Original Research Article

Estimation of the calorific power of a heating element

ABSTRACT
The present paper determines the calorific power of a heating source, consisting of a laterally isolated aluminum cylinder, which incorporates an internal electrical resistance for controlling the heat, adjusted to a preset temperature. For this, a glass vessel covering the heating surface and containing a certain mass of water is placed on the cylinder, while the thermal evolution of the water, is monitored by means of a thermocouple probe. We describe a simple model, based on the resolution of the differential equation for the heat balance associated with water, incorporating two gain and loss coefficients (the latter directly associated with the heating power required) and its application to the stationary response achieved at a given time with the final temperature steady. In this way and for different preset temperatures ranging from 30 to 70 ° C (in each case, the actual temperature of the source is determined by infrared thermometer), a table of power values is obtained, whose results can be used in other experiments that require it, such as those related to measures heat conductivities in discoidal samples.

Keywords: Heat capacity; heat balance; losses coefficient

1. Introduction

In thermal physics laboratories, it is sometimes necessary to determine the calorific power of heating sources without prior characterization. Such is the situation that arises when attempting to determine heat conductivities [1,2] corresponding to discoidal samples heated at different temperatures on each side, generating a heat flow to the opposite face and transmitted to another system, and whose thermal monitoring is possible to determine. This paper presents an experimental procedure to obtain the calorific power of a heating source with controllable temperature for different preset values, by means of a simple method based on the calorific balance between the source and a body in mutual contact [3], when the steady state is reached. The model incorporates two coefficients corresponding to gains and losses, respectively, for the heated element, the first being the coefficient directly related to the heating power and whose determination for each temperature allows the thermal characterization of the heating element.

2. Theory

Let us consider the case of a mass of water contained in a glass vessel on a heating element at temperature θ_c > room temperature θ_a, (see figure 1). Taking into account heat gains and losses from the system (glass vessel+water+accessories), expressed by λ_1 and λ_2 coefficients respectively the heating power transferred, is given by

$$\frac{\delta Q}{dt}=\left(mc_a+k\right)\frac{d\theta}{dt}=C\frac{d\theta}{dt}=\left(\frac{\delta Q}{dt}\right)_1+\left(\frac{\delta Q}{dt}\right)_2=\lambda_1\left(\theta_c-\theta\right)+\lambda_2\left(\theta_a-\theta\right) \tag{1}$$

where $\left(\delta Q/dt\right)_1$ is the supplied power to the system by the heated element and $\left(\delta Q/dt\right)_2$ is the power lost by the same, with $C\equiv mc_a+k$ the heat capacity of the calorimetric system (water+glass vessel+accessories), θ its temperature, m the water mass in the glass vessel, c_a the water specific heat and *k* the water equivalent of the system.

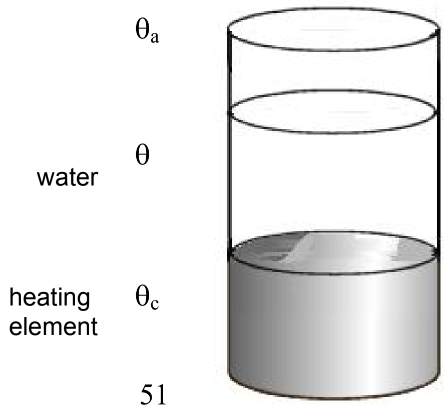

Fig. 1. System scheme heating element and glass vessel with water

Grouping the terms in the above equation, $-(\lambda_1+\lambda_2)\theta+\lambda_1\theta_c+\lambda_2\theta_a=C(d\theta/dt)$, and renaming $\alpha\equiv\lambda_1\theta_c+\lambda_2\theta_a$ and $\beta\equiv\lambda_1+\lambda_2$, gives

$$C\frac{d\theta}{dt}=\alpha-\beta\theta \quad\Rightarrow\quad \frac{Cd\theta}{\alpha-\beta\theta}=dt \tag{2}$$

Integrating between the initial temperature of the liquid (θ_o) and temperature (θ) at instant t, gives:

$$\int_{\theta_o}^{\theta}\frac{C\cdot d\theta}{\alpha-\beta\theta}=-t \quad\Rightarrow\quad \ln\left[\frac{\alpha-\beta\theta}{\gamma-\alpha\theta_o}\right]=-\frac{\beta t}{C} \quad\Rightarrow\quad \alpha-\beta\theta=(\alpha-\beta\theta_o)\exp(-\gamma t)\quad\left(\gamma=\frac{\beta}{C}>0\right)\quad\Rightarrow$$
$$\Rightarrow\quad \theta=\frac{\alpha}{\beta}-\left(\frac{\alpha}{\beta}-\theta_o\right)\exp(-\gamma t) \tag{3}$$

Therefore and according to the boundary conditions applicable to the system, ($t\to 0$, $\theta=\theta_o$ (initial temperature) and $t\to\infty$, $\theta=\theta_f=(\alpha/\beta)$ (final temperature)), equation (3) can be rewritten as

$$\theta=\theta_f-(\theta_f-\theta_o)\exp(-\gamma t)=A-B\exp(-\gamma t)\quad\begin{cases}A=\theta_f=\dfrac{\lambda_1\theta_c+\lambda_2\theta_a}{\lambda_1+\lambda_2},\\ B=\theta_f-\theta_o=\dfrac{\lambda_1(\theta_c-\theta_o)+\lambda_2(\theta_a-\theta_o)}{\lambda_1+\lambda_2},\\ \gamma=\dfrac{\beta}{C}=\left(\dfrac{\lambda_1+\lambda_2}{C}\right)\end{cases} \tag{4}$$

and consequently

$$(\lambda_1+\lambda_2)\theta_f=\lambda_1\theta_c+\lambda_2\theta_a \quad\Rightarrow\quad \frac{\lambda_2}{\lambda_1}=\frac{\theta_f-\theta_c}{\theta_a-\theta_f}=\theta_f^* \tag{5}$$

where a final reduced temperature θ_f^* has been introduced. In addition

$$C\cdot\gamma=\lambda_1+\lambda_2 \tag{6}$$

both equations (5) and (6) allow λ_1 and λ_2 coefficients to be obtained from adjustable parameters γ and θ_f, giving

$$\left.\begin{aligned}\lambda_1&=\frac{C\gamma}{1+\theta_f^*}\\ \lambda_2&=\lambda_1\theta_f^*\end{aligned}\right\} \tag{7}$$

According to (1), equations (7) show the energy gain and loss coefficients of the system, as well as the value of the power supplied by the heating element. It sufficient time elapsed to reach the steady state ($\theta=\theta_f$), the net heat exchange is cancelled out and, consequently, $(\delta Q/dt)=0$. Then, the power supplied by the heat source is

$$\left(\frac{\delta Q}{dt}\right)_1=\lambda_1\left(\theta_c-\theta_f\right) \tag{8}$$

2.1. Obtaining the k value: mixtures method.

To calculate the power supplied by the heating element, as described above, it is necessary to know the value of the water equivalent of the system. To do this, we use the mixtures method [4], which consists of introducing into a glass vessel containing water at room temperature, a known metal body previously heated in a recipient of boiling water (100 °C at normal atmospheric pressure) [5]. In this way, when the body is introduced into the problem system, the water temperature increases. After a time interval Δt, the body is removed from the system and the water cools down towards environment temperature. figure 2 outlines the process with its three stages: preheating, heating and post-heating. In our case the duration of these stages was 20, 1 and 20 minutes, respectively, for which the corresponding thermogram is shown in figure 3.

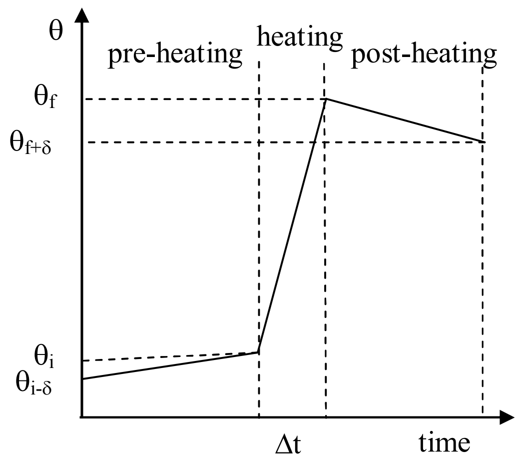

Fig. 2. Thermal evolution diagram of the water in a mixture process. θ_i: initial temperature, θ_f: final temperature, δ: pre and post-heating time interval.

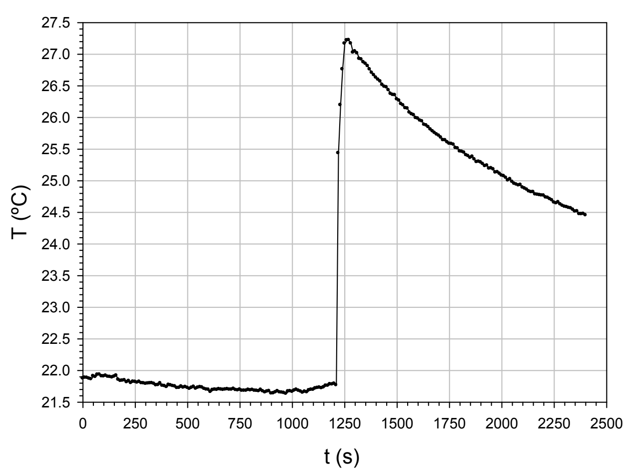

Fig. 3. Thermogram obtained to calculate the water equivalent value of our system, obtained by introducing a metal body at 100 ºC

The energy balance for the heating stage is described by the equation

$$Mc_M(\theta_f-\theta_i)=(m_a c_a+k)(\theta_f-\theta_i)+\dot{Q}_p\Delta t \quad (9)$$

where M is the metal body mass, c_M is its specific heat [6], m_a is the water mass, c_a its specific heat, θ_i and θ_f are the initial and final temperature corresponding to the time interval Δt, and $\dot{Q}_p$ refers to heat losses per unit of time in this stage, which can be determined as the arithmetic mean of the losses in the pre and post-heating stages:

$$\dot{Q}_p=\left[\left(\dot{Q}_p^{pre}+\dot{Q}_p^{post}\right)/2\right]$$

Preheating stage: thermal stabilization of the glass vessel+water system tending to temperature θ_a,

$$\dot{Q}_p^{pre}=(m_a c_a+k)\left[\frac{\left[(\theta_i-\theta_{i-\delta})\right]}{20}\right]$$

where $\theta_{i-\delta}$ and θ_i are the initial and final temperature of this stage.

Post-heating stage: removal of the metal body and thermal monitoring cooling process of the water,

$$\dot{Q}_p^{post}=(m_a c_a+k)\left[\frac{\left[(\theta_{f+\delta}-\theta_f)\right]}{20}\right]$$

where θ_f and $\theta_{f+\delta}$ are the initial and final temperature of this stage.

Thus, the energy balance given by equation (9) is

$$Mc_M(\theta_f-\theta_i)=(m_a c_a+k)\left[(\theta_f-\theta_i)+\frac{\left[(\theta_f-\theta_{i-\delta})+(\theta_{f+\delta}-\theta_i)\right]}{40}\right] \quad (10)$$

This equation allows us to determine the value of the equivalent k, when the mass M of the metal used and its specific heat c_M are known.

3. Experimental device

We have designed a device (figure 4) with an aluminum cylinder of 2 cm thickness as a heating element isolated with armaflex, containing an electrical resistance as heating element. The control temperature θ_c (temperature setting) is measured with a type J thermocouple probe (iron–constantan, with - 210 to 1200 °C range) incorporated into the cylinder. The resistance and thermocouple are connected to a control device, equipped with display showing the instantaneous values of θ_c^* (nominal control temperature). This temperature is the magnitude controlling the heat flow, while the heating element directs and regulates the heating process of the system. At every instant another type J thermocouple records the temperature θ of the deionized water (40 cm^3) contained in a glass vessel (wall 2 mm), placed on top of the heater cylinder. In the steady-state, the temperature reaches a value θ_f for each considered θ_c^*. The real temperature values on this surface of the heating cylinder (θ_c), for each of the nominal control values selected (θ_c^*), were measured with an infrared thermometer of configurable emissivity (Optris® LS with dual focus infrared light) [7], provided with a laser marking device, which points to the upper part of the cylinder (or element whose surface temperature is required) projecting a cross. This acts as a guide for the measurement distance (distance of the object (D) and size of the area of focal measurement (S), whose d: s ratio is in our case 75:1 the disc in studio) (figure 5).

4. Results and discussion

Appling eq. (10) to 40 cm^3 water permits the determination of the water equivalent of the calorimetric system (k = 14.623 g) and taking into account that $C\equiv mc_a+k$, it follows that C = 64.623 cal g^{-1} $°C^{-1}$. This value, together with the γ parameter of the exponential fitting of the corresponding thermograms for the system, (over a range of temperatures θ_c^* from 30 to 70 ºC, at 5 ºC intervals) allows calculation of the coefficients λ_1 and λ_2 (eq. (7)), and, consequently, according to eq. (8) determination of the corresponding heating power. Figure 6, shows the thermogram corresponding to the value θ_c^* = 50° C.

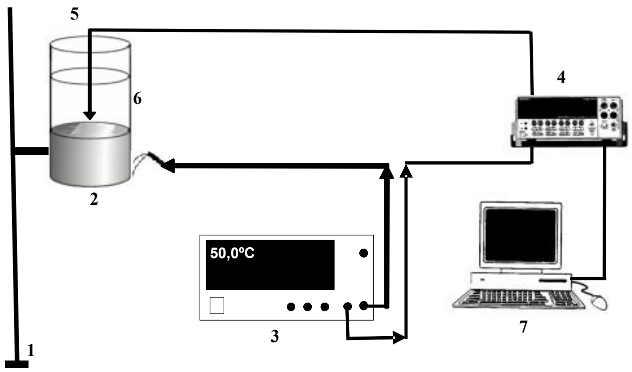

Fig. 4. Experimental set up for monitoring the evolution of the heating power of the heater element: 1) Support. 2) Heater cylinder with insulation. 3) Thermal control device (temperature of the heating element). 4) Multimeter. 5) Type J thermocouple probe in water. 6) Glass vessel. 7) PC.

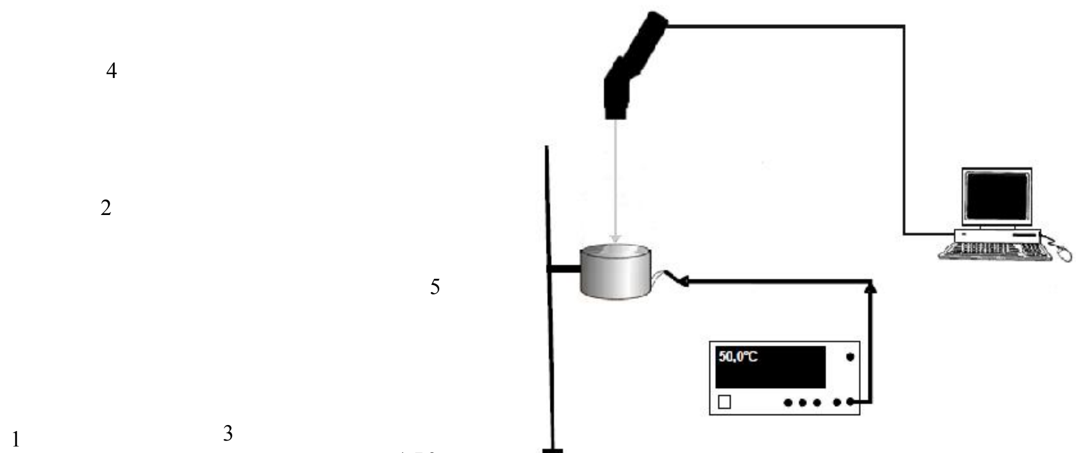

Fig. 5. Experimental set up for the measurement of the surface temperature of the heating element (θ_c): 1) Support. 2) Heater cylinder with insulation wrap. 3) Thermal control device of temperature setting (temperature of the heating element). 4) IR thermometer. 5) PC.

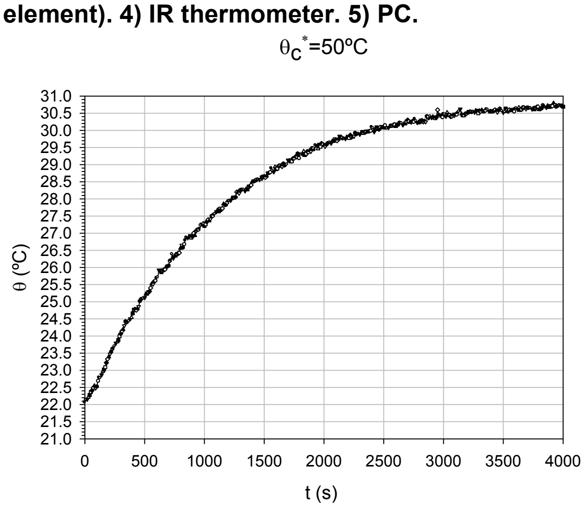

Fig. 6. Thermal evolution of the calorimetric ensemble (glass+water+accessories) at a nominal temperature θ_c^*=50 ºC

Table 1 shows the values of measured temperature for every control temperature θ_c^* considered [30-70 ºC]. In all cases, 4000 s was sufficient to reach the steady state.

Table 1. Thermal evolution of the system for the different control temperatures θ_c^* considered.

	Nominal control temperature θ_c^* (ºC)								
	30	35	40	45	50	55	60	65	70
t (s)	**Temperature θ (ºC) according to time**								
0.00	22.43	21.36	21.17	22.00	22.06	23.23	23.11	23.45	23.19
500.00	22.57	22.75	23.35	24.44	25.12	26.65	27.13	28.18	28.84
1000.00	22.69	23.61	24.78	26.02	27.14	28.94	29.74	31.01	31.99
1500.00	22.78	24.20	25.70	27.06	28.49	30.42	31.42	32.79	33.90
2000.00	22.85	24.61	26.29	27.73	29.37	31.38	32.51	33.90	35.06
2500.00	22.90	24.88	26.66	28.17	29.94	32.01	33.22	34.59	35.75
3000.00	22.94	25.00	26.90	28.46	30.32	32.41	33.67	35.02	36.20
3500.00	23.00	25.13	27.06	28.65	30.56	32.67	33.97	35.17	36.48
4000.00	22.98	25.19	27.17	28.80	30.69	32.81	34.16	35.41	36.64

Figure 7 illustrates the different thermograms for the θ_c^* values shown in Table 1.

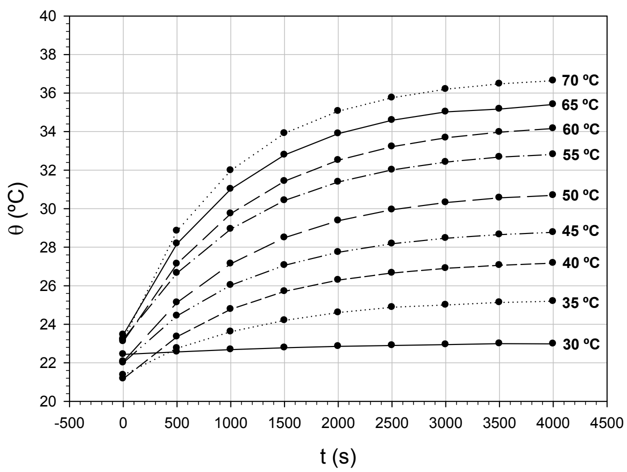

Fig. 7. Fitting experimental curves (eq. (3)) for every control temperature θ_c^*. The experimental points shown correspond to 500 s intervals between 0 and 4000 s.

Table 2: Experimental values for the fitting parameters corresponding to different control temperature θ_c^* shown in Table 1. r, correlation coefficient.

θ_c^* (ºC)	A (s)	B (s)	$\gamma\times10^4$ (s^{-1})	r^2
30	24.890	0.6605	5.006	0.9969
35	25.334	3.964	8.420	0.9998
40	27.355	6.185	8.772	0.9999
45	29.000	7.000	8.555	1.0000
50	31.027	8.977	8.405	0.9999
55	33.167	9,947	8,576	0.9999
60	34.510	11.400	8.708	1.0000
65	35.640	12.180	9.728	0.9999
70	36.810	13.570	10.38	0.9998

Table 3 shows experimental results for characteristic temperatures, initial (θ_o), room (θ_a) and final (θ_f), as well as the loss and gain coefficients, and heating power calculated from eq. (8).

Table 3: Characteristic temperatures, gain and loss coefficients and calorific power for each control temperature θ_c and θ_c^*.

θ_c^*	θ_c	θ_a	θ_o	θ_f	$\Delta\theta=(\theta_c-\theta_f)$	$\gamma C=(\lambda_1+\lambda_2)$	$\theta_f^*=(\lambda_1/\lambda_2)$	λ_1 (w/ºC)	λ_2 (w/ºC)	Pot (w)
30	29.5	18,9	22.4	23.0	6.5	0.0324	1.5854	0.0125	0.0198	0.3400
35	34.1	21.3	21.4	25.2	8.9	0.0544	2.2821	0.0166	0.0378	0.6168
40	39.4	21.9	21.2	27.2	12.2	0.0567	2.3019	0.0172	0.0395	0.8755
45	44.4	22.0	22.0	28.8	15.6	0.0553	2.2941	0.0168	0.0385	1.0944
50	49.5	21.9	22.1	30.7	18.8	0.0543	2.1364	0.0173	0.0370	1.3609
55	54.3	23.1	23.2	32.8	21.5	0.0554	2.2165	0.0172	0.0382	1.5485
60	59.1	23.1	23.1	34.1	24.9	0.0563	2.2727	0.0172	0.0391	1.7969
65	63.7	23.5	23.5	35.4	28.3	0.0629	2.3782	0.0186	0.0443	2.2014
70	68.4	23.5	23.2	36.6	31.8	0.0671	2.4275	0.0196	0.0475	2.6014

Figure 8 shows the final temperature (θ_f) *versus* control temperature values (θ_c) of the heating source and the straight line fitted by less squared analysis.

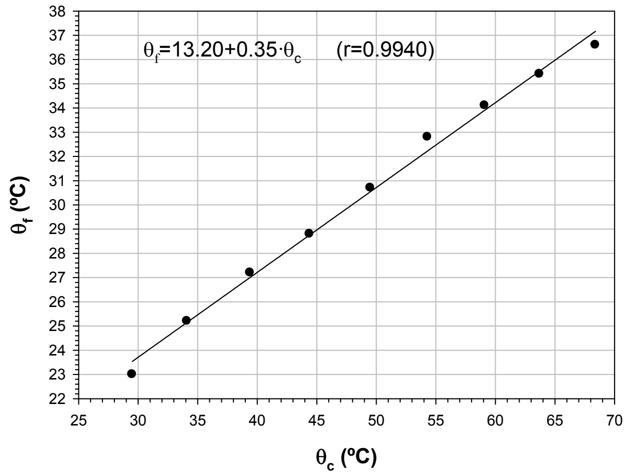

Fig. 8. Final temperature (θ_f) *versus* control temperature (θ_c) of the heating element.

The behaviour of calculated powers against the final temperature θ_c when steady state is reached, is shown in figure 9.

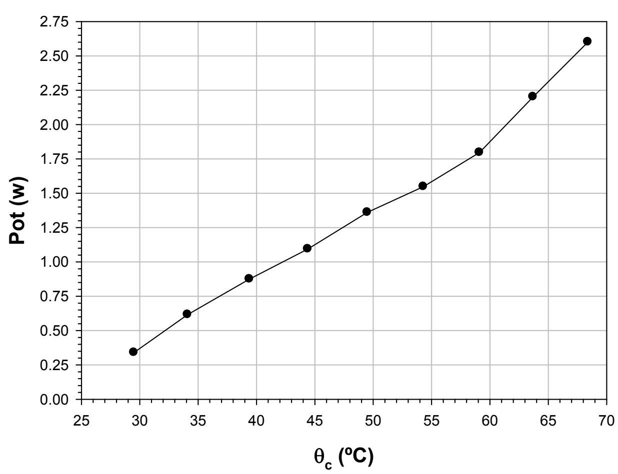

Fig. 9. Power for the heating system *versus* control temperature θ_c.

The values obtained from eq. (7) for the gain (λ_1) and loss (λ_2) coefficients *versus* the control temperature (θ_c) of the heating element are shown in figure 10. Note the existence of plateau for values 35<θ_c<60 °C, with an average value (<λ_1>= (170±2)·10^{-4} w/ºC) based on data of Table 3. The mean power for the plateau according to eq. (8) and taking into account the equation in figure (8), gives

$$< \mathrm{Pot} > = < \lambda_1 > \left[0{,}65 \cdot \theta_c \text{-} 13{,}20\right] \qquad (12)$$

This equation allows determination of <Pot> for every control temperature θ_c. The results are listed in Table 4 and illustrated in figure 11.

Table 4: Mean heating power for the different control temperatures considered. Also, the final associated temperatures are shown.

θ_c	34.1	39.4	44.4	49.5	54.3	59.1
θ_f	25.2	27.2	28.8	30.7	32.8	34.1
<Pot> ± 0,03 (w)	0.64	0.88	1.12	1.35	1.57	1.80

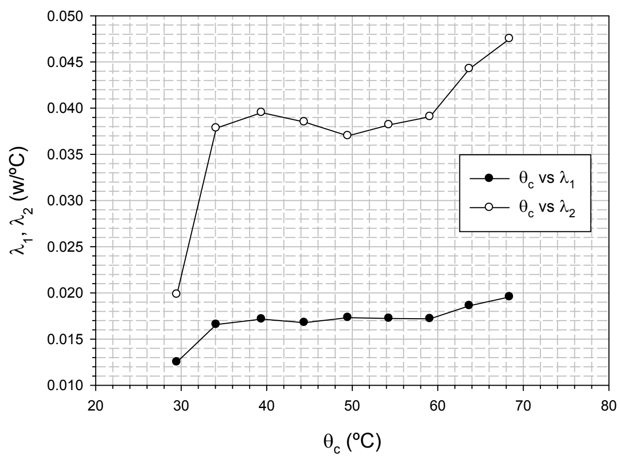

Fig. 10. Gain (λ_1) and loss (λ_2) coefficients *versus* control temperature of the heating source.

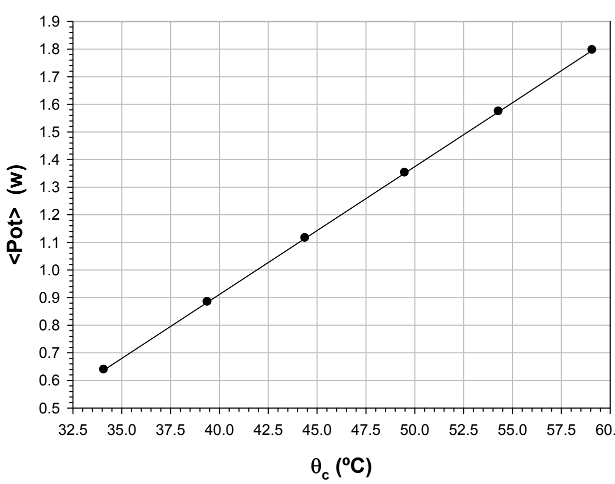

Fig. 11. Mean power values <Pot> as a function of the control temperature θ_c.

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