Original Research Article

Intermittency of regular and chaotic motion in the dynamic system with multiple Lorenz attractors

ABSTRACT: The article considers the regime of intermittency "quasi-periodic motion – chaos" in a dynamic system with compound chaotic multiattractor uniting several attractors of Lorenz. The possibility of changes in the parameters of this mode in a wide range by the variation of a single parameter.

Keywords: *multiattractor, composite multiattractor, intermittency, chaotic motion, quasi-periodic motion, Lorenz attractor.*

1. INTRODUCION

The study of the unpredictable alternation of regular and chaotic behaviour of dynamical systems is one of the important problems of nonlinear dynamics. This phenomenon is known as intermittency, is associated with different types of interaction of attractors, and manifests itself, in particular, in the the form of intermittency "quasiregular motion - chaos" [1-4], and intermittency of "chaos-chaos" [5-8].

One of the classes of dynamical systems with continuous time, which is characterized by the alternation of regular and chaotic regimes are the system of ordinary differential equations, the movement of which occurs in the so-called compound (or composite) chaotic multiattractors, which is a limit set consisting of a number of regions of attraction (local chaotic attractors), in all of which phase trajectory stays for long enough, making chaotic oscillations, and, from time to time, making the transitions between neighboring regions [8-10]. The dynamics of such systems is an alternation of two types of movement – chaotic on the local attractors and, as a rule, the regular – during the transitions between them.

Usually, because of the short duration of the episodes of transition movements from one local attractor to another, the observation of intermittency in such systems is difficult, resulting in their dynamics appears as a collection of chaotic fluctuations on the local attractors and fast erratic hopping of movement from one of them to another. However, in some cases, may be a considerable increase of the time of transitions, with the result that the intermittency manifests itself quite clearly.

2. INTERMITTENCY “QUASIREGULAR MOTION - CHAOS” IN THE DYNAMIC SYSTEM WITH MULTIPLE LORENZ ATTRACTORS

For example, consider the following dynamic system have the amounts of the composite chaotic multiattractor consisting of attractors of Lorenz [8]:

$$\begin{cases} \dfrac{dx}{d\tau} = A\left[H(\mu x + y) - \mu x - x\right]; \\ \dfrac{dy}{d\tau} = x(B - z) - H(\mu x + y) + \mu x; \\ \dfrac{dz}{d\tau} = \left[H(\mu x + y) - \mu x\right]x - Cz\ . \end{cases} \qquad (1)$$

where

$$H(\xi)=\xi+(d+1)\left\{P\left(\xi+s+h+\frac{h}{d}\right)+P\left(\xi+s-h-\frac{h}{d}\right)-\right.$$
$$\left.-\sum_{m=0}^{M}\left[P\left(\xi+s-(2m-1)\left(h+\frac{h}{d}\right)\right)+\frac{h}{d}\right]-\sum_{n=0}^{N}\left[P\left(\xi+s+(2n-1)\left(h+\frac{h}{d}\right)\right)-\frac{h}{d}\right]\right\}, \qquad (2)$$

$$P(\xi)=\frac{1}{2}\left(\left|\xi+\frac{h}{d}\right|-\left|\xi-\frac{h}{d}\right|\right),$$

– replicates (reduplicate) operator creates copies of the attractor of the original dynamical system, ordered by coordinate $\xi=\mu x+y$, where μ is a real constant, and their merger into a single multiattractor. It represents a nonlinear function consisting of *1+M+N* line segments of unit slope, connected by more steep intermediate segments with slope *-d*.

The number of local attractors in the multiattractor of system (1), (2) is equal to the number of line sections with a single slope. Each of them is inside its own region of phase space (phase cell), with a length of *2h* in the coordinate ξ. The constant s accounts for the asymmetry of the local attractors relative to the center of your cell. The coefficient d determines the width of the transition layer the phase space between adjacent cells (equal to *2h/d*) [8].

Let *A=10.5, B=33.2189, C=3/8*, *M=1, N=0, h=22, d=10, s*=0. In this case, the replicate operator is a nonlinear function of the variable ξ containing two line segments with unit slope, connected by an intermediate segment with a slope *-d* (Fig.1), and the system (1) has the simplest composition multiattractor containing two local chaotic attractors (Fig.2).

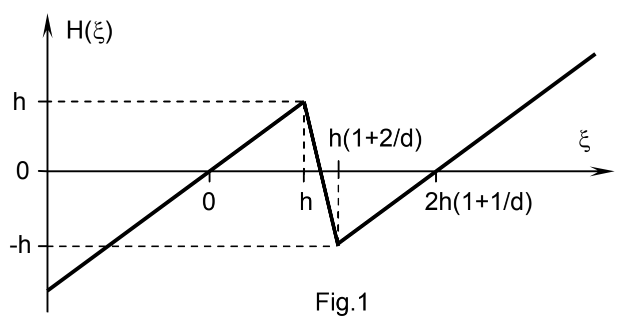

Fig.1

Consider the evolution of such multiattractor when you change the value of constants μ. When $\mu < -0.2$, transitions of the phase point between the local attractors occur along short smooth segments (Fig.2, a). In the result the phases of regular movement look like as a fast direct transition of the phase point from one of the local chaotic at-tractor on the other.

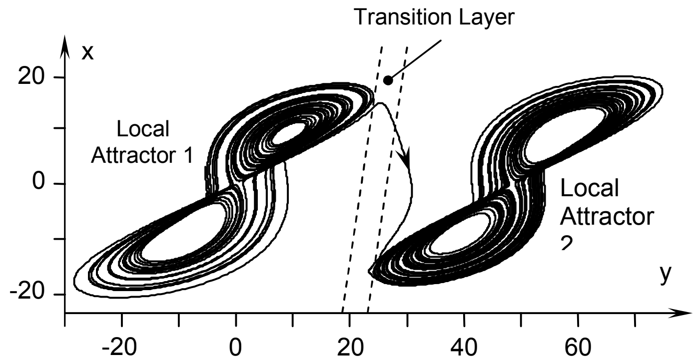

Fig.2,a. Example of the transition movement in the system (1), (2) from local chaotic attractor 1 on the local chaotic attractor 2 when μ = *-0.2.*

However, if you increase the value of this parameter to -0.15 phase trajectories begin to twist around the unstable cycle, which owes its existence to nonlinearity of the replicate function. First, when $\mu \approx -0.15$, trajectories manage to do a maximum of one turn before it gets into the region of attraction of one of the local attractors and are attracted to it (Fig.2, b).

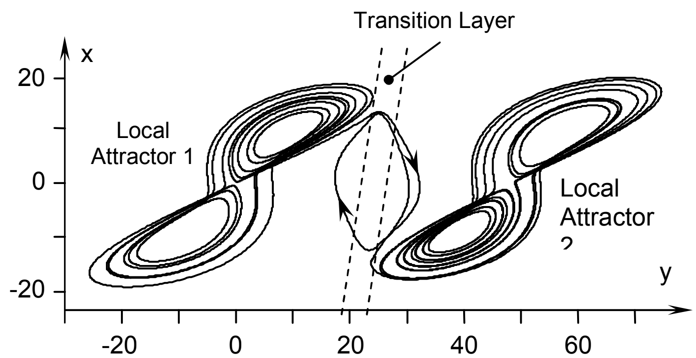

Fig.2,b. Example of the transition movement in the system (1), (2) from local chaotic attractor 1 on the local chaotic attractor 2 when μ = *-0.15.*

With the increase of this coefficient the maximum number of turns of the trajectories increases. Accordingly, increasing the average time of regular motion in the neighborhood of this cycle. In the timing diagram appear long sections of quasiperiodic oscillations (Fig.3). When $\mu \approx -0.1$ cycle becomes stable. Now the phase trajectory, once finding himself in region of its attraction can not leave. That is, the case $\mu \geq -0.1$ corresponds to the global metastability of the system (1), (2). A movement that begun on any of the local chaotic attractors, through the end time will always reach a stable cycle corresponding to regular oscillations.

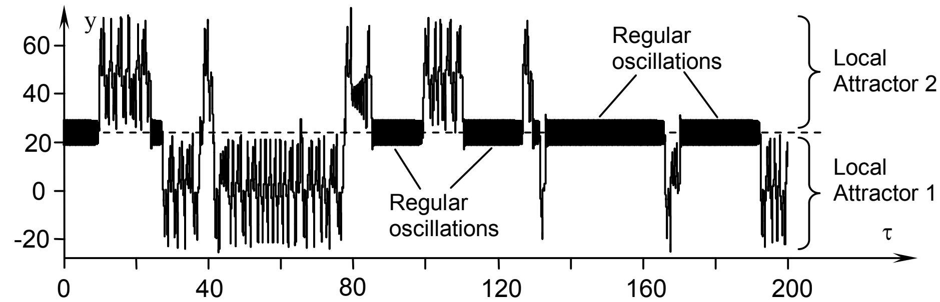

Fig.3, a. An example of time dependencies of dynamic variable y of system (1), (2) when μ = *-0.109*.

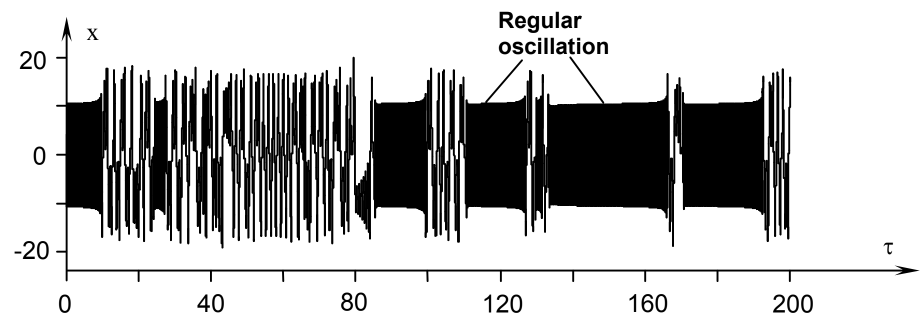

Fig.3, b. An example of time dependencies of dynamic variable *x* of system (1), (2) when μ = *-0.109*.

Thus, in the interval of values of the coefficient μ from about -0.15 to -0.1 for the chosen values of the other constants, the system (1), (2) shows a typical example of intermittent dynamics. If the value μ close to -0.1 observed long laminar phases of motion, during which the number of revolutions of the phase trajectory around the unstable cycle can be very large (Fig.3).

The same behavior of the system (1), (2) is observed in the General case of an arbitrary number of local attractors in the composition of multiattractor [8].

3. STATISTICAL CHARACTERISTICS

Random variables that can be investigated by statistical methods to description of the phenomenon of intermittency in dynamical systems that have multiple chaotic multiattractor are the duration of individual episodes of motion on the chaotic attractors and in the vicinity of the regular attractors, part of multiattractor.

In the presentase the most interest are the dependence of the relative total time of the regular movements of the value of the constant μ and frequency distribution of durations of regular and chaotic motions.

The relative total duration of regular motion is equal to $to_{reg} = \lim\limits_{T_\Sigma \to \infty} \frac{\sum_i T_{reg\,i}}{T_\Sigma}$, where T_Σ – total time of observation, $T_{reg\,i}$ – duration of the i-th episode of a regular movement.

The frequency distribution in this case represents the relationship "the number of episodes of movement on the selected attractor – the duration of these episodes" for the observation time T_{Σ} at $T_{\Sigma} \to \infty$.

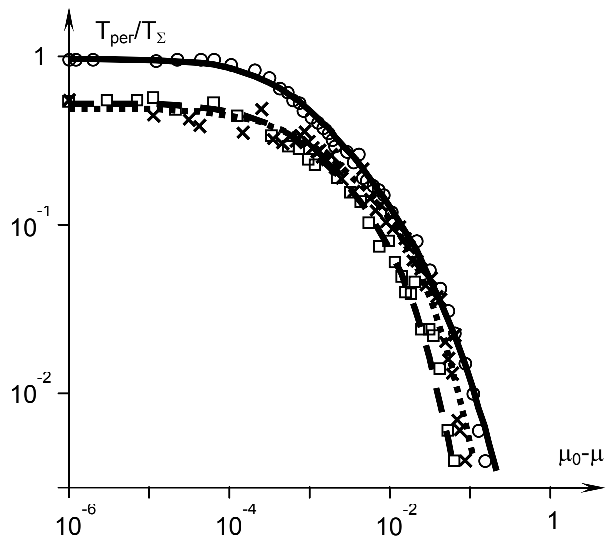

Fig.4. The dependence of the relative total time to the regular movements of the value of the constants μ at *d=10* (□ - numerical data, dashed line – approximation by function (3)), *d=100* (x - numerical data, small dashed line – approximation by function (3)), $d=\infty$ (o - numerical data, solid line – approximation by function (3)). μ_0 – limit constant value μ, above which the regular oscillations become stable (for *d=10* $\mu_0 \approx -0.10088$, for *d=100* $\mu_0 \approx -0.09966$, for $d=\infty$ $\mu_0 \approx -0.1002$).

The dependence $to_{reg}(\mu)$ for three values of the slope of the intermediate segment of the replicate function ($d=10$, $d=100$, $d=\infty$) is shown in Fig.4. A characteristic feature of this dependence is the existence of the limit of the maximum value to_{reg} when $d<\infty$. For example, for *d=10* and *d=100* the percentage of time consumed on a regular traffic may not exceed approximately *0.55*. In the case of discontinuous replicate function the upper limit of to_{reg} equal to *1*.

Note that these dependences are satisfactorily approximated by functions of the form

$$to_{reg} = \frac{\alpha}{(|\mu| - \beta)^{\delta} |\mu|^{\lambda}}, \qquad (3)$$

where – $\alpha, \beta, \delta, \lambda$ – are positive constants

For the dependence corresponding to d=10 (Fig.4), these constants have the following values: $\alpha=1.6 \cdot 10^{-8}$, $\beta=0.1005$, $\delta=0.45$, $\lambda=6$. For the dependence corresponding to d=100, these constants have the following values: $\alpha=3 \cdot 10^{-6}$, $\beta=0.0993$, $\delta=0.35$, $\lambda=4$. For the dependence corresponding to $d=\infty$, they are equal $\alpha=1.5 \cdot 10^{-4}$, $\beta=0.09975$, $\delta=0.6$, $\lambda=1.8$.

Frequency distribution of durations of episodes of motion on the chaotic attractors is shown in Fig.5. They show that the duration of motion on the chaotic attractors are concentrated within a limited interval within which appreciable secondary concentration ravnodushie with each other the highsThe values of the maximums are approximately uniformly distributed throughout the interval. The equality of intervals between the peaks is due to the fact that the visit of the phase point of the intersection area of the chaotic attractor with the boundary of its phase cell is mostly quasi-periodic character. Any pronounced dependence of these distributions from μ not observed.

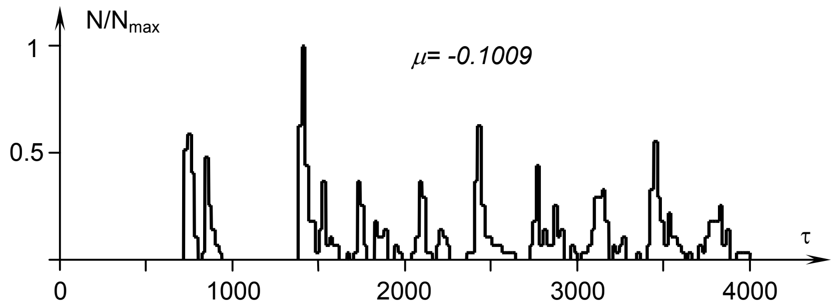

Fig.5, a. Frequency distribution of durations of episodes of movement on the local chaotic attractor 1.

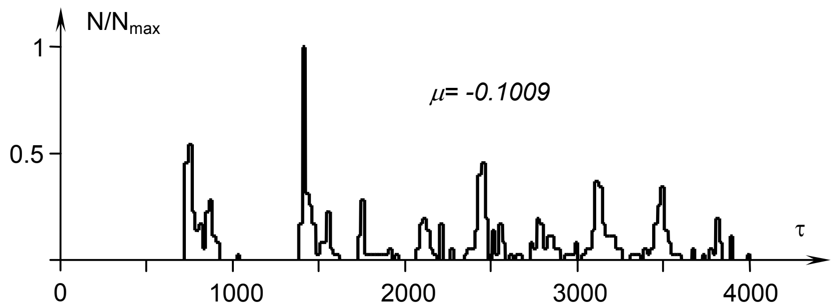

Fig.5, b. Frequency distribution of durations of episodes of movement on the local chaotic attractor 2.

In Fig.6 shows the frequency distribution of durations of episodes of regular motion, including at least one rotation of the trajectory around the unstable cycle, with μ*=0.1009, 0.109* and *0.125*, which, according to Fig.4, corresponding to values of relative total duration of regular movement toper approximately equal to *0.55, 0.1* and *0.03*. It is seen that these distributions have an exponential character. That is, the duration of episodes of regular movement in General are concentrated near the minimum value, which is equal to time of one rotation of the phase trajectory around the unstable cycle ($\tau_{turn}\approx 90$). Also it is clearly seen that the distributions consists of significantly more highly expressed compared to the distributions in Fig.6, the individual concentrations, separated by equal intervals of τ_{turn} /2, which is a direct consequence of the quasi-periodic nature of the regular movement. (The fact that neighboring maxima separated by intervals of length exactly τ_{turn} /2, due to the fact that for every revolution, the trajectory passes through the vicinity of two areas of contact of regular manifolds with chaotic attractors).

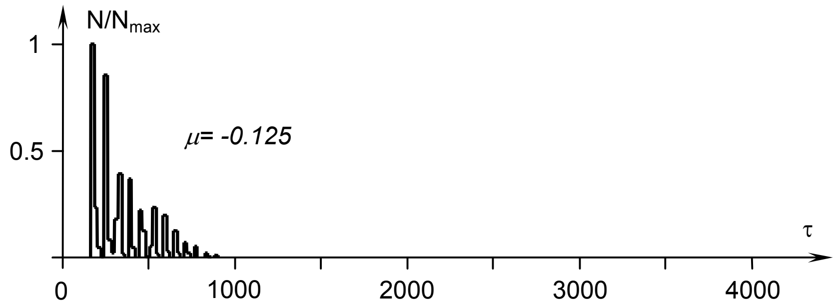

Fig.6, c.

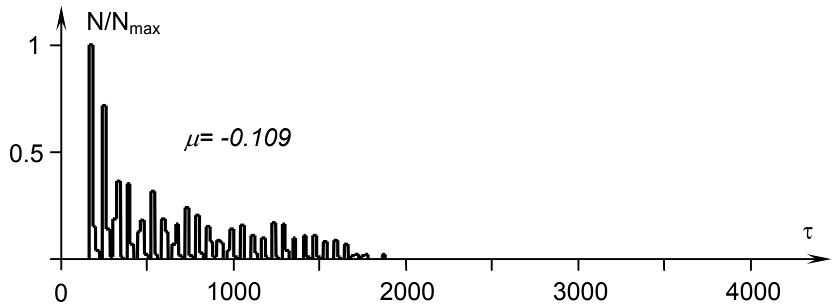

Fig.6, b.

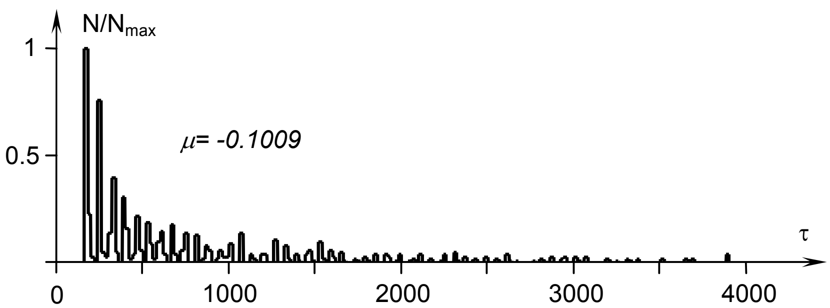

Fig.6, a.

A comparison of these distributions corresponding to different values of the constant μ, shows their strong dependence on to_{reg}. With the reduction in relative overall duration of regular motion the distribution of the lengths of its intervals is substantially compressed by the ordinate. From Fig.6 seen that when μ changing from *-0.1009* to *-0.125* (in this case to_{reg} is reduced from *0.55* to *0.03* – see Fig.5) maximum observed length of intervals of regular motion is reduced four times – from *4000* to *1000*.

3. THE MECHANISM OF INTERMITTENCY

The reason for the alternation between chaotic and laminar phases of the movement in the system (1), (2) is the coexistence of interacting attractors of two types (chaotic and regular) that are in a metastable state, and having such a mutual position that the phase trajectory, leaving the attractor of the same type always appears in the region of attraction of the attractor of another type.

Metastability of regular motion due to instability of the corresponding limit cycle. Metastability of the local chaotic attractors induced by the choice of size of the containing cell of the phase space, so that each of them had crosses the boundaries of its cell, causing the phase trajectory gets the opportunity to leave a local attractor through the area of its intersection with the border of the cell [8].

Therefore, the mechanism for intermittent oscillations in dynamic systems that have composite chaotic multiattractors, can be described as follows.

For example, the initial conditions chosen in the domain of attraction of one of the local chaotic attractors. Then, the phase point coming on this attractor will have some time to make chaotic motion on it, until it leaves it through the intersection with the boundary of the phase cell. Getting off a chaotic attractor she gets into the region of attraction of the unstable limit cycle and starts a quasi-periodic motion in its surroundings. Because of the instability cycle, the magnitude of the momentum of the phase trajectory around it over time begins to grow (Fig.3,b) with simultaneous displacement of the region of rotation of the phase trajectories at the unstable manifold – until the phase trajectory crosses the border of the region of attraction of one of the local attractors and be attracted to it. Further, the movement continues on a chaotic attractor, while the phase trajectory will go beyond the boundaries of the containing it cell of the phase space and does into the region of attraction of the cycle, and again started to make momentum around it. The result is a typical pattern of intermittency "quasi-periodic motion – chaos" (Fig.3).

4. CONCLUSION

Thus, the alternation of regular and chaotic types of motion underlying the dynamics of systems with composite chaotic multiattractor, can manifest itself very clearly, and the degree of its manifestation controlled by a small number of control parameters (in the considered case, by the value of a single constant μ).

REFERENCES

1. Pomeau Y, Manneville P. Intermittent transition to turbulence in dissipative dynamical systems. Commun. Math. Phys. 1980;74:189-197.
2. Kim CM, Yim GS, Kim YS, Kim JM, Lee HW. Experimental evidence of characteristic relations of type-I intermittency in an electronic circuit. Phys. Rev. E. 1997;56(3):2573–2577.
3. Baptista MS, Caldas IL. Type-II intermittency in the driven double scroll circuit. Phys. D. 1999;132:325–338.
4. Cavalcante HLDS, Rios Leite JR. Averages and critical exponents in type-III intermittent chaos. Phys. Rev. E. 2002;66(026210):5.
5. Anishchenko VS, Neiman AB. The Increase of duration of correlations in intermittency of the “chaos-chaos”. Technical physics Letters. 1987;13(17):1063-1066. Russian.
6. Anishchenko VS. Interaction of strange attractors. The intermittency of “chaos-chaos”. Technical physics Letters. 1984;10(10):629-633. Russian.
7. Afraimovich VS, Rabinovich MI, Ugodnikov AD. Critical point and "phase transitions" in the behavior of non-Autonomous stochastic anharmonic oscillator. J. of Appl. 1983;38(2):64-67.
8. Prokopenko VG. Reduplication of chaotic attractors and construction composite multiattractors. Nonlinear dynamics. 2012;8(3):483-496. Russian.
9. Prokopenko VG. The Formation of Composite Chaotic Multiattractors Containing Inhomogeneities. Technical Physics. 2017;62(8):1139–1147.
10. V.G. Prokopenko. The generator of chaotic oscillations, RF Patent No. 2403672. Bull. Izobret. No. 31, 2010. Russian.
11. V.G. Prokopenko. The generator of chaotic oscillations, RF Patent No. 2421877. Bull. Izobret. No. 17, 2011. Russian.

12. V.G. Prokopenko. The generator of chaotic oscillations, RF Patent No. 2540817. Bull. Izobret. No. 4, 2015. Russian.