

# Entropy Is the Sum of Heat Capacities

## ABSTRACT

An attempt is made to explain the meaning of entropy in thermodynamics. A new concept of heat capacity is defined. For it, the temperature change is measured from 0 kelvin. It is supposed that the entropy of a substance is the sum of these heat capacities in the formation of the substance from 0 kelvin to the actual temperature. This conclusion agrees with experimental data.

**Keywords:** *entropy; heat capacity; enthalpy*

## 1. INTRODUCTION

The physical meaning of entropy in thermodynamics is still unknown [1]. In this paper, an effort is made to explain what entropy is.

## 2. THEORY

The change in entropy is given by this equation:

$$dS = \frac{\delta q}{T} \quad (1)$$

where  $\delta q$  is the heat introduced into the substance, and  $T$  is the absolute temperature. It has the following meaning. One introduces some heat at the temperature  $T - dT$  and the temperature of the substance increases to  $T$  ( $\delta q$  and  $dT$  are sufficiently small). One can notice that (1) can be rewritten as

$$dS = \frac{\delta q}{T - T_0} = \frac{\delta q}{\Delta T} = C_0(T) \quad (2)$$

where  $T_0 \rightarrow 0$  K. Equation (2) has the following meaning. The change in entropy  $dS$  is the heat capacity of the substance  $C_0(T)$  when the heating is calculated in the interval from 0 to  $T$  kelvin. This means that 0 K is the reference point, and one takes into account the heat introduced into the substance in the interval from 0 to  $T$  that is needed to increase its temperature from  $T - dT$  to  $T$ . It differs from the usual heat capacity where the reference point is  $T - dT$ .

Therefore, the absolute value of the entropy will be

$$S \approx \sum_i \frac{\delta q_i}{T_i - T_0} = \sum_i C_0(T_i) \quad (3)$$

where  $T_i = 1, 2, 3, \dots, K$ , and  $\delta q_i$  is the quantity of heat needed to increase the temperature from  $T_{i-1}$  to  $T_i$ . In the continuous case, (3) turns into the well-known formula:

$$S = \int_{T_0}^T \frac{C_P(T) dT}{T - T_0} \quad (4)$$

where  $C_P$  is the isobaric heat capacity.

There is the first mean value theorem for definite integrals [2]. If the functions  $g(x)$  and  $f(x)g(x)$  are integrable functions on  $[a, b]$ ,  $f(x)$  is a bounded and a continuous function, and  $g(x)$  does not change sign on  $[a, b]$ , then there exists  $c$  in  $[a, b]$  such that

$$\int_a^b f(x)g(x)dx = f(c)\int_a^b g(x)dx. \quad (5)$$

Therefore, one can rewrite (4):

$$S = \left\langle \frac{1}{T} \right\rangle \int_{T_0}^T C_P(T)dT = \frac{H}{T_{\text{mean}}} \quad (6)$$

where  $T_{\text{mean}}$  is the temperature which corresponds to the mean value of the inverse temperature, and  $H$  is the enthalpy.

In [3], the heat is considered for 77 solids needed to raise a substance from absolute zero temperature to standard conditions ( $T^\circ = 298.15$  K and atmospheric pressure):

$$H^\circ = \int_0^{T^\circ} C_P(T)dT \quad (7)$$

Here, the superscript  $^\circ$  denotes the standard conditions. It is remarkable that this enthalpy is very nearly proportional to the standard entropy at 298.15 K for a wide variety of solids:

$$\frac{S^\circ}{H^\circ} = 0.0066 \text{ K}^{-1}. \quad (8)$$

Monatomic, diatomic, triatomic, heavy polyatomic ionic solids, and complex covalent solids have data points on or near the best-fit line given by (8). The value  $0.0066 \text{ K}^{-1}$  corresponds to  $T_{\text{mean}} = 151.5 \text{ K} \approx 298/2 \text{ K}$ . It is notable that the average function  $\langle 1/T \rangle$  has such a simple form. This result agrees with the above definition of entropy, (3); it is plausible to assume that

$$S(T) \approx \sum_i \frac{\delta q_i}{0.5T - T_0}. \quad (9)$$

Here one can draw an analogy with the Lagrangian and Eulerian specifications of the flow field [4]. In the former, the observer follows an element of fluid as it moves, in the latter, the fluid's motion is observed from a fixed reference point. The traditional heat capacity corresponds to the Lagrangian method, and the heat capacity  $C_0$  corresponds to the Eulerian one (the observer is at 0 K).

In [5,6] it was inferred that in the description of natural phenomena their development from 0 K must be taken into account. It is not enough to describe the phenomenon at the current moment, it is necessary to include its history from zero kelvin (*i.e.*, its development from 0 K to the current temperature). This leads to new terms in the resulting equations.

### 3. CONCLUSION

One can define a different concept of heat capacity for which the temperature change  $\Delta T$  is counted from zero kelvin:  $\Delta T = T - T_0$  where  $T_0 \rightarrow 0 \text{ K}$ . Then, the entropy difference  $dS$  is nothing but this heat capacity:  $\delta q / \Delta T$ .

### REFERENCES

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