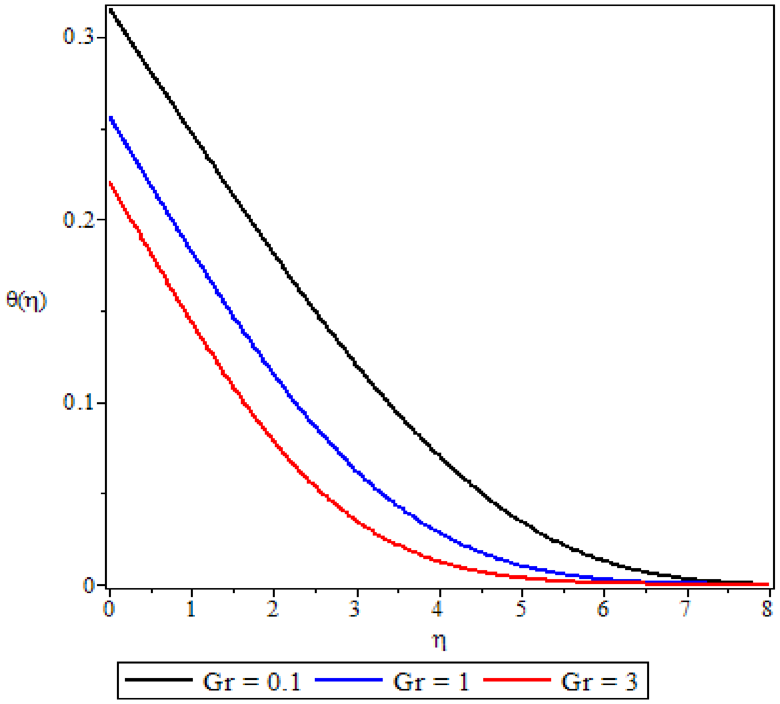

Fig. 4.6 temperature profile for Gr

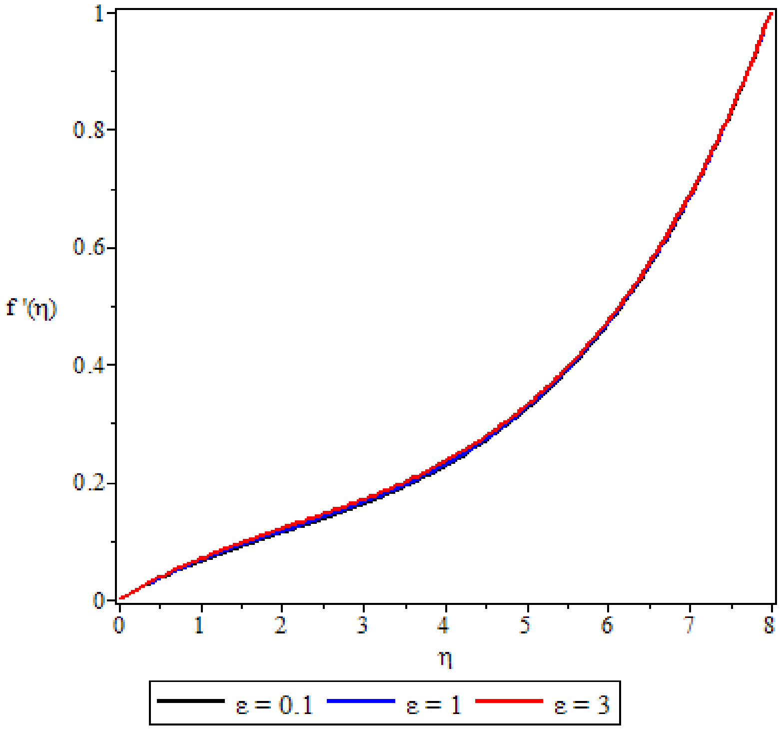

Fig. 4.7 velocity profile for ε

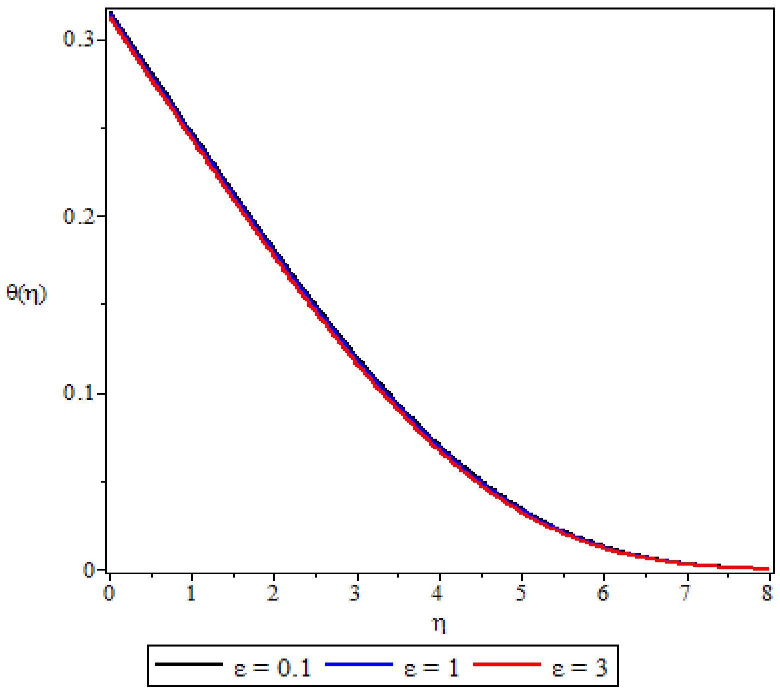

Fig.4.8 temperature profile for ε

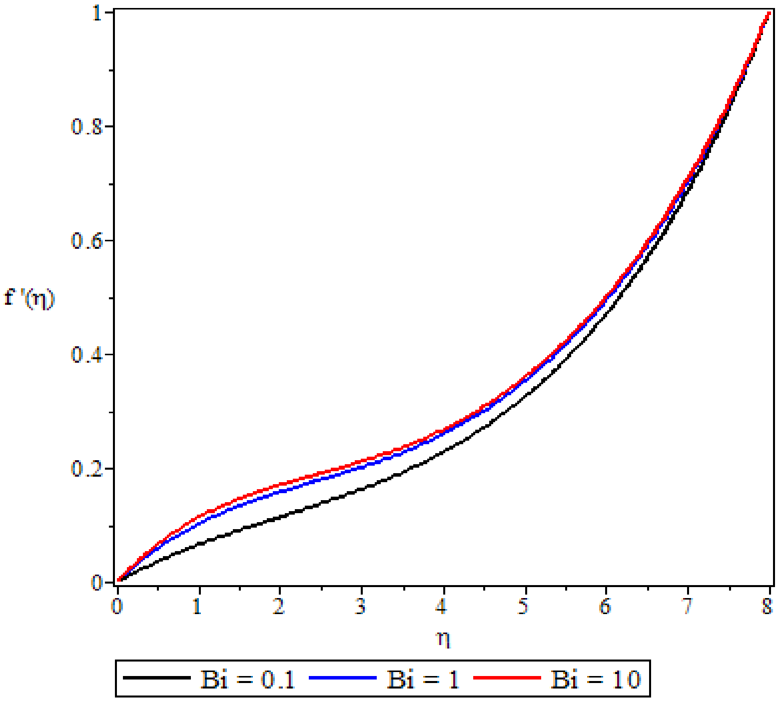

Fig. 4.9 velocity profile for Bi

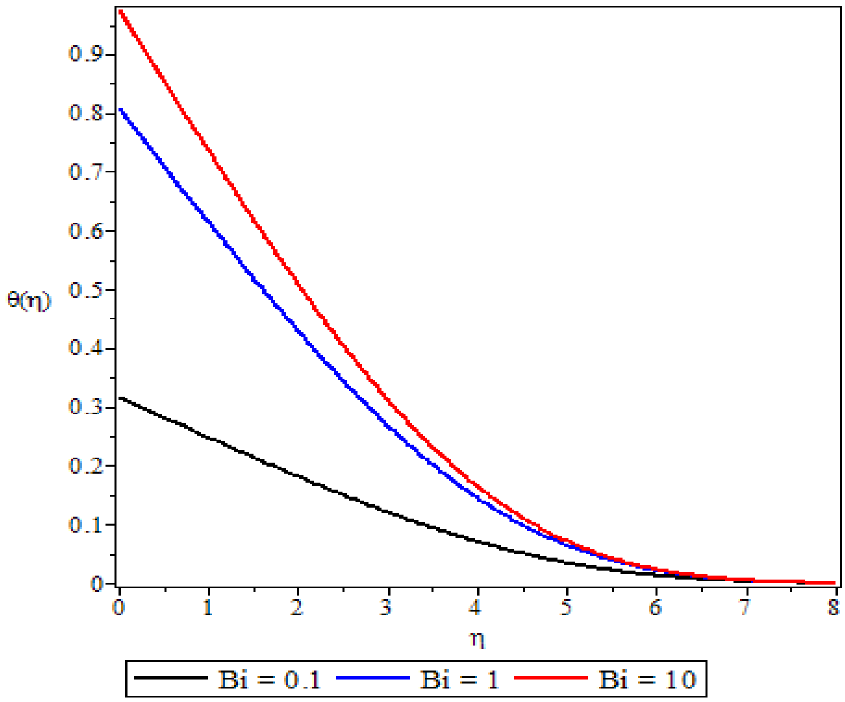

Fig. 4.10 : Temperature profile for B_i

5. CONCLUSION

From the numerical solutions and graphical representations, the combined effects of increasing the Prandtl number and the Grashof number tend to reduce the thermal boundary layer thickness. Fluid temperature increases due to increase in magnetic field intensity while it decreases due to increase in electrical conductivity parameter. Fluid velocity increases due to increase in electrical conductivity parameter while it decreases due to increase in magnetic field intensity which is in full agreement with the physical phenomenon.

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