NATURAL CONVECTIVE HEAT TRANSFER IN A LAMINAR FLOW OVER AN IMMERSED CURVED SURFACE

ABSTRACT

Numerical solutions of unsteady laminar free flow of a viscous fluid past an immersed curved surface were presented in this research study. The two-dimensional fluid flow in consideration was incompressible. Flows of this nature are commonly encountered in engineering studies such as Aerodynamics e.g. aero planes and Hydrodynamics e.g. ships. The continuity, the momentum and thermal energy equations were non-dimensionalised and the solutions of the dimensionless governing equations approximated using finite-difference method, since these equations were non-linear and hence could not be solved using analytical methods. The velocity and temperature fields were studied by varying various parameters in the equations governing the fluid flow. The results obtained in tabular form were presented graphically for comprehensive and easier interpretation. From the results, it was found out that the dissipation of heat increases with increase in the length of the curvature within the boundary layer. As the length of the curvature is increased, the amount of heat dissipated within the boundary layer also goes high. Also when the Reynolds number was increased, this led to decrease in heat dissipation within the boundary layer. These findings would assist Engineers in making appropriate designs and estimate improvements in equipment that require minimal resistance to the fluid motion.

1 BACKGROUND INFORMATION

Natural Convective heat transfer over an immersed curved surface is receiving research attention due to its wide applications in designing of devices such as flying planes, submarines, pumps, cooling fans among many others.

In the study of models of the turbulent boundary layer with pressure gradient, Barenblatt *et al* (2002), posted that at large Reynolds number, the turbulent boundary layer consists of two separate layers in which the shape of the vortex fields is different. Interestingly, both showed similar characteristics. The first layer has vertical structure that is common to all developed shear flows. In this layer, the effect of viscosity is transmitted to the mainstream body through streaks that separates the viscous sub layers. The second layer possess the remaining part of the neighbouring region of the boundary layer.

Gupta *et al* (2003) investigated heat transfer along the surface with a longitudinal curvature in laminar fluid flow and concluded that as the curvature changes from concave to convex, the Nusselt number decreases for Eckert number being small and increases if the Eckert number is increased to unity.

Bradshaw *et al* (2006) extended the study on the use of the algebraic analogy to the curved shear layers and the effects of the curvature on the mixing length if the shear layer thickness exceeds 1/300 of the radius of the curvature. In their study they concluded that large effects occurred in compressible fluid flows.

From the investigations conducted by Khoshevis *et al* (2007) on effects of the concave curvature on turbulent fluid flows, it was found that turbulent intensities as well as shear stresses are high on concave surfaces compared to a flat surface under similar conditions. In their study, they concluded that the de-stabilizing effects on the boundary layer of the concave surface leads to increase in turbulence between the fluid particles similar to the way concave curvature would cause the flow to be destabilized.

Mugambi *et al* (2008) in their investigation on the forces produced by the fluid motion on a sub-merged finite curved plates established the relationship between geometrical shape of the curvature and the variation of drag force of specific velocities of the viscous fluid.

George *et al* (2009) in their study on the convective heat transfer over curved surface established that as fluid flows over an immersed curved surface, some work is done against viscous effect and energy spent is converted into heat. The vortices formed in the boundary layer due to high velocity gradient is swept outwards from the boundary layer. They established that the rate of heat transfer is considerably high at points close to the convex surface within the boundary layer thickness. This, as a result leads to a decrease in fluid viscosity.

Kioi et *al* (2011) in their study noted that when the Reynolds number is high, the heat dissipation in the boundary layer also goes high. Their study concluded that when the Reynolds number is increased, the consequence is decrease in drag. When the Reynolds number decreased, the effect of drag goes high. At high Reynolds number the lift is increased and vise versa, hence a direct proportionality of the two quantities.

Mawira *et al* (2014) investigated the convective transfer of heat in a laminar boundary layer over an immersed curved surface. In their study, they established the pressure gradient affects the velocity and temperature profiles in the laminar boundary layer in that when the fluid pressure was decreased in the direction of the flow, this led to increase in velocity. The study concluded that when the surface area of the curvature was increased, the velocity and temperature of the fluid increased and vise versa.

From the above discussed research investigations and findings, it is clear that limited or little attention has been paid on the extent to which varying the length and the nature of the surface of the curvature would affect the velocity and temperature profiles along the unsteady laminar fluid flow. This was the motivation of this research study.

2 STATEMENT OF THE RESEARCH PROBLEM

Many researches in the past has laid more emphasis mass transfer, effect of varying the curvature radius on velocity and temperature distributions on the fluid flow but little has been done on the extent to which varying the length of the curvature and inertia forces would affect the temperature and velocity profiles along the immersed curved surface in consideration. This thus formed the basis of this research study.

2.1 JUSTIFICATION

The cost of maintenance brought about by degradation of equipment and a machine whose parts comes in contact with a fluid has been a major economical threat to manufacturers in

Engineering sector. Heat produced due to viscosity on the body surface has led to degradation of equipment and machines which has led to high cost of maintenance being incurred. Rise in temperature decreases the viscosity of the fluid and vise versa, thus need to design bodies with optimal curvature lengths and appropriate materials that could withstand such variations.

An aquatic animal like fish that solely depends on their effective swimming ability is affected by variations in fluid physical conditions such as temperature and velocity.

3 OBJECTIVES OF THE RESEARCH STUDY

3.1 General objective of the study

The aim of our study is to investigate the problem of natural convective heat transfer in a laminar flow over an immersed curved surface.

3.2 Specific objectives

1. To determine the effect of varying the length of the curvature on velocity and temperature profiles

2. To study the effect of inertia forces on velocity and temperature distributions along the boundary layer of an immersed curved surface

4 EQUATIONS GOVERNING THE FLUID FLOW

4.1 Equation of continuity

The equation is based on the law of conservation of mass, which states that matter cannot be created nor destroyed. The rate at which mass enters the system is equal to the rate at which mass leaves the system in a control volume. The general expression representing mass conservation is:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0 \qquad (1.0)$$

In Cartesian co-ordinate form, the equation (1.0) is expressed as;

$$\frac{\partial \rho}{\partial t} + \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] = 0 \qquad (1.1)$$

For two-dimensional fluid flow with constant density, w=0 and $\frac{\partial \rho}{\partial t}$ = 0 and thus equation (1.1) reduces to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1.2}$$

Equation (1.2) is our two-dimensional continuity equation in the velocity boundary layer under consideration.

4.2 Momentum equation

This equation is formulated from the Newton's second law of motion, which states that the rate of change of momentum of a body or matter is equal to the net external force applied to that particular body. These external forces that acts on the body are of two categories:

i) **Body forces**

These are forces acting on a body from an external source. They are usually expressed as forces per unit mass e.g gravitational force, magnetic force or electric fields and centrifugal forces.

ii) **Surface force**

The surface forces are due the interaction between the body and the matter in the immediate contact with it. The viscous stresses at any point in the velocity boundary layer were resolved into the two components; the normal stress which was always perpendicular to the surface and shear stress which was always tangential to the surface in consideration.

The momentum equation along x- axis is generally given as:

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] + \rho\frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \left[\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}\right] + \rho F_x \tag{1.3}$$

Along the y-direction we have:

$$\rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] + \rho\frac{\partial v}{\partial t} = -\frac{\partial P}{\partial y} + \left[\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x}\right] + \rho F_y \tag{1.4}$$

The viscous stresses and shear stresses in two dimensions are given by;

$$\sigma_{xx} = 2\mu\frac{\partial u}{\partial x} - \frac{2\mu}{3}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] \quad (1.5a)$$

$$\sigma_{yy} = 2\mu\frac{\partial v}{\partial y} - \frac{2\mu}{3}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] \quad (1.5b)$$

$$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right] \quad (1.5c)$$

Substituting (1.5a), (1.5b), (1.5c) into equations (1.3) and (1.4), we obtain momentum equation along the x-axis and y- axis as:

Along the x-axis;

$$\rho\frac{\partial u}{\partial t} + \rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left\{\mu\left[2\frac{\partial u}{\partial x} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right]\right\} +$$

$$\frac{\partial}{\partial y}\left\{\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right\} + \rho F_x \quad (1.6a)$$

Along the y- axis;

$$\rho\frac{\partial v}{\partial t} + \rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial P}{\partial y} + \frac{\partial}{\partial y}\left\{\mu\left[2\frac{\partial v}{\partial y} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right]\right\} +$$

$$\frac{\partial}{\partial x}\left\{\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right\} + \rho F_y \quad (1.6b)$$

Since $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, equations (1.6a) and (1.6b) reduced to

$$\rho\frac{\partial u}{\partial t} + \rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial P}{\partial x} + 2\mu\frac{\partial^2 u}{\partial x^2} + \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y}\right) + \rho F_x \quad (1.7a)$$

and

$$\rho \frac{\partial v}{\partial t} + \rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial P}{\partial y} + 2\mu\frac{\partial^2 v}{\partial x^2} + \mu\left(\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial^2 v}{\partial y^2}\right) + \rho F_y \qquad (1.7a)$$

respectively.

From the boundary layer approximations made earlier, the boundary layer thickness under consideration was very small to the extent that the velocity component tangential to the surface was much larger than that perpendicular to the surface. Hence the gradients perpendicular to the surface were larger than those along the surface i.e $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial v}{\partial x} \lll \frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial t} = 0$ since the fluid flow is assumed to be steady along the y- direction.

From these approximations, equation (1.7a) reduces to:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} + F_x$$

But $\frac{\mu}{\rho} = v$ and thus the above equation further reduces to:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\frac{\partial^2 u}{\partial y^2} + F_x \qquad (1.8a)$$

Also from the approximations made earlier, equation (1.7b) reduced to:

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial y} + F_y \qquad (1.8b)$$

From Bernoulli's equations, we have

$$P + \frac{1}{2}\rho u^2 = \text{constant} \qquad (1.9)$$

The curved surfaces provided both adverse and favourable pressure gradients whose tangential components of the velocity of the outer flow reveals a power law dependence on the streamwise x measured along the curved surface boundary as;

$$\frac{u}{c} = x^{m} \tag{2.0}$$

Where c was a positive velocity coefficient and m was an integer obtained from the angle of inclination from a horizontal plane

Differenciating partially equation (1.9) with respect to x, we obtain

$$\frac{\partial P}{\partial x} + \rho u \frac{\partial u}{\partial x} = 0 \tag{2.1}$$

Which implied that;

$$-\frac{1}{\rho}\frac{\partial P}{\partial x} = u\frac{\partial u}{\partial x} \tag{2.2}$$

But from the power law dependence,

$$u\frac{\partial u}{\partial x} = m\, c^{2}\, x^{2m-1} \tag{2.3}$$

Hence equation (1.8a) became

$$\frac{\partial u}{\partial t} = P_t + v\frac{\partial^2 u}{\partial y^2} + F_x \qquad \text{where } P_t = m\, c^{2}\, x^{2m-1} \tag{2.4}$$

$$\text{And} \quad \frac{\mu}{\rho} = v$$

Since the body under consideration had both concave and convex surfaces, the concave part of the body brought about an unstable effect which was determined by $\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}$. The curved surface as a curvature was defined by a quadratic equation of the form

$$bx^{2} + c(x) - y = 0 \tag{2.5}$$

where $0< b < 1$ was set to ensure that surface radius of the curvature was large enough and the end points were set at specific co-ordinates values when solving for a particular case of which length of the plate curvature were determined analytically.

The concave wall extended a destabilizing influence on the momentum exchange. Prandtl, who is considered to be the father of fluid mechanics proposed to account for the effect of the curvature through multiplying the length of the concave curved surface by a factor f. This factor f was a function given by;

$$f\left(\frac{\partial u}{\partial y}\right) = -\frac{k_r\, u}{4} + 1\left(\frac{\partial u}{\partial y}\right), \text{ which on simplifying further yielded:} \qquad (2.6)$$

$$f = -\frac{1}{4}\frac{k_r\, u}{\left(\frac{\partial u}{\partial y}\right)} + 1 \qquad (2.7)$$

he also deduced that the boundary layer equation on the curved surface was written as ;

$$\rho k_r\, u^2 = h_1 \frac{\partial P}{\partial y} \quad \text{, which was re-written as}$$

$$\frac{1}{\rho}\frac{\partial P}{\partial y} = \frac{k_r\, u^2}{h_1} \qquad (2.8)$$

Where k_r and h_1 are curvature parameters which were defined as

$$k_r = -\frac{1}{c(x)} \qquad (2.9)$$

$$h_1 = 1 + k_r\, y \qquad (2.10)$$

where c(x) was the radius of the curvature.

Equation (1.8b) was re-written as

$$F_y = \frac{1}{\rho}\frac{\partial P}{\partial y} \qquad (2.11)$$

Comparing equation (2.8) and (2.11) , we have;

$$\frac{k_r\, u^2}{h_1} = F_y \qquad (2.12)$$

Body forces, F_x and F_y which were purely due to the gravitational pull and which was assumed to be a constant in both cases. This led to a crucial assumption that:

$$F_x = F_y \tag{2.13}$$

From equations (2.12) and (2.13), it was resolved that

$$\frac{k_r\, u^2}{h_1} = F_x \tag{2.14}$$

Equation (2.14) was replaced in equation (2.5), a result which gave us a generalized equation of conservation of momentum for fluid flow over an immersed curved surface as;

$$\frac{\partial u}{\partial t} = P_t + v\frac{\partial^2 u}{\partial y^2} + \frac{k_r\, u^2}{h_1} \tag{2.15}$$

since $h_1 = 1 + k_r\, y$, the term $\frac{k_r\, u^2}{h_1}$ in equation (3.3) was written in Taylor series as

$$k_r\, u^2\, (1 + k_r\, y)^{-1} = k_r\, u^2\, (1 - k_r\, y + k_r^2 y + \ldots..) \tag{2.16}$$

And therefore, equation (2.15) yielded

$$\frac{\partial u}{\partial t} = P_t + v\frac{\partial^2 u}{\partial y^2} + k_r\, u^2\, (1 - k_r\, y + k_r^2 y + \ldots..) \tag{2.17}$$

The flow was along the x- axis. This implies that $y \cong 0$ and for every small value of k_r we have $(1 - k_r\, y + k_r^2 y + \ldots..) = 0$. Consequently, equation (2.17) reduced to

$$\frac{\partial u}{\partial t} = P_t + v\frac{\partial^2 u}{\partial y^2} + k_r\, u^2 \tag{2.18}$$

4.3 The Energy equation

This equation is derived from the First Law of Thermodynamics that asserts the mutual equivalence between heat and mechanical work. The law states that the amount of heat added to the system, dQ is equal to the sum work done, dW in the system plus the of change in the internal energy, dE of the system .

In mathematical expression, the equation can be written as:

$$dQ = dE + dW \tag{2.19}$$

where $dW = pdV = pd(\frac{1}{\rho})$ for a unit mass.

Equation (2.19) reduced to

$$dQ = dE + pd(\frac{1}{\rho}) \qquad (2.20)$$

The 1st law of thermaldynamics for a fluid flow with constant thermal conductivity K, zero internal generation and negligible compressibility effect, the equation reduced to;

$$\rho\, C_p \frac{Dh}{Dt} = K\nabla^2 T + \mu ø, \qquad (2.21)$$

where μø was the internal heating due to the viscous dissipation while for an incompressible two-dimensional fluid flow, the viscous dissipation function was

$$ø = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \qquad (2.22)$$

By considering unsteady incompressible flow in a control volume, the standard thermal energy equation for the thermal boundary layer was given by

$$\rho v\,\frac{\partial h}{\partial y} + \rho\frac{\partial h}{\partial t} + \rho u\frac{\partial h}{\partial x} = (\mu ø + q) + \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \left(u\frac{\partial P}{\partial x} + v\frac{\partial P}{\partial y}\right) \qquad (2.23)$$

where h was the enthalpy and q was the rate of heat dissipation.

Now the enthalpy h was given by

$$h = E + P\left(\frac{1}{\rho}\right) \qquad (2.24)$$

then, the first order derivative of enthalpy became

$$dh = dE + \left(\frac{1}{\rho}\right) dP + pd\left(\frac{1}{\rho}\right) \qquad (2.25)$$

But $dQ = dE + dW = dE + pd\left(\frac{1}{\rho}\right)$ and for a unit mass and a single species fluid,

$dQ = Tds$. Therefore we have

$$dE = Tds - pd\left(\frac{1}{\rho}\right) \qquad (2.26)$$

In view of (2.26), equation (2.25) became

$$dh = Tds + \left(\frac{1}{\rho}\right)dP + pd\left(\frac{1}{\rho}\right) - pd\left(\frac{1}{\rho}\right) \qquad (2.27)$$

hence

$$dh = Tds + \left(\frac{1}{\rho}\right)dP \qquad (2.28)$$

Assuming that $u\frac{\partial P}{\partial x}$ and $v\frac{\partial P}{\partial y}$ were negligible and $dh = CpdT$, equation (2.23) reduced to

$$C_p\rho\frac{\partial T}{\partial t} + C_p\rho\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + q \qquad (2.29)$$

For a fluid flowing over a body with a curved surface, the heat transfer area was the length of the of the curved surface and increase in the heat transfer area intensified the natural convective heat transfer along the surface of the fluid flow. The convection equation was expressed as

$$q = KAdT \qquad (2.30)$$

where $dT = (T_\infty - T_s)$ was the difference in temperature between the body surface and the bulk fluid. A was the area of the surface .

In this case, the area of the surface was the length of the curved surface and for this concave surface which had a destabilizing effect, the effect of the curved surface was taken into account by multiplying the area, A by a dimensionless factor given by the equation (2.7). This resulted to:

$$q = AfK\,dT \qquad (2.31)$$

Where q was the heat transferred per unit time.

On replacing f, equation (2.31) reduced to

$$q = k\left(1 - \frac{1}{4}\frac{k_r\, u}{(\frac{\partial u}{\partial y})}\right) A\ (T_\infty - T_s) \qquad (2.32)$$

From Newton's law of cooling, the local heat flux was given by

$$q_s'' = h\ (T_\infty - T_s) \qquad (2.33)$$

Where h was the local convection coefficient.

Since the flow conditions varied from one point to another on the curved surface, both q_s'' and h also varied along the curved surface.

For any particular distance x from the edge of the curved surface, q_s'' was found by applying the Fourier's Law to the fluid. This was done at y = 0 and was given as:

$- q_s'' = k\frac{\partial T}{\partial y}$, which was re-written as:

$$q_s'' = - k\frac{\partial T}{\partial y} \qquad (2.34)$$

The local convection heat transfer was expressed as

$$h = - k\frac{\partial T}{\partial y}(T_\infty - T_s)^{-1} \qquad (2.35)$$

At the thermal boundary layer, the rate of heat conduction along the y- direction was larger than that along the x- axis i.e $\frac{\partial T}{\partial y} >> \frac{\partial T}{\partial x}$

Then the equation of 1st Law of thermodynamics (2.24) reduced to

$$C_p\rho\left(u\,\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) + C_p\rho\frac{\partial T}{\partial t} = q + k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \qquad (2.36)$$

From the above approximations, equation (2.36) reduced to

$$C_p\rho\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + q \qquad (2.37)$$

But the value of q was replaced with equation (2.32) in order to take care of the curvature effects and hence on substituting equation (2.32) in equation (2.37) yielded

$$C_p \rho \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + k \left(1 - \frac{1}{4} \frac{k_r u}{\left(\frac{\partial u}{\partial y}\right)} \right) A \left(T_\infty - T_s \right) \qquad (2.38)$$

Equation (2.38) gave the equation of energy for convective heat transfer over an immersed curved surface.

5 DESCRIPTION OF THE FLOW

In this research work, a two dimensional laminar unsteady flow of a fluid over an immersed curved surface was studied. Since the body had both convex and concave surfaces there existed two non- zero pressure gradients.

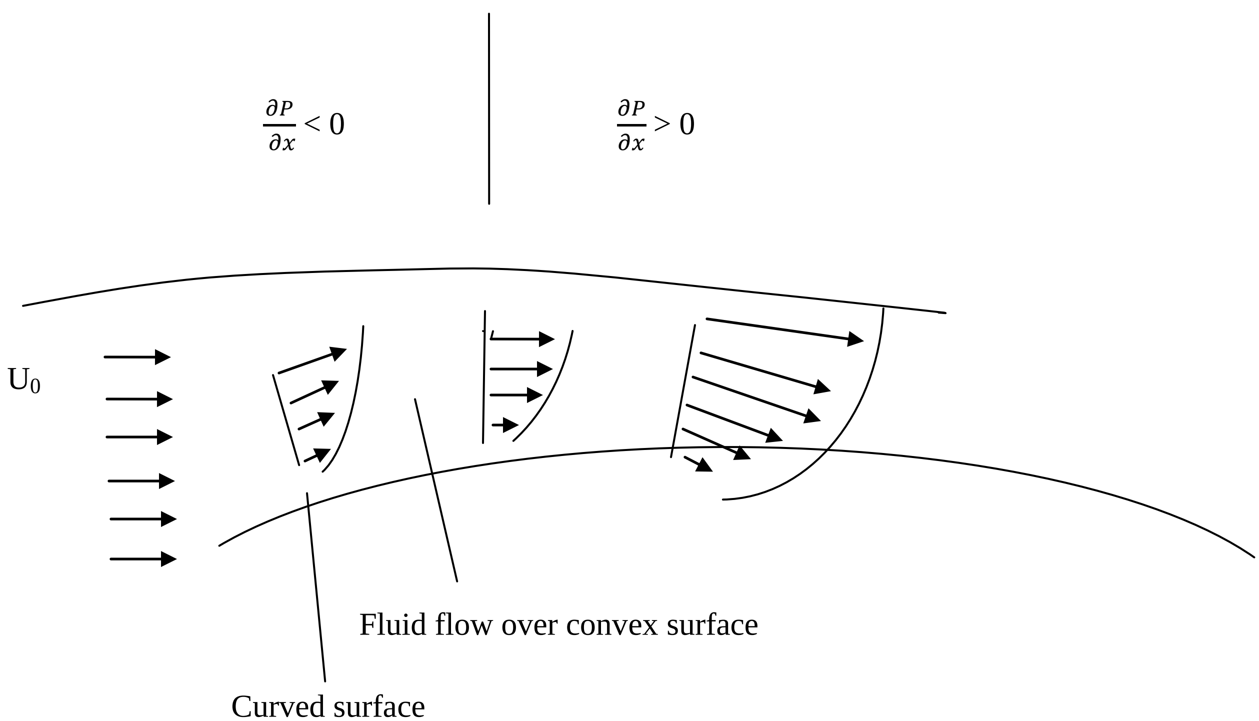

Figure 1. Effects of pressure gradients

6 NON-DIMENSIONALIZING THE EQUATIONS GOVERNING THE FLOW

As defined earlier, Dimensional analysis is a method which describes a natural phenomenon by a dimensionally correct equation with certain variables which affects the phenomenon.

In our research work, we let L, V, P and T to be the characteristic length, velocity, pressure and temperature respectively. The following transformations were used to reduce our equations in a dimensionless form;

$$\frac{x}{x^*} = L, \quad \frac{y}{y^*} = L, \quad \frac{u}{u^*} = V, \quad \frac{v}{v^*} = V, \quad \frac{p}{p^*} = P, \quad T^*(T_\infty - T_s)^{-1} = T - T_s$$

$$t^*L = tV \quad \text{or} \quad t = \frac{t^*V}{\mathrm{L}}$$

6.1 Equation of Continuity

For this particular fluid flow, the equation of continuity was given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.39)$$

On non-dimensionalising , the equation of continuity became

$$\frac{\partial (u^*V)}{\partial (x^*L)} + \frac{\partial (v^*V)}{\partial (y^*L)} = 0 \qquad (2.40)$$

Or $$\frac{V}{L}\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*}\right) = 0 \qquad (2.41)$$

Or $$\left(\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*}\right) = 0 \qquad (2.42)$$

6.2 The Momentum Equation

The equation of conservation of momentum for this flow problem was given by

$$\frac{\partial u}{\partial t} = \mathrm{P_t} + v\frac{\partial^2 u}{\partial y^2} + k_r u^2 \qquad (2.43)$$

On non-dimensionalising, the equation became

$$\frac{\partial (u^* V)}{\partial (\frac{t^* L}{V})} = \mathrm{P}^* \mathrm{P_t} + v\frac{\partial^2 (u^* V)}{\partial (y^* L)^2} + k_r (u^* V)^2 \qquad (2.44)$$

Hence the equation became

$$\frac{V^2}{L}\frac{\partial u^*}{\partial t^*} = \mathrm{P}\,\mathrm{P_t}^* + \frac{vV}{L^2}\frac{\partial^2 u^*}{\partial y^{*2}} + k_r V^2 u^{*2} \qquad (2.45)$$

Multiplying both sides by $\frac{L}{V^2}$ we have

$$\frac{\partial u^*}{\partial t^*} = \frac{PL}{V^2}\,\mathrm{P_t}^* + \frac{v}{LV}\frac{\partial^2 u^*}{\partial y^{*2}} + k_r L u^{*2} \qquad (2.46)$$

This gave the equation of momentum in non-dimensional form

6.3 The Energy Equation

The equation of conservation of energy was given by

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{C_p\rho}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{C_p\rho}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{kA}{C_p\rho}(T_\infty - T_s)\left(1 - \frac{1}{4}\frac{k_r u}{(\frac{\partial u}{\partial y})}\right) \qquad (2.47)$$

From the boundary boundary approximations the above equation reduced to

$$\frac{\partial T}{\partial t} = \frac{k}{C_p\rho}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{C_p\rho}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{kA}{C_p\rho}(T_\infty - T_s)\left(1 - \frac{1}{4}\frac{k_r u}{(\frac{\partial u}{\partial y})}\right) \qquad (2.47)$$

From the non-dimensional form of T, we had

$T^* = \frac{T - T_S}{(T_\infty - T_S)}$, which on making T the subject of the formulae yielded

$T = T^*(T_\infty - T_s) + T_s$ and thus the equation of energy became

$$\frac{\partial\,[T^{*}(T_{\infty}-T_{S})\,+T_{S}]}{\partial(\frac{t^{*}L}{\mathrm{V}})}=\frac{k}{C_{p}\rho}\frac{\partial^{2}\,[T^{*}(T_{\infty}-T_{S})\,+T_{S}]}{\partial(y^{*}L)^{2}}+\frac{\mu}{C_{p}\rho}\left(\frac{\partial(u^{*}V)}{\partial(y^{*}L)}\right)^{2}+\frac{kA}{C_{p}\rho}(T_{\infty}\text{-}T_{s})\left(1-\frac{1}{4}\frac{k_{r}\,(u^{*}V)}{\left(\frac{\partial(u^{*}V)}{\partial(y^{*}L)}\right)}\right)$$

$$(2.48)$$

This equation became

$$\frac{V(T_{\infty}-T_{S})}{L}\;\frac{\partial T^{*}}{\partial t^{*}}=\frac{k}{C_{p}\rho}\frac{(T_{\infty}-T_{S})}{L^{2}}\;\frac{\partial^{2}\,T^{*}}{\partial {y^{*}}^{2}}+\frac{\mu\,V^{2}}{C_{p}\rho\;\;L^{2}}\left(\frac{\partial u^{*}}{\partial y^{*}}\right)^{2}+\frac{kA}{C_{p}\rho}(T_{\infty}\text{-}T_{s})\left(1-\frac{1}{4}\frac{k_{r}\,u^{*}L}{\left(\frac{\partial u^{*}}{\partial y^{*}}\right)}\right)\qquad(2.49)$$

Diving all through by the term $\frac{V(T_{\infty}-T_{S})}{L}$, we obtained

$$\frac{\partial T^{*}}{\partial t^{*}}=\frac{k}{C_{p}\rho LV}\frac{\partial^{2}\,T^{*}}{\partial {y^{*}}^{2}}+\frac{\mu V}{C_{p}\rho L(T_{\infty}-T_{S})}\left(\frac{\partial u^{*}}{\partial y^{*}}\right)^{2}+\frac{kLA}{C_{p}\rho V}\left(1-\frac{1}{4}\frac{k_{r}\,u^{*}L}{\left(\frac{\partial u^{*}}{\partial y^{*}}\right)}\right)\qquad(2.50)$$

Multiplying the term $\frac{\mu V}{C_{p}\rho L(T_{\infty}-T_{S})}\left(\frac{\partial u^{*}}{\partial y^{*}}\right)^{2}$ by V in the numerator and the denominator, we obtained

$$\frac{\partial T^{*}}{\partial t^{*}}=\frac{k}{C_{p}\rho LV}\frac{\partial^{2}\,T^{*}}{\partial {y^{*}}^{2}}+\frac{\mu V.V}{C_{p}\rho L.V(T_{\infty}-T_{S})}\left(\frac{\partial u^{*}}{\partial y^{*}}\right)^{2}+\frac{kLA}{C_{p}\rho V}\left(1-\frac{1}{4}\frac{k_{r}\,u^{*}L}{\left(\frac{\partial u^{*}}{\partial y^{*}}\right)}\right)\qquad(2.51)$$

The equation (2.52) represented the equation of conservation of energy in non-dimensional form

7 NUMERICAL METHOD OF SOLUTION

In carrying out this study, we shall solve the governing equations using the finite difference method. The advantage of this method is based on its convergence and its ability to take less memory. The distinguishing feature of a finite difference method is the approximation of partial derivatives in the governing equations with finite differences relating the values of the unknown function at a set of the neighboring grid points at various levels.

7.1 REPRESENTATION OF THE RESULTS

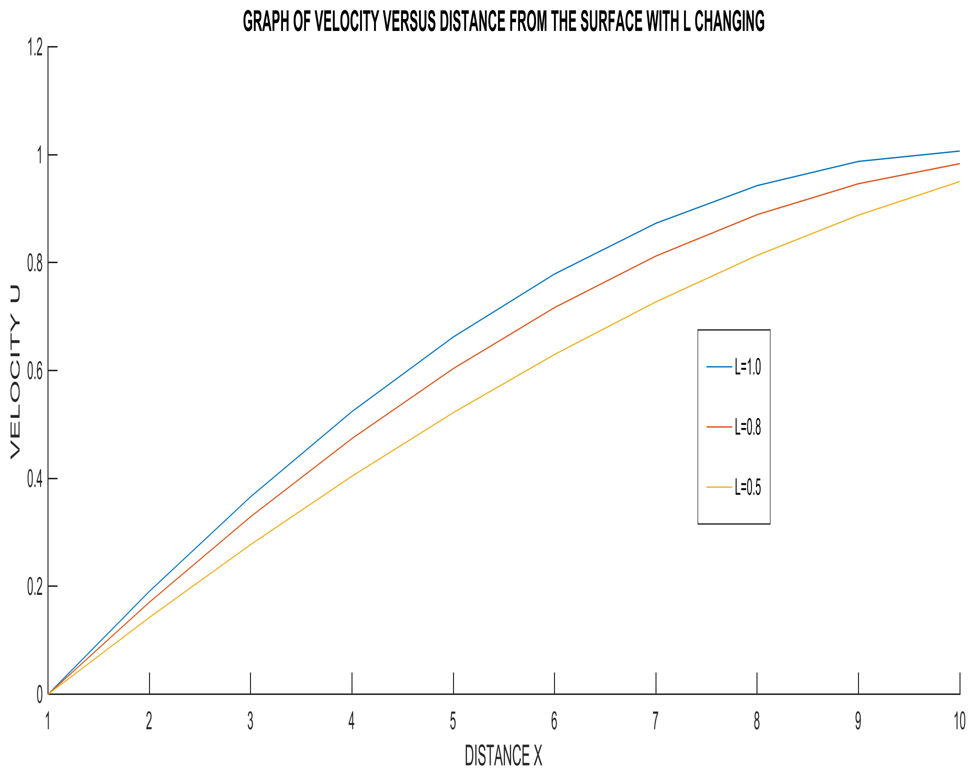

Figure 2: velocity profiles for Re=1.3, Pe=1, V=1, Kr = 1, Ec = 2, A = 2, Pt = 1

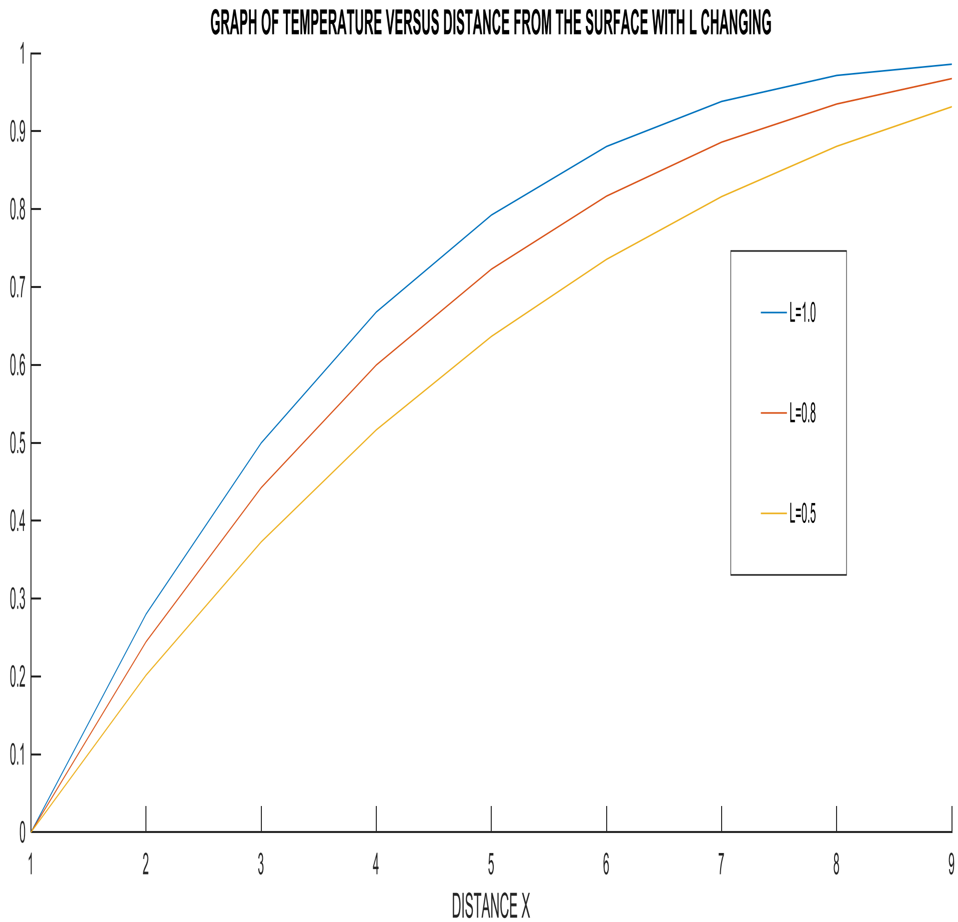

Figure 3: velocity profiles for Re=1.3, Pe=1, V=1, Kr = 1, Ec = 2, A = 2, Pt = 1

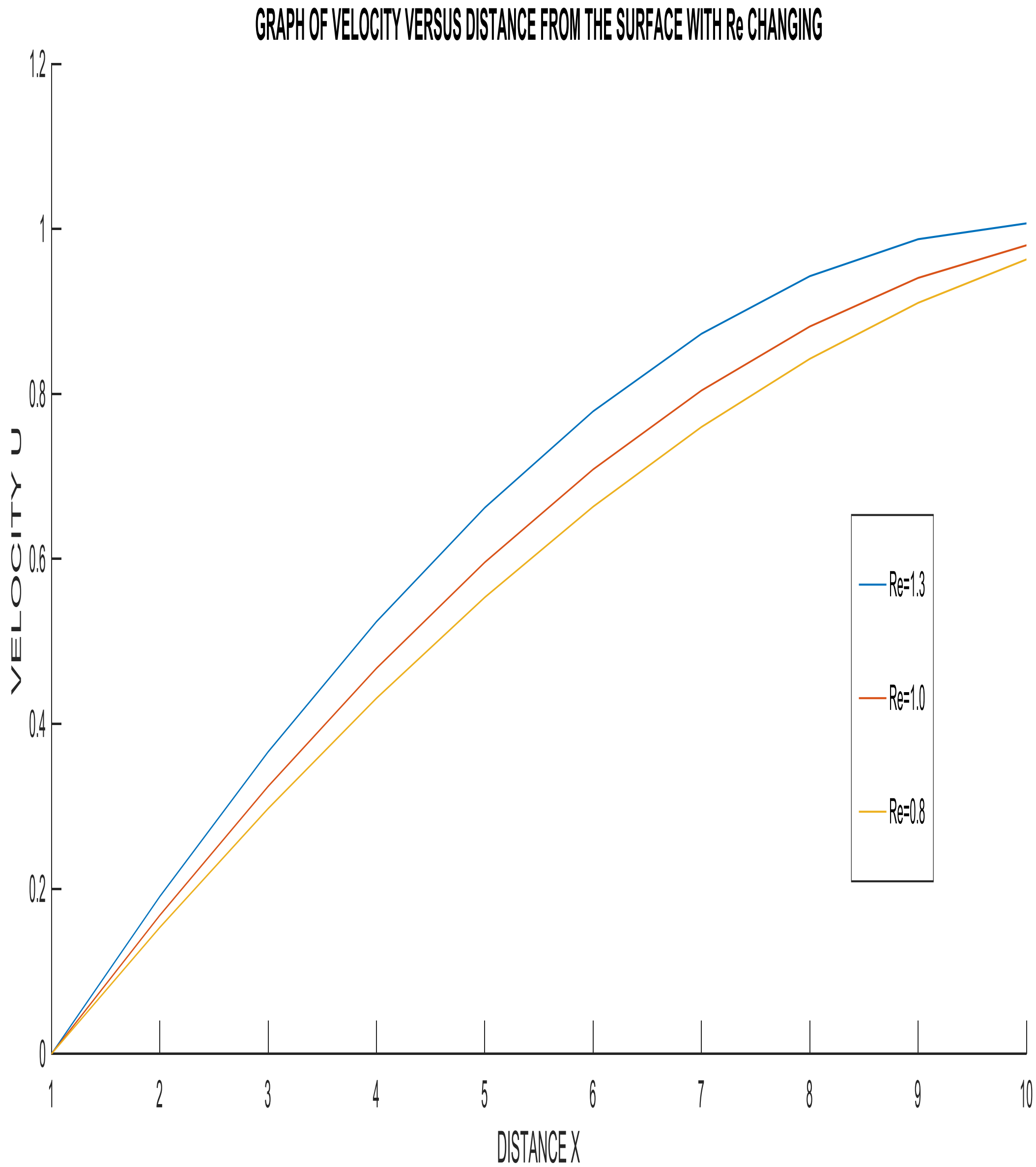

Figure 4: velocity profile for L=1, Pe=1, V= 1, Kr= 1, Ec= 2, A = 2, Pt= 1

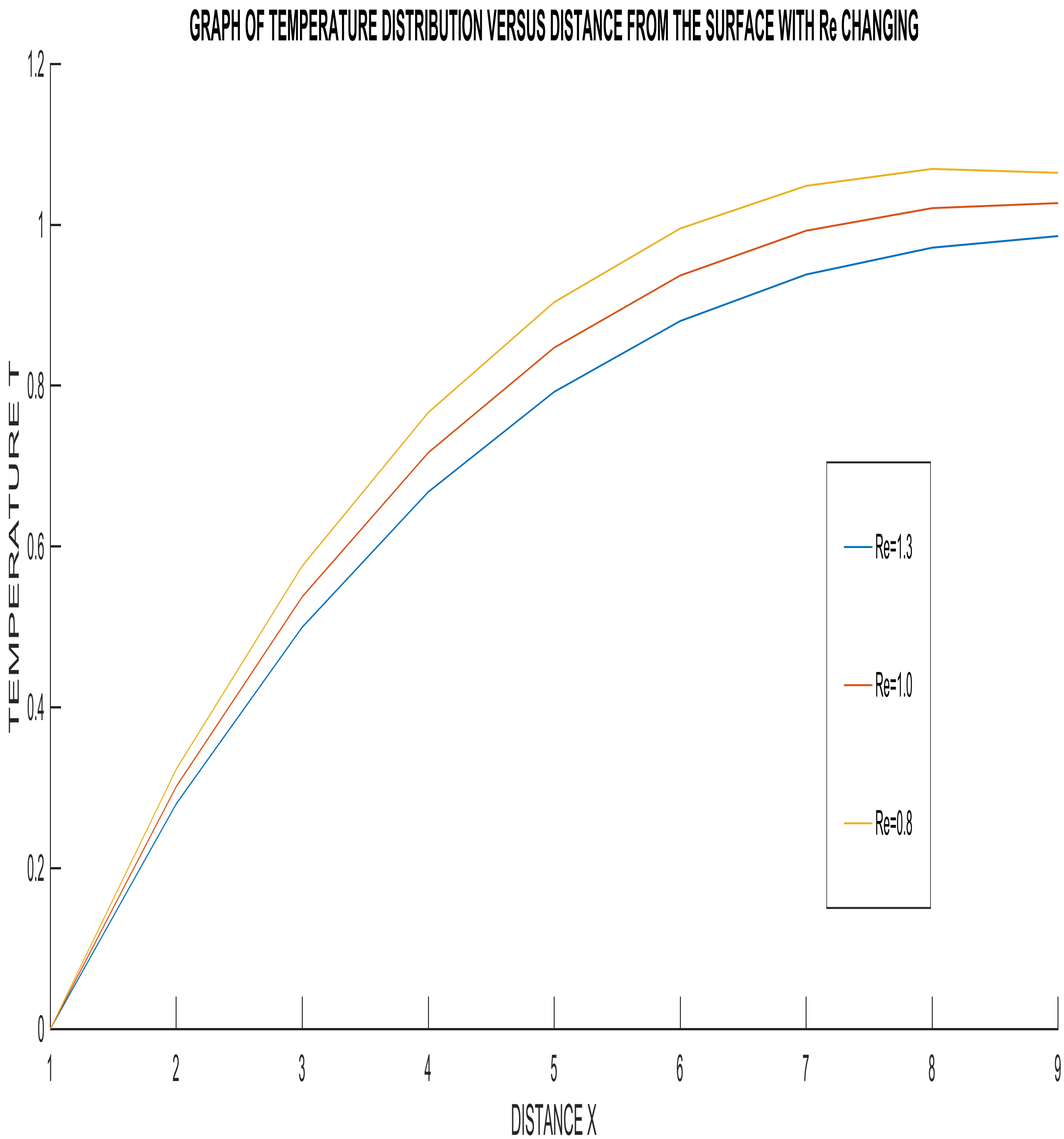

Figure 5: Temperature profile for L=1, Pe = 1, V = 1, Kr= 1, Ec= 2, A = 2, Pt= 1

7.2 DISCUSSION OF THE RESULTS

From figure 2, when the length of the curvature was increased form L= 0.5 to L= 1.0 , the free stream velocity was accompanied by a considerable increase from 0.275501 to 0.360971 as shown on the graph.

This is because as the length of the curvature increases, the velocity gradient also increases. Increase in velocity gradient increases the velocity of the fluid flow in consideration. When the length of the curvature was increased, the velocity gradient also increased and when the length of the curvature was reduced, the velocity gradient also decreased.

More so, when the velocity gradient is increased, the kinetic energy of the fluid particles went high at the boundary layer which implied that the fluid particles possessed high velocities.

From Figure 3, we note that when the length of the curvature was increased from L = 0.5 to L = 1.0, the heat dissipation in the boundary layer increased from 0.392678 to 0.572599.

This is because increase in the length of the curvature increases the velocity gradient which led to increase in shear stresses. The friction between the fluid particles and the surface in consideration was brought about by these shear stresses. In return, this friction force led to the dissipation of heat in the boundary layer. This was due to the fact that the shear stress is directly proportional to velocity gradient. i.e $\tau = \mu\frac{\partial u}{\partial y}$

From Figure 4, we note that as the Reynolds number increases from 0.7 to 1.3, a direct consequence of the increase in inertia forces occurred leading to increase in velocity from 0.297405 to 0.367155. When the Reynolds number is high, the inertia forces tend to dominated over the viscous force and consequently, the friction of the fluid particles and the surface in consideration was very minimal. This resulted to increase in velocity of the fluid flow. At large inertia forces, the velocity of the fluid tend to be high since low viscous forces implied that little or minimal friction existed between the fluid particles and the surface in consideration.

From Figure 5, we note that when the Reynolds number is increased from 0.8 to 1.3, the heat dissipation in the boundary layer reduced from 0.613144 to 0.508381.

This is because when the value of the Reynolds number was small, the inertia forces were very minimal. The viscosity of the fluid thus dominated over the inertia forces and consequently, the friction of the fluid particles with surface was high resulting to increase in heat dissipation within the boundary layer. When Reynolds number was high, the viscous forces were very minimal since inertia forces dominated in the fluid flow. Consequently, the friction of the fluid particles with the surface was minimal and this resulted to minimal dissipation of heat within the boundary.

7.3 CONCLUSION

Numerical investigations of the convective heat transfer in a laminar boundary layer over an immersed curved surface ha been carried out. The variations of the length of the curvature as well as the Reynolds number affected the velocity and temperature profiles in the laminar boundary layer.

When the length of the curvature was increased, this led to velocity and temperature rise. This matched the theoretical explanation since increase in velocity gradient increases the velocity of the fluid flow. Also at high velocity gradients, the shear stresses are high which brings about the friction between the fluid particles and the surface. Consequently, heat is dissipated. It thus follows that the length of the curvature is directly proportional to the velocity and temperature distribution.

It was also observed that at large Reynolds number, the inertia forces were high compared to the viscous effect of the fluid and consequently, the fluid velocity went high. This is in line with theoretical explanation, since at low viscosity, minimal shear stresses exist between the fluid particles and the surface and thus the velocity of the fluid is favoured. At low Reynolds number, the viscosity of the fluid is high since there are minimal inertia forces. Consequenltly, the fluid velocity is low. At high Reynolds number, the amount of heat dissipated at the boundary layer was minimal due to minimal friction between the fluid particles and the surface.

It therefore follows that Reynolds number is directly proportional to the velocity distribution and inversely proportional to the temperature distribution in the boundary layer.

8 RECOMMENDATIONS

It is recommended that further investigations be done in the following areas:

1. Compressible fluid flow over immersed curved surface
2. Convective heat transfer on turbulent fluid flows over immersed curved surface
3. Use of finite element method for solving the problem for more accurate results
4. Study of the same orientation but in three-dimensional aspect

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