APPROXIMATE K-STATE SOLUTIONS OF THE DIRAC EQUATION FOR MODIFIED ECKART PLUS INVERSE SQUARE POTENTIAL MODEL IN THE PRESENCE OF SPIN AND PSEUDO-SPIN SYMMETRY WITHIN THE FRAMEWORK OFNIKIFAROV-UVAROVMETHOD

ABSTRACT

Spin and pseudospin symmetries of the Dirac equation for Modified Eckart plus Inverse square potential within a zero tensor interaction are investigated using the parametric Nikiforov-Uvarov method which is based on the solutions of general second-order linear differential equations with special functions. The bound state eigen value was obtained with some few cases of potential considerations.

Keywords: Modified Eckart plus Inverse Square Potential; Dirac Equation; Spin and Pseudospin Symmetry; Nikiforov-Uvarov Method.

1. INTRODUCTION

The exact solutions of wave equations are still an interesting problem in fundamental quantum mechanics. Unfortunately, there are only a few potentials for which the Schrodinger, Dirac, Klein-Gordon, and Duffin-Kemmer-Petiau (DKP) equations can be exactly solved. Several potential models have been introduced to explore the relativistic and nonrelativistic energy spectra and the corresponding wave functions [1–5]. Jia*et al*. [6] have derived the bound-state solution of the Klein-Gordon equation under unequal scalar and vector kink-like potentials.The solutions of the Dirac equation under pseudospin and spin symmetries with a number of potential models have been investigated by many researchers. These potentials include the ManningÄRosen [7], Eckart [8], Hylleraas [9], Deng-Fang [10], Méobious square [11], Tietz [12], hyperbolical [13], Yukawa and inversely quadratic Yukawa [14, 15] potentials. The spin and pseudospin symmetries under various phenomenological potentials have been investigated using various methods, such as the NikiforovÄUvarov (NU) method [16], supersymmetric quantum mechanics (SUSYQM) [17], and others [18]. On the other hand, we are now almost sure that the spin and pseudospin symmetries of the Dirac equation play a significant role in nuclear and hadronic spectroscopy[19, 20]. The tensor interaction has attracted a great attention as it removes the degeneracy between the doublets [20]. In most studies, due to the mathematical structure of the problem, the tensor interaction is considered as the Coulomb-like [19, 20] or Cornell interaction. Hassanabadi *et al.* were the first to introduce the Yukawa tensor interaction [21]. The investigation has shown that tensor interaction removes the degeneracy between two states in the pseudospin and spin doublets. The effect of tensor coupling under spin and pseudospin symmetries has been studied only for the Coulomb-like interaction until recently that Hassanabadi et al. [21] introduced the Yukawa tensor interaction. Our research group has recently solved the eigen functions of Dirac, Klein-Gordon and Schrodinger using combined or superposed potentials. These include Manning-Rosen plus shifted Deng-fang potential [22], Manning-Rosen plus Yukawa Potential [23], Generalized Woods-Saxon plus Mie-Type Nuclei Potential [24], with Kratzer plus Reduced Pseudoharmonic Oscillator potential [25] and so on.

In the present study, we obtain the approximate analytical solutions of the Dirac equation for the vector Modified Eckart plus Inverse square potentials under zero tensor interaction within the framework of spin and pseudospin symmetry limits.
This paper therefore, is organized as follows. Section 1 covers the introduction, in section 2, we review the NU method, Section 3 is devoted to the Dirac equation for spin and pseudospin symmetries, Special case of the potential is discussed in Section 4, and finally, we give a brief conclusion.

2. REVIEW ON NIKIFAROV-UVAROV METHOD

The main equation which is closely associated with the method is given in the following form (Nikiforov and Uvarov, 1988).

$$\psi''(s) + \frac{\tilde{\tau}(s)}{\sigma(s)}\tau'(s) + \frac{\tilde{\sigma}(s)}{\sigma^2(s)}\psi(s) = 0 \qquad (1)$$

Where $\sigma(s)$ and $\tilde{\sigma}(s)$ are polynomials at most second-degree, $\tilde{\tau}(s)$ is a first-degree polynomial and $\psi(s)$ is a function of the hypergeometric-type.

In order to find the exact solution to Eq. (2), we set the wave function as

$$\psi(x) = \emptyset(s)\mathcal{X}(s) \qquad (2)$$

and on substituting Eq. (3) into Eq. (2), then Eq. (3) reduces to hypergeometric-type,

$$\sigma(s)\mathcal{X}''(s) + \tau(s)\mathcal{X}'(s) + \lambda\mathcal{X}(s) = 0 \qquad (5)$$

where the wave function $\emptyset(s)$ is defined as the logarithmic derivative

$$\frac{\emptyset'(s)}{\emptyset(s)} = \frac{\pi(s)}{\sigma(s)'} \qquad (6)$$

Where $\pi(s)$ is at most first-order polynomial?

The hypergeometric-type function $\emptyset(s)$ whose polynomial solutions are given by the Rodrigues relation

$$\emptyset(s) = \frac{B_n}{\rho(s)}\frac{d^n}{ds^n}[\sigma^n(s)\rho(s)] \qquad (7)$$

Where B_n is the Normalization constant and the weight function $\rho(s)$ most satisfy the condition

$$\frac{d}{ds}[\sigma^n(s)\rho(s)] = \tau(s)\rho(s) \qquad (8)$$

Where

$$\tau(s) = \check{\tau}(s) + 2\pi(s) \qquad (9)$$

In order to accomplish the condition imposed on the weight function $\rho(s)$, it is necessary that the classical or polynomials $\tau(s)$ be equal to zero to some point of an interval (a, b) and its derivative at this interval at $\sigma(s) > 0$ will be negative, that is

$$\frac{d\tau(s)}{ds} < 0 \qquad (10)$$

Therefore, the function $\pi(s)$ and the parameters λrequired for the NU method are defined as follows:

$$\pi(s) = \frac{\sigma' - \check{\tau}}{2} \pm \sqrt{\left(\frac{\sigma' - \check{\tau}}{2}\right)^2 - \tilde{\sigma} + k\sigma} \qquad (10)$$

Where $\lambda = k + \pi'(s)$

The parametric generalization of the NU method is given by the generalized hypergeometric-type equation as

$$\psi''(s) + \left(\frac{(c_1 - c_2 s)}{s(1 - c_3 s)}\right)\psi'(s) + \left(\frac{-\xi_1 s^2 + \xi_2 s - \xi_3}{s^2(1 - c_3 s)^2}\right)\psi(s) = 0 \qquad (11)$$

Equation (11) is solved by comparing it with Eq. (2) and the following polynomials are obtained:

$$\check{\tau}(s) = (c_1 - c_2 s), \;\; \sigma(s) = s(1 - c_3 s), \;\; \tilde{\sigma}(s) = -\xi_1 s^2 + \xi_2 s - \xi_3 \qquad (12)$$

Now substituting Eq. (12) into Eq. (11), we find

$$\bar{\sigma}(s) = c_4 + c_5 s \pm \sqrt{[(c_6 - c_3 k_\pm)s^2 + (c_7 + k_\pm)s + c_8]} \qquad (13)$$

Where $c_4 = \frac{1}{2}(1 - c_1)$, $c_5 = \frac{1}{2}(c_2 - 2c_3)$, $c_6 = c_5^2 + \xi_1$, $c_7 = 2c_4 c_5 - \xi_2$, $c_8 = c_4^2 + \xi_3$, $c_9 = c_3 c_7 + c_3^2 c_8 + c_6$, $c_{10} = c_1 + 2c_4 + 2\sqrt{c_8}$, $c_{11} = c_2 - 2c_5 + 2(\sqrt{c_9} + c_3\sqrt{c_8})$, $c_{12} = c_4 + \sqrt{c_8}$, $c_{13} = c_5 - (\sqrt{c_9} + c_3\sqrt{c_8})$ (14)

The resulting value of *k* in Eq. (13) is obtained from the condition that the function under the square root be square of a polynomials and it yields,

$$k_\pm = -(c_7 + 2c_3 c_8) \pm 2\sqrt{c_9 c_8} \qquad (15)$$

Where $c_9 = c_3 c_7 + c_3^2 c_8 + c_6$

The new $\pi(s)$ for k becomes

$$\pi(s) = c_4 + c_5 s - [(\sqrt{c_9} + c_3\sqrt{c_8})s - \sqrt{c_8}] \qquad (16)$$

Using Eq. (8), we obtain

$$\tau(s) = c_1 + 2c_4 - (c_2 - 2c_5)s - 2\left[\left(\sqrt{c_9} + c_3\sqrt{c_8}\right)s - \sqrt{c_8}\right] \quad (17)$$

We obtain the energy equation as

$$(c_2 - c_3)n + c_3 n^2 - (2n+1)c_5 + (2n+1)\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) + c_7 + 2c_3c_8 + 2\sqrt{c_8c_9} = 0 \quad (18)$$

While the wave function is given as

$$\Psi_n(s) = N_{n,l} S^{c_{12}} (1 - c_3 s)^{-c_{12} - \frac{c_{13}}{c_3}} P_n^{\left(c_{10}-1, \frac{c_{11}}{c_3} - c_{10} - 1\right)} (1 - 2c_3 s) \quad (19)$$

Where P_n is the orthogonal polynomials.

Given that $P_n^{(\alpha,\beta)} = \sum_{r=0}^{n} \frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{\Gamma(\alpha+r+1)\Gamma(n+\beta-r+1)(n-r)!r!} \left(\frac{x-1}{2}\right)^r \left(\frac{x+1}{2}\right)^{n-r}$ (20)

This can also be expressed in terms of the Rodriguez's formula

$$P_n^{(\alpha,\beta)}(x) = \frac{1}{2^n n!}(x-1)^{-\alpha}(x+1)^{-\beta}\left(\frac{d}{dx}\right)^n \left((x-1)^{n+\alpha}(x+1)^{n+\beta}\right) \quad (21)$$

3. BOUND STATE SOLUTION OF THE DIRAC EQUATION

The Schrodinger like differential equation for the upper radial spinor component of the Dirac equation is given as

$$\left\{-\frac{d^2}{dr^2} + \frac{k(k+1)}{r^2} + \frac{1}{\hbar^2 c^2}[MC^2 + E_{nk} - \Delta(r)]\left[MC^2 - E_{nk} + \sum(r)\right]\right\} F_{nk}(r) = \frac{\frac{d\Delta r}{dr}\left(\frac{d}{dr} + \frac{k}{r}\right)}{[MC^2 + E_{nk} - \Delta(r)]} F_{nk}(r) [22]$$

Where $\Delta(r) = V(r) - S(r)$ and $\sum(r) = V(r) + S(r)$ are the differences and the sum of the potentials V(r) ans S(r), respectively.

In the presence of the SS, that is, the difference potential $\Delta(r) = V(r) - S(r) = C_s = costant$ or $\frac{d\Delta r}{dr} = 0$. Then the above equation becomes

$$\left\{-\frac{d^2}{dr^2} + \frac{k(k+1)}{r^2} + \frac{1}{\hbar^2 c^2}[MC^2 + E_{nk} - C_s]\sum(r)\right\} F_{nk}(r) = [E_{nk}^2 - M^2C^4 + C_s(MC^2 - E_{nk}]F_{nk}(r) [23]$$

Similarly, under PSS conditions, $\sum(r) = V(r) + S(r) = C_{ps} = constant$ or $\frac{d\sum(r)}{dr} = 0$

$$\left\{-\frac{d^2}{dr^2}+\frac{k(k+1)}{r^2}+\frac{1}{\hbar^2c^2}\left[MC^2-E_{nk}+C_{ps}\right]\Delta(r)\right\}G_{nk}(r)$$
$$=\left[E_{nk}^2-M^2C^4+C_{ps}(MC^2-E_{nk}\right]G_{nk}(r)[24]$$

The Modified Eckart Potential is given as

$$V(r)=-\left(\frac{V_oe^{-\alpha r}}{(1-e^{-\alpha r})^2}\right) \qquad [25]$$

The Inverse Square Potential, $V(r)=\frac{A}{r^2}$ [26]

Applying the transformation $S=e^{-\alpha r}$ and pekeris-type approximation.The superposed potential can be represented as MEISP

$$V(s)=-\left(\frac{V_os}{(1-s)^2}\right)+\frac{4A\alpha^2}{(1-s)^2} \qquad [27]$$

Applying the pekeris-type approximation and after lengthy algebra, we obtained the following second order differential equation for Spin Symmetry in the presence of Spin-Orbit Coupling term

$$\frac{d^2R(s)}{ds^2}+\frac{(1-s)}{(1-s)s}\frac{dR(s)}{ds}$$
$$+\frac{1}{(1-s)^2s^2}[(\beta^2+P)s^2+(-2\beta^2-2P-Q)s+(\beta^2-H-P-\lambda)]R(s)$$
$$=0[28]$$

Where

$-\beta^2=\left(\frac{E^2-M^2}{4\alpha^2}\right),\ \lambda=(k(k+1)),\ P=\left(\frac{E-M}{4\alpha^2}\right)C_s,\ Q=\left(\frac{E+M-C_s}{4\alpha^2}\right)V_o,\ H=\left(\frac{E+M-C_s}{4\alpha^2}\right)A,$

$$c_1=c_2=c_3=1,c_4=0,c_5=-\frac{1}{2},c_6=\frac{1}{4}+\beta^2+P,c_7=-2\beta^2-2P-Q,$$

$$c_8 = 2\beta^2 - H - \lambda + P, c_9 = \frac{1}{4} - \lambda - H - Q, c_{10} = 1 + 2\sqrt{2\beta^2 - H - \lambda + P}, c_{11} = 2 + 2\left(\sqrt{\frac{1}{4} - \lambda - H - Q} + \sqrt{2\beta^2 - H - \lambda + P}\right), c_{12} = \sqrt{2\beta^2 - H - \lambda + P}, c_{13} = -\frac{1}{2} - \left(\sqrt{\frac{1}{4} - \lambda - H - Q} + \sqrt{2\beta^2 - H - \lambda + P}\right), \varepsilon_1 = 2\beta^2 + B + P + K, \varepsilon_2 = 4\beta^2 - \emptyset + B + H, \varepsilon_3 = 2\beta^2 - 2J - K + H$$

Using the eigenvalue equation, the energy eigen spectrum of MEISP is found to be

$$\beta^2 = \left[\frac{(Q+P+2H+2\lambda) - \left(n^2+n-\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4}-\lambda-H-Q}}{\left(n+\frac{1}{2}\right)+2\sqrt{\frac{1}{4}-\lambda-H-Q}}\right]^2 - (H+P+\lambda) \qquad [29]$$

3.1.SPIN SYMMETRY

The above equation can be solved explicitly and the energy eigen spectrum under the Spin Symmetry$\boldsymbol{k(k+1)}$**,**MEISP

$$E^2 - M^2 = 4\alpha^2\left\{\left[\frac{\left(\left(\frac{E+M-C_S}{4\alpha^2}\right)V_O + \left(\frac{E-M}{4\alpha^2}\right)C_S + 2\left(\frac{E+M-C_S}{4\alpha^2}\right)A + k(k+1)\right) - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right)V_O - \left(\frac{E+M-C_S}{4\alpha^2}\right)A - k(k+1)}}{\left(n+\frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right)V_O - \left(\frac{E+M-C_S}{4\alpha^2}\right)A - k(k+1)}}\right]^2\right\} - \left(\left(\frac{E-M}{4\alpha^2}\right)C_s + \left(\frac{E+M-C_s}{4\alpha^2}\right)A + k(k+1)\right) \qquad [30]$$

3.2.PSEUDO-SPIN SYMMETRY

For Pseudo-Spin consideration$\boldsymbol{k(k-1)}$**,** the explicit energy of the MEISP becomes

$$E^2 - M^2 = 4\alpha^2 \left\{ \left[\frac{\left(\left(\frac{E+M-C_S}{4\alpha^2}\right) V_o + \left(\frac{E-M}{4\alpha^2}\right) C_S + 2\left(\frac{E+M-C_S}{4\alpha^2}\right) A + k(k-1) \right) - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right) V_o - \left(\frac{E+M-C_S}{4\alpha^2}\right) A - k(k-1)}}{\left(n+\frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right) V_o - \left(\frac{E+M-C_S}{4\alpha^2}\right) A - k(k+1)}} \right]^2 \right\} - \left(\left(\frac{E-M}{4\alpha^2}\right) C_s + \left(\frac{E+M-C_s}{4\alpha^2}\right) A + k(k-1) \right)$$

[31]

4. DISCUSSION

We consider the following cases of potential from equations [30] *and* [31]

(I) When $V_o = 0$, Dirac equation for Inverse square potential for Spin and Pseudo-spin symmetry is obtained as follows

SPIN SYMMETRY

$$E^2 - M^2 = 4\alpha^2 \left\{ \left[\frac{\left(\left(\frac{E-M}{4\alpha^2}\right) C_S + 2\left(\frac{E+M-C_S}{4\alpha^2}\right) A + k(k+1) \right) - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right) A - k(k+1)}}{\left(n+\frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right) A - k(k+1)}} \right]^2 \right\} - \left(\left(\frac{E-M}{4\alpha^2}\right) C_s + \left(\frac{E+M-C_s}{4\alpha^2}\right) A + k(k+1) \right)$$

[32]

PSEUDO-SPIN SYMMETRY

$$E^2 - M^2 = 4\alpha^2 \left\{ \left[\frac{\left(\left(\frac{E-M}{4\alpha^2}\right) C_S + 2\left(\frac{E+M-C_S}{4\alpha^2}\right) A + k(k-1) \right) - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right) A - k(k-1)}}{\left(n+\frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right) A - k(k+1)}} \right]^2 \right\} - \left(\left(\frac{E-M}{4\alpha^2}\right) C_s + \left(\frac{E+M-C_s}{4\alpha^2}\right) A + k(k-1) \right)$$

[33]

(II) When $A = 0$,Dirac equation for Modified Eckart potential for Spin and Pseudo-spin symmetry is obtained as follows

SPIN SYMMETRY

$$E^2 - M^2 = 4\alpha^2 \left\{ \left[\frac{\left(\left(\frac{E+M-C_S}{4\alpha^2}\right)V_o + \left(\frac{E-M}{4\alpha^2}\right)C_S + k(k+1)\right) - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right)V_o - k(k+1)}}{\left(n+\frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right)V_o - k(k+1)}} \right]^2 \right\} - \left(\left(\frac{E-M}{4\alpha^2}\right) C_s + k(k+1)\right) \quad [34]$$

PSEUDO-SPIN SYMMETRY

$$E^2 - M^2 = 4\alpha^2 \left\{ \left[\frac{\left(\left(\frac{E+M-C_S}{4\alpha^2}\right)V_o + \left(\frac{E-M}{4\alpha^2}\right)C_S + k(k-1)\right) - \left(n^2+n+\frac{1}{2}\right) - (2n+1)\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right)V_o - k(k-1)}}{\left(n+\frac{1}{2}\right) + 2\sqrt{\frac{1}{4} - \left(\frac{E+M-C_S}{4\alpha^2}\right)V_o - k(k+1)}} \right]^2 \right\} - \left(\left(\frac{E-M}{4\alpha^2}\right) C_s + k(k-1)\right) \quad [35]$$

5. CONCLUSION

In this paper, we obtained the approximate analytical solutions of the Dirac equation for the Modified Eckart plus Inverse Square potential for zero tensor interaction within the framework of pseudospin and spin symmetry limits using the NU technique. We have obtained the energy levels in a closed form and some special case of the potential has been discussed.

REFERENCES

[1] Y. Xu, S. He, and C. S. Jia, "Approximate analytical solutions of the Klein-Gordon equation with the P¨oschl-Tellerpotential including the centrifugal term," *PhysicaScripta*, vol. 81, Article ID 045001, 2010.

[2] X. Y. Gu and S. H. Dong, "Energy spectrum of the Manning- Rosen potential including centrifugal term solved by exact and proper quantization rules ," *Journal ofMathematical Chemistry*, vol. 49, pp. 2053–2062, 2011.

[3] M. R. Pahlavani and S. A. Alavi, "Solutions of Woods- Saxon potential with Spin-Orbit and centrifugal terms through
Nikiforov-Uvarov method," *Communications in Theoretical Physics*, vol. 58, p. 739, 2012.

[4] G. F. Wei and S. H. Dong, "A novel algebraic approach to spin symmetry for Dirac equation with scalar and vector second
P¨oschl-Tellerpotentials ," *European Physical Journal A*, vol. 43, pp. 185–190, 2010.

[5] C.-Y. Chen, D.-S.Sun, and F.-L.Lu, "The relativistic bound states of the Hartmann potentials," *PhysicaScripta*, vol. 74, no. 4, article 001, pp. 405–409, 2006.

[6] C. S. Jia, X. P. Li, and L. H. Zhang, "Exact solutions of the Klein- Gordon equation with position-dependent mass for mixed vector and scalar kink-like potentials," *Few-Body Systems*, vol. 52, pp. 11–18, 2012.

[7]. *Falaye B. J. et al.* Bound State Solutions of the ManningÄRosen Potential // Can. J. Phys. 2013.V. 91. P. 98.

[8]. *Jia C. S., Guo P., Peng X. L.* Exact Solution of the DiracÄEckart Problem with Spin and PseudospinSymmetries // J. Phys. A: Math. Gen. 2006. V. 39. P. 7737.

[9]. *Hassanabadi H. et al.* Approximate Any *l*-State Solutions of the Dirac Equation for Modiˇed Deformed Hylleraas Potential by Using the NikiforovÄUvarov Method // Chin. Phys. B. 2012. V. 21. P. 120302.

[10]. *Maghsoodi E., Hassanabadi H., Zarrinkamar S.* Spectrum of Dirac Equation under DengÄFan Scalar and Vector Potentials and a Coulomb Tensor Interaction by SUSYQM // Few-Body Syst. 2012. V. 53. P. 525.

[11]. *Maghsoodi E. et al.* Arbitrary-State Solutions of the Dirac Equation for a Méobius Square Potential Using the NikiforovÄUvarov Method // Chin. Phys. C. 2013. V. 37. P. 04105.

[12]. *Hassanabadi H., Maghsoodi M., Zarrinkamar S.* Relativistic Symmetries of Dirac Equation and the Tietz Potential // Eur. Phys. J. Plus. 2012. V. 127. P. 31.

[13]. *Ikot A. N. et al.* Approximate Relativistic κ-State Solutions to the Dirac-Hyperbolic Problem with Generalized Tensor Interactions // Intern. J. Mod. Phys. E. 2013. V. 22. P. 1350048.

[14]. *Ikhdair S.M., Falaye B. J.* Approximate Relativistic Bound States of a Particle in Yukawa Field with Coulomb Tensor Interaction // Phys. Scripta. 2013. V. 87. P. 035002.

[15]. *Hamzavi M., Ikhdair S.M., Ita B. I.* Approximate Spin and Pseudospin Solutions to the Dirac Equation for the Inversely Quadratic Yukawa Potential and Tensor Interaction // Phys. Scripta. 2012. V. 85. P. 045009.

[16]. *Nikiforov A. F., Uvarov V. B.* Special Functions of Mathematical Physics. Basel: Birkhauser, 1988.

[17]. *Cooper F., Khare A., Sukhatme U.* Supersymmetry and Quantum Mechanics // Phys. Rep. 1995. V. 251. P. 267.

[18]. *Hassanabadi H., Yazarloo B.H., Zarrinkamar S.* Exact Solution of KleinÄGordon Equation for HuaPlus Modiˇ edEckart Potentials // Few-Body Syst. 2013. V. 54. P. 2017.

[19]. *Aydogdu O., Sever R.* Exact Pseudospin Symmetric Solution of the Dirac Equation for PseudoharmonicPotential in the Presence of Tensor Potential // Few-Body Syst. 2010. V. 47. P. 193.

[20]. *Ikot A. N.* Solutions of Dirac Equation for Generalized Hyperbolical Potential Including Coulomb-Like Tensor Potential with Spin Symmetry // Few-Body Syst. 2012. V. 53. P. 549.

[21]. *Hassanabadi H., Maghsoodi E., Zarrinkamar S.* Spin and Pseudospin Symmetries of Dirac Equation and the Yukawa Potential as the Tensor Interaction // Commun. Theor.Phys. 2012. V. 58. P. 807.

[22]. H. Louis., B.I. Ita., N. Nelson., P.I. Amos., I. Joseph and I. Opara (2017): Analytic spin and Pseudospin solutions to the Dirac equation for the Manning-Rosen plus shifted Deng-fang potential and Yukawa-like tensor interaction. WSN 81(2) 292-304.

[23]. H. Louis., B.I. Ita**.,**P.I. Amos., T.O. Magu and N.A. Nzeata-Ibe (2017): Bound State Solutions of the Klein-Gordon Equation with Manning-Rosen plus Yukawa Potential Using Pekeris-like Approximation of the Coulomb Term and Nikiforov-Uvarov Method. *Physical Science International Journal*. 15(3): 1-6, 2017

[24]. Louis, Hitlerand Benedict, IseromIta(2017): Bound State Solutions of the s-wave Schrodinger Equation for Generalized Woods-Saxon plus Mie-Type Nuclei Potential within the framework of Nikifarov-Uvarov Method. WSN 77 (2) 378-384.

[24]. H. Louis*,.B.I. Ita., B.E. Nyong., T.O.Magu, S. Barka and N.A. Nzeata-Ibe**(2016):** Radial solution of the s-wave D-dimensional Non-Relativistic Schrodinger equation for generalized manning-Rosen plus Mie-type nuclei potentials within the framewoark of parametric Nikifarov-Uvarov Method. *Journal of Nigerian Association of Mathematical Physics* vol.36, No. 2, (July, 2016) pp.193-198.

[25]. H. Louis*****, B.I. Ita., B.E. Nyong.,,T.O. Magu, and N.A. Nzeata-Ibe**(2016):** Approximate solution of the N-dimensional radial Schrodinger equation with Kratzer plus Reduced Pseudoharmonic Oscillator potential within the framework of Nikifarov-Uvarov Method. *Journal of Nigerian Association of Mathematical Physics*.vol.36, No. 2. (July, 2016) Pp. 199-204