Original Research Article

Circuit Design of the Polynomial Chaotic Sun System

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ABSTRACT

This report is about circuit design of chaotic systems. Specifically, the polynomial chaotic Sun system is used to demonstrate the concept and the circuit output signals agree well with chaotic signals from the abstract mathematical model.

Keywords: Chaotic systems, circuit model.

1. INTRODUCTION

Chaos is a phenomenon that could be modeled with nonlinear systems of equations[1]. A nonlinear, aperiodic and continuous-time system is said to be chaotic if it exhibits sensitive dependence on initial conditions; this behaviour makes it practically impossible to predict a future state of the system given that we cannot with pinpoint accuracy ascertain the initial states[2]. Having a positive Lyapunov exponent is also an indication that a nonlinear system is chaotic. Resurgence of interest in chaotic systems started after an MIT professor in 1963, applied it to weather forecasting[3]. With chaos synchronization[4], engineering applications have soared. More recently, chaotic systems have been used in private communications[5][6]. Chaotic systems have also found applications in many other areas such as ecology[7][8], robotics[9], lasers[10][11], neural networks[12][13], chemical reaction[14], cryptosystem[15], finance and economy[16][17], medicine and biology[18][19], and so on. Polynomial chaotic Sun system[20] is one example of very many such systems. The system is interesting for many reasons including the fact that it has twelve parameters, six nonlinearities and six linear terms.

Engineering applications of chaotic systems often involve design and hardware implementation, hence the motivation for the present study. In this report, we present a circuit design of the novel polynomial chaotic Sun system; to the best of our knowledge, circuit design of this system has not been investigated. The polynomial chaotic Sun system is stated as:

$$\begin{aligned} \dot{x} &= p_1 x + p_2 y + p_3 yz + p_4 y^2 \\ \dot{y} &= p_5 x + p_6 y + p_7 z + p_8 xy + p_9 y^2 \\ \dot{z} &= p_{10} z + p_{11} xy + p_{12} y^2, \end{aligned} \tag{1}$$

where the parameter values are $p_1 = -2, p_2 = 10, p_3 = -1, p_4 = 2, p_5 = 18, p_6 = -8, p_7 = 8, p_8 = -1, p_9 = -2, p_{10} = -2, p_{11} = 2, p_{12} = 1$. x, y, z are time dependent state variables. A dot on a state variable implies derivative with respect to time, for example $\dot{x} = {dx}/{dt}$.

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METHODOLOGY

The amplitudes and scales of the state variables differ markedly and are also too large for a reasonable DC supply; to take care of these issues, we make the following transformations.

$$x \rightarrow 6x; \quad y \rightarrow 4y; \quad z \rightarrow 11z\,. \tag{2}$$

And we obtain a more physically viable polynomial chaotic Sun system which reads:

$$\begin{aligned} \dot{x} &= p_1 x + \frac{2}{3}p_2 y + \frac{22}{3}p_3 yz + \frac{8}{3}p_4 y^2 \\ \dot{y} &= \frac{3}{2}p_5 x + p_6 y + \frac{11}{4}p_7 z + 6p_8 xy + 4p_9 y^2 \\ \dot{z} &= p_{10} z + \frac{24}{11}p_{11} xy + \frac{16}{11}p_{12} y^2 \end{aligned} \tag{3}$$

Numerical data from the solutions of eq.(3) is plotted in Fig 1 and the dynamics agree with the chaotic Sun system as originally reported in Ref.[20].

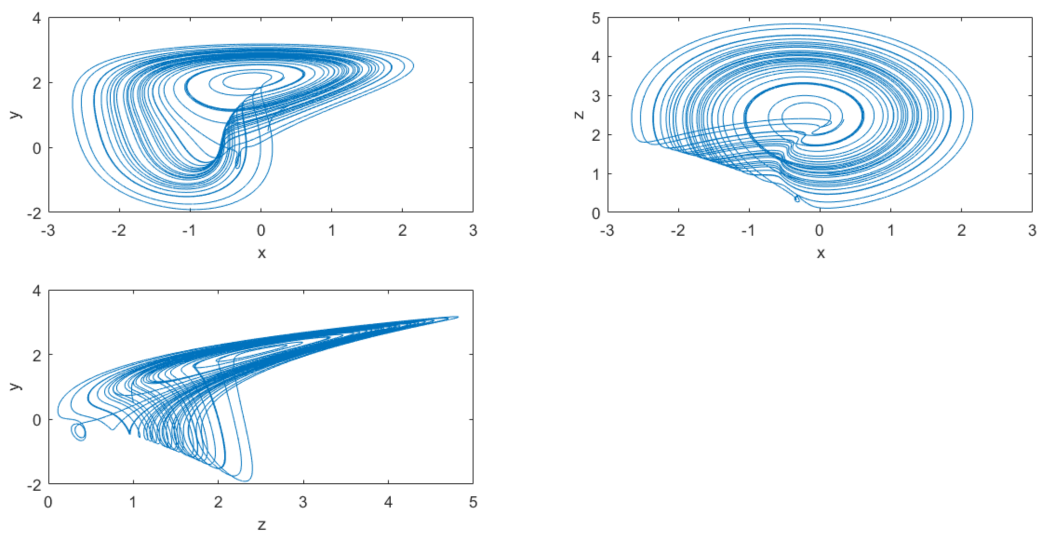


Fig 1. Phase portraits from numerical solutions of the scaled polynomial chaotic Sun system. Matlab *ode45* with automatic time step and initial conditions $x(0) = 0.1, y(0) = 0.5, z(0) = 1.0$ was used.

Now, let $v(x), v(y)$ and $v(z)$ be physical signals (e.g. electric voltages) corresponding to the mathematical objects x, y and z respectively. Then the electronic circuit model corresponding to eq.(3) is given as follows.

$$\begin{aligned} \dot{v}(x) &= -\frac{1}{R_1C_1}v(x) + \frac{1}{R_2C_1}\frac{r_1}{r_2}v(y) - \frac{1}{R_3C_1}v(y)v(z) + \frac{1}{R_4C_1}v(y)^2 \\ \dot{v}(y) &= \frac{1}{R_5C_2}\frac{r_3}{r_4}v(x) - \frac{1}{R_6C_2}v(y) + \frac{1}{R_7C_2}\frac{r_5}{r_6}v(z) - \frac{1}{R_8C_2}v(x)v(y) - \frac{1}{R_9C_2}v(y)^2 \\ \dot{v}(z) &= -\frac{1}{R_{10}C_3}v(z) + \frac{1}{R_{11}C_3}v(x)v(y) + \frac{1}{R_{12}C_3}v(y)^2 \end{aligned} \tag{4}$$

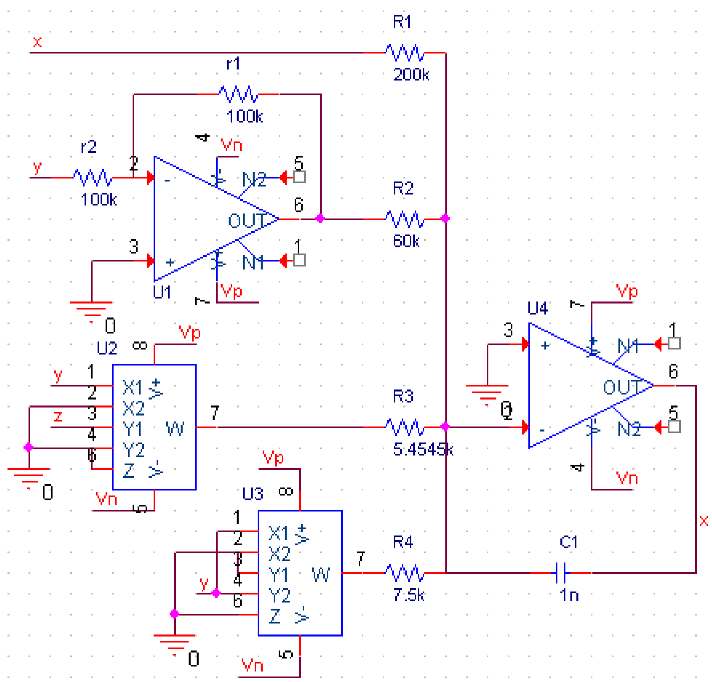
A circuit design for the model is displayed in Fig.2 and choosing a scale of 2500, the following formula gives the relationship between parameters of the chaotic Sun system and the electric circuit parameters.

$$R_i = \frac{1/C_j}{k \times |p_i| \times 2500} \times \frac{1}{10^{n-1}} \tag{5}$$

where for a reasonably chosen circuit capacitance, $C_j (= 1nF$ in our case), R_i is a circuit resistance corresponding to system parameter p_i, k is a scalar from the transformation and n is the (polynomial) power of the i^{th} term; for example, looking at eq.(1) and eq.(3), it is clear that for $p_4 = 2$, and $k = 8/3$, the power of the corresponding variable y, is $n = 2$ and the corresponding circuit resistance R_4 is thus calculated as:

$$R_4 = \frac{{}^{1}/_{10^{-9}}}{8/3 \times 2 \times 2500} \times \frac{1}{10} = 7.5k\,;$$

other R_i are computed in a similar fashion using eq.(5). Note that the r_i as indicated in the circuit only serve to provide the appropriate gains after signal inversion.



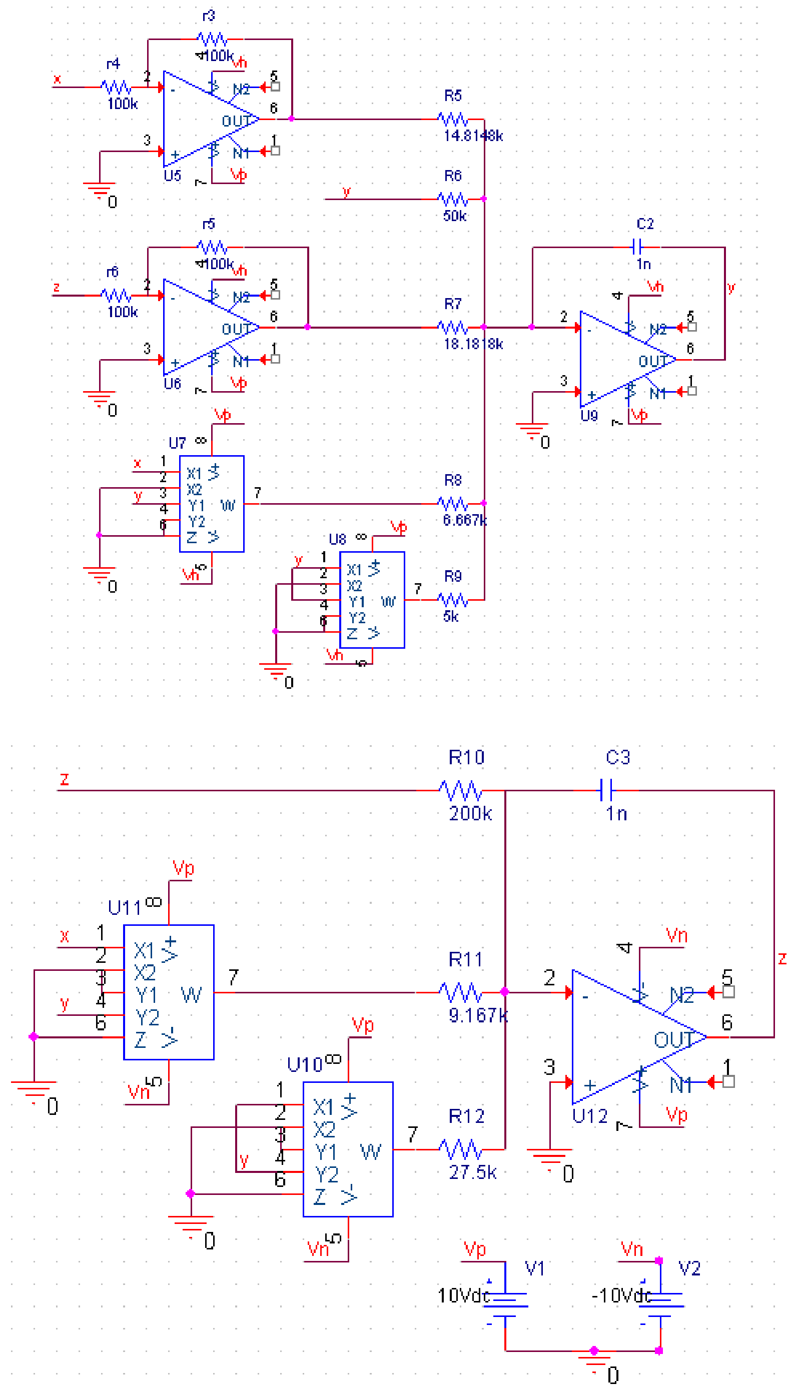
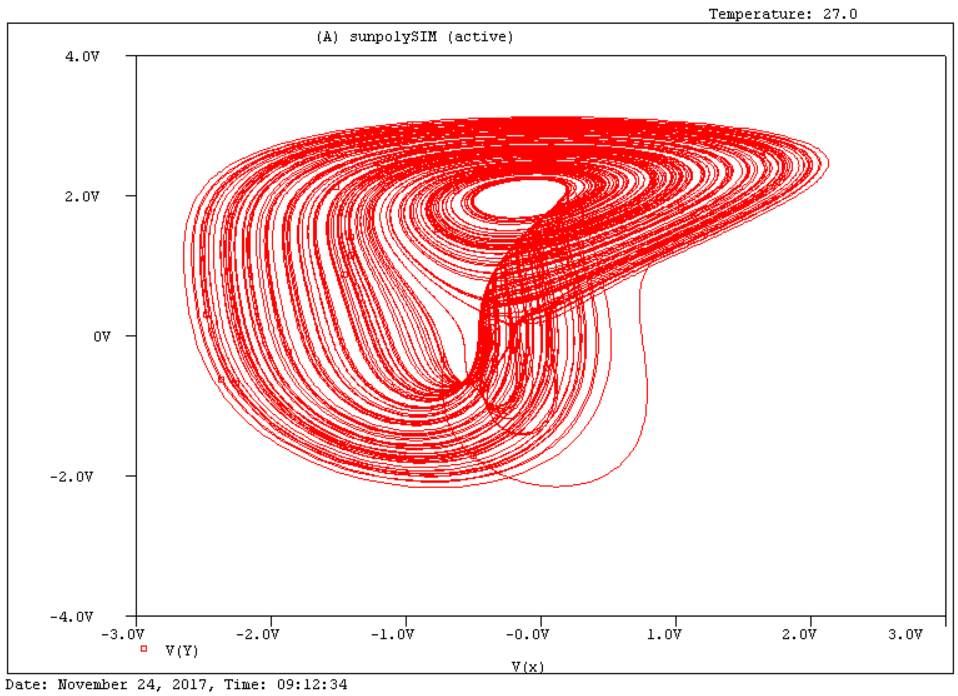
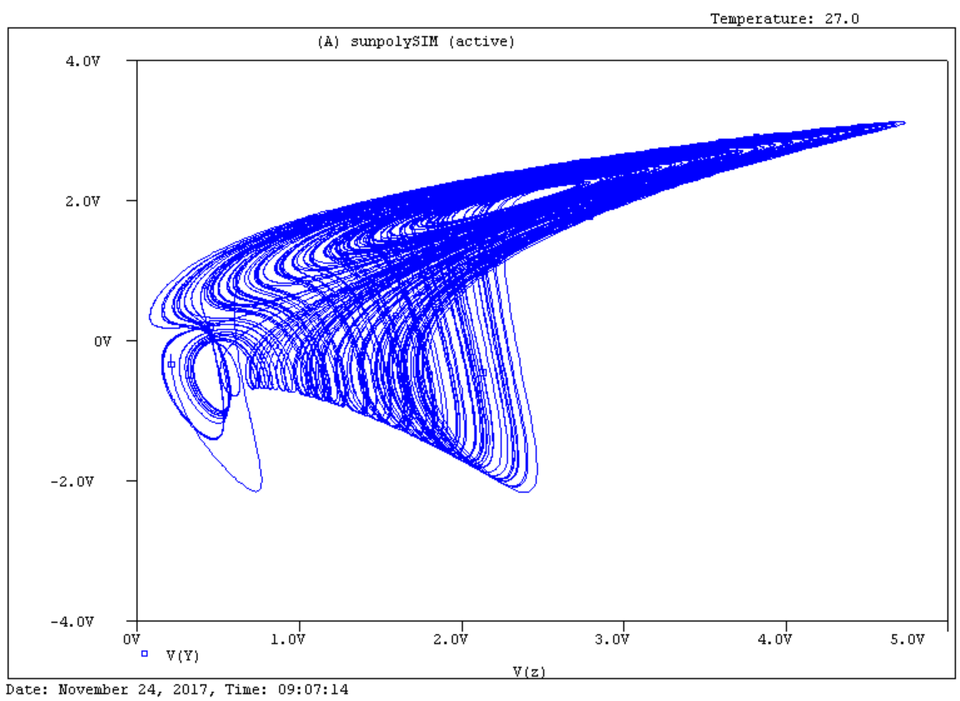


Fig 2. Circuit design of the polynomial chaotic Sun system. OrCAD-Capture was used for the design.

3. RESULTS AND DISCUSSION

The circuit output signals are displayed in Fig. 3 and Fig. 4. As can be seen, the emanating electric voltage signals are highly erratic and follow trajectories very similar to the polynomial chaotic Sun's attractor [20]. In fact, running two simultaneous simulations with initial voltages of the capacitors for a circuit all set to $zero\ volts$ and the other, with initial voltages of the capacitors set at $(0.01, 0, 0)\ volts$, indicates that the signals from the circuit design are chaotic as shown by Fig. 4.





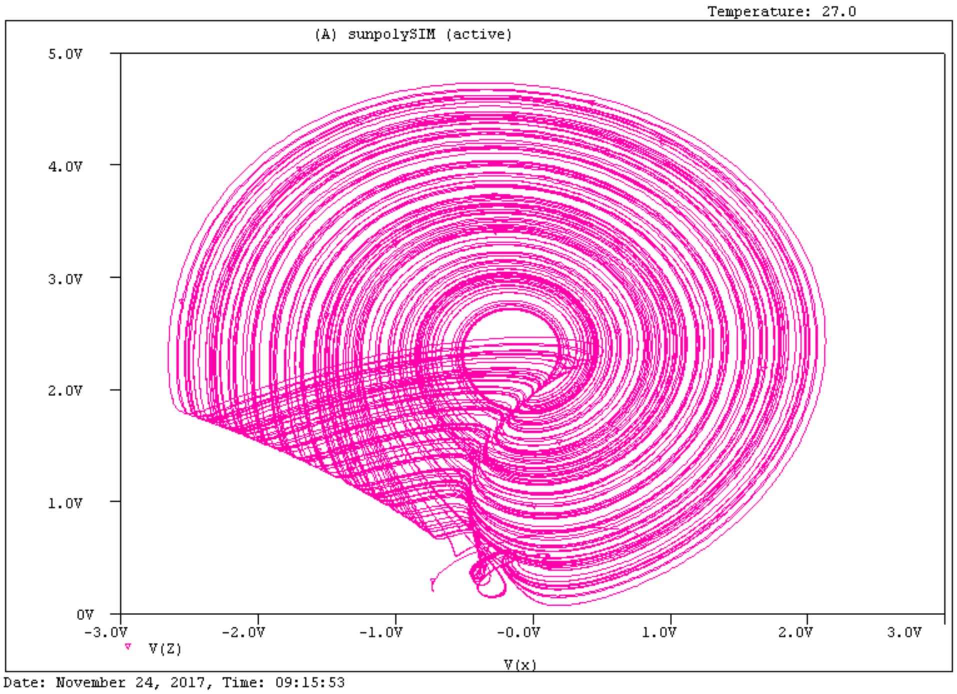
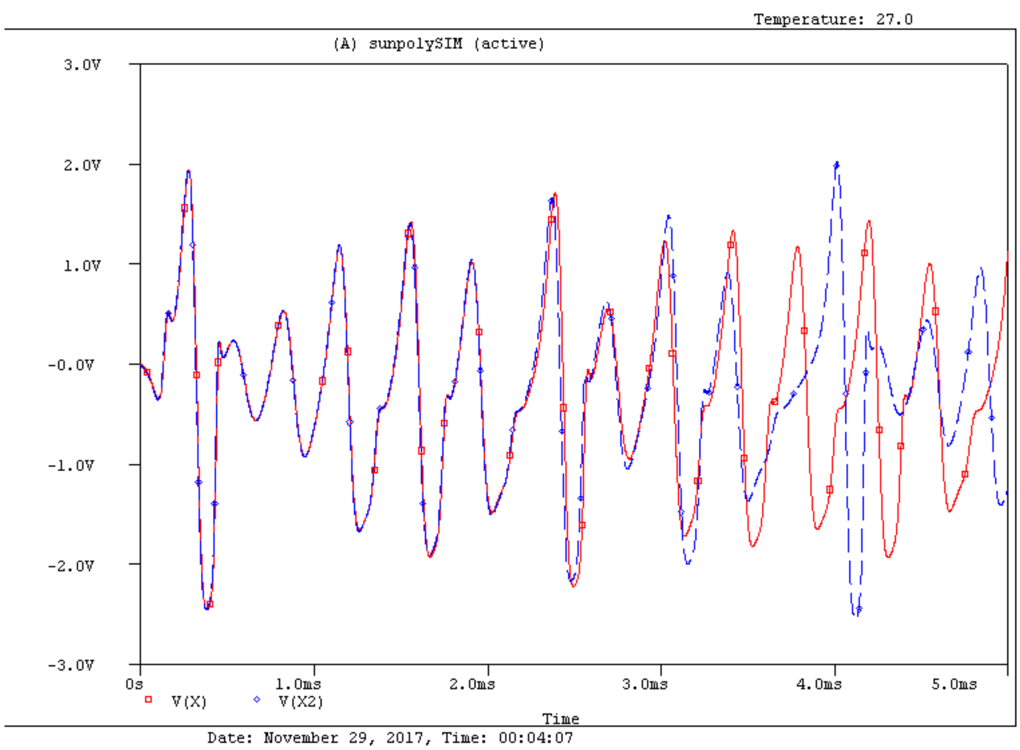


Fig 3. Analog circuit outputs of the polynomial chaotic Sun system. Simulation results are from PSpice A/D with initial conditions of capacitors corresponding to $v(x), v(y)$ and $v(z)$ set respectively to $0, 0.2$ *and* 0.



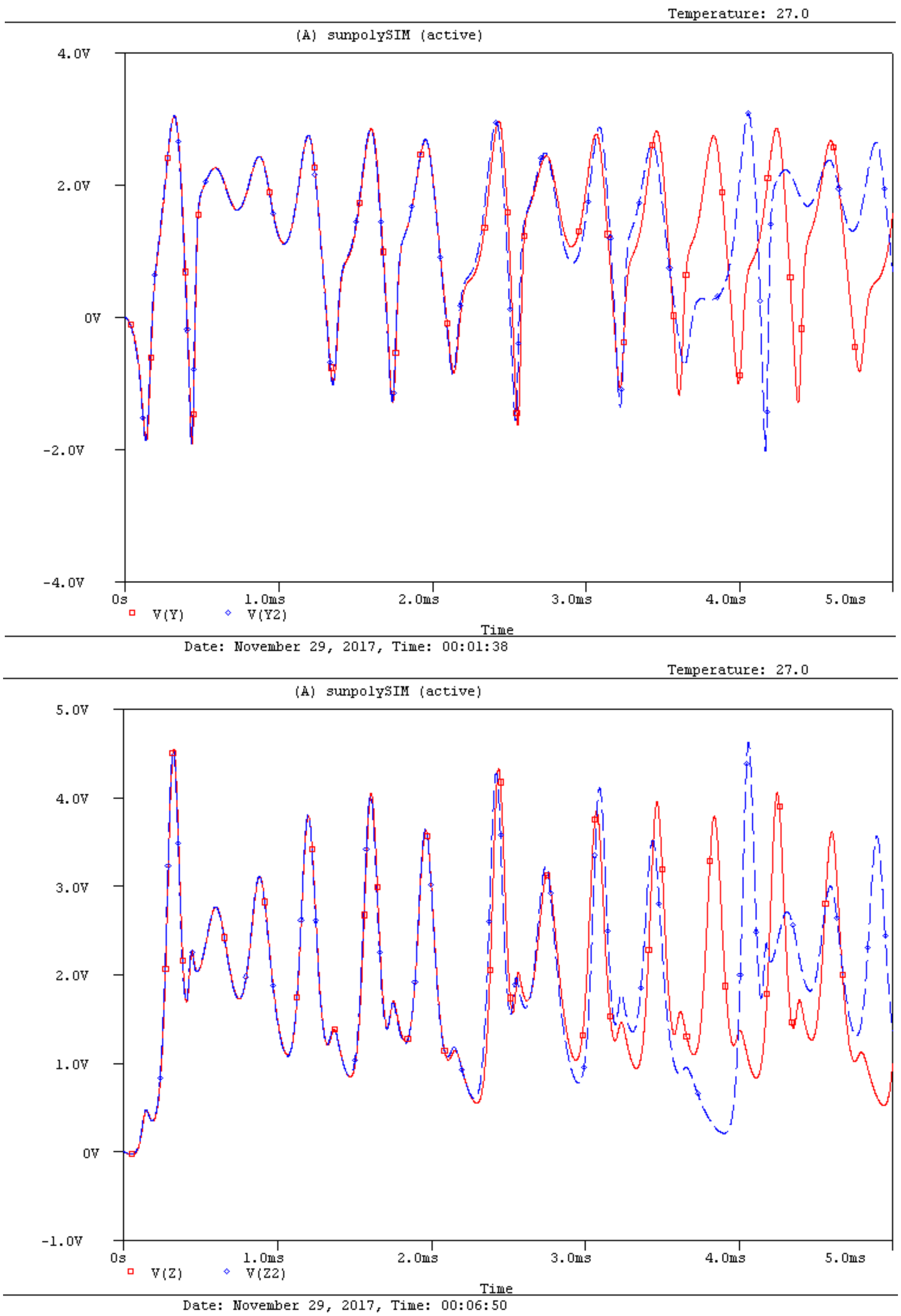


Fig 4. Circuit output signals from PSpice A/D show sensitivity to initial stored voltages of the capacitors. $V(X), V(Y), V(Z)$ were initially set at $(0,0,0) volts$. $V(X2), V(Y2), V(Z2)$ were initially set at $(0.01,0,0) volts$.

4. CONCLUSION

The circuit output signals as shown in Fig.3 agree completely with numerical results from the abstract mathematical model plotted in Fig.1 and with Ref.[20]. Hence the circuit design effectively represents the polynomial chaotic Sun system. Also, the circuit operation is near room temperature.

COMPETING INTERESTS

There are no competing interests associated with this research.

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