Toy model of quantum cosmology with evolving dark energy

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Abstract

Qualitatively we assume that, at any stage of cosmic evolution, 1) Space-time curvature follows, $GM_t \cong R_t c^2$ where M_t and R_t represent the cosmic mass and radius respectively. 2) Planck scale Hubble parameter plays a crucial role in cosmic evolution. 3) Ratio of cosmic mass and volume is equal to the ordinary matter density. With further research, a unified model of 'quantum cosmology' with evolving dark energy can be developed.

Proceeding further, we define the Planck scale Hubble parameter, $H_{pl} \cong \sqrt{\frac{c^5}{G\hbar}} \approx 1.86 \times 10^{43}\ \text{sec}^{-1}$ and apply it to cosmological data fitting and prediction in the form of $\gamma_t \cong \left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right]$ where H_t is the running Hubble parameter. At any stage of cosmic evolution: 1) Ratio of ordinary matter density and critical density is, $(\Omega_{OM})_t \cong \left(\frac{1+\sqrt{\gamma_t}}{2}\right)\left(\frac{1}{1+\gamma_t}\right)$. 2) Ratio of dark matter density and critical density is, $(\Omega_{DM})_t \cong \left(\frac{1+\sqrt{\gamma_t}}{2}\right)^2\left(\frac{1}{1+\gamma_t}\right)$. 3) Ratio of dark energy density and critical energy density is, $(\Omega_{DE})_t \cong 1-\left[(\Omega_{OM})_t+(\Omega_{DM})_t\right]$. 4) Ratio of dark matter density and ordinary matter density is, $\frac{(\Omega_{DM})_t}{(\Omega_{OM})_t} \cong \left(\frac{1+\sqrt{\gamma_t}}{2}\right)$. 5) Cosmic radius and (ordinary) mass are: $R_t \cong \sqrt{\frac{2}{(\Omega_{OM})_t}}\left(\frac{c}{H_t}\right)$ and $M_t \cong \sqrt{\frac{2}{(\Omega_{OM})_t}}\left(\frac{c^3}{GH_t}\right)$ respectively. 6) Thermal wavelength and temperature are : $(\lambda_{max})_t \cong \left(\frac{1+\sqrt{\gamma_t}}{2}\right)\sqrt{R_t\,(R_t)_{pl}}$ and $T_t \cong \frac{2.898\times10^{-3}\ Km}{(\lambda_{max})_t}$ respectively where $(R_t)_{pl} \cong 2\sqrt{\frac{G\hbar}{c^3}}$. 7) Observed anisotropy in current CMBR temperature can be understood with the relational condition: $\left(\frac{(\Omega_{OM})_0}{(\Omega_{DM})_0}\right)_{galaxy}$ is greater than or less than $\left(\frac{1+\sqrt{\gamma_0}}{2}\right)^{-1}$. 8) For $(z+1) \leq 1100$, cosmic scale factor and age are: $\left(\frac{1}{z+1}\right) \approx \left[\exp\left(\frac{\gamma_0-\gamma_t}{2}\right)\right]^{-1} \approx \frac{T_0}{T_t} \approx \sqrt{\frac{H_0}{H_t}}$ and $t \approx \frac{(1+z)^{-\frac{3}{2}}}{H_0} \approx \frac{\sqrt{z+1}}{H_t} \approx \left(H_t^{0.75}H_0^{0.25}\right)^{-1}$ respectively where $H_t \cong \frac{H_{pl}}{\gamma_0-[2\ln(z+1)]} \approx (1+z)^2 H_0$. 9) Cosmic expansion velocity is, $V_t \cong \sqrt{\frac{2}{(\Omega_{OM})_t}}\,c$.

Keywords: Big bang; Planck scale Hubble parameter, Quantum cosmology; Mach's principle; Holographic principle; Observational cosmology; Dark energy;Thermal radiation isotropy and anisotropy; Redshift; Cosmic age; Scale factor;

Nomenclature

At any stage of cosmic evolution,

1. γ_t = Newly defined number.
2. $(\Omega_{OM})_t$ = Ratio of ordinary matter density and critical density.
3. $(\Omega_{DM})_t$ = Ratio of dark matter density and critical density.
4. $(\Omega_{DE})_t$ = Ratio of dark energy density and critical energy density.
5. H_t = Hubble parameter, M_t = Ordinary cosmic mass and R_t = Cosmic radius.
6. $(\lambda_{max})_t$ = Cosmic thermal wavelength and $T_t = \frac{2.898 \times 10^{-3} K.m}{(\lambda_{max})_t}$ = Cosmic temperature.
7. z = Cosmic redshift and $a_t = \frac{1}{z+1}$ = Cosmic scale factor.
8. V_t = Cosmic expansion velocity.
9. $(d_g)_0$ = Current galactic distance from the point of big bang.
10. $(v_g)_0$ = Current galactic receding speed from and about the point of big bang.

At Planck scale,

1. γ_{pl} = Defined Planck scale $\gamma = 1$.
2. (Ω_{pl}) = Defined Planck scale ratio of ordinary matter density and critical density = $\frac{1}{2}$.
3. (Ω_{pl}) = Defined Planck scale ratio of dark matter density and critical density = $\frac{1}{2}$.
4. (Ω_{DE}) = Defined Planck scale ratio of dark energy density and critical energy density = 0.
5. H_{pl} = Defined Planck scale Hubble parameter = $\sqrt{\frac{c^5}{G\hbar}}$.
6. R_{pl} = Planck size = $\sqrt{\frac{G\hbar}{c^3}}$ and $(R_t)_{pl}$ = Planck scale cosmic radius = $2\sqrt{\frac{G\hbar}{c^3}} = 2R_{pl}$.
7. M_{pl} = Planck mass = $\sqrt{\frac{\hbar c}{G}}$ and $(M_t)_{pl}$ = Planck scale cosmic mass = $2\sqrt{\frac{\hbar c}{G}} = 2M_{pl}$.
8. $(\lambda_{max})_{pl}$ = Planck scale cosmic thermal wavelength = $(R_t)_{pl} = 2R_{pl}$ and $T_{pl} = \frac{2.898 \times 10^{-3} K.m}{(\lambda_{max})_{pl}}$ = Planck scale cosmic temperature.
9. V_{pl} = Planck scale cosmic expansion velocity.

1 Introduction

By modelling the observed universe as an imaginary quantum gravitational evolving sphere, we try to develop a toy model of quantum cosmology. With reference to the currently believed cosmic density break up and Planck scale critical density, we proposed an empirical relation for understanding/predicting the quantitative percentages of past and future cosmic density breakups. Proceeding further and by considering the proposed set of assumptions, we tried our level best in fitting the current cosmological physical parameters. Further study, may help in understanding the actual nature of 'dark energy'.

1.1 Observable universe - a quantum gravitational object

Photons and black holes can be considered as the best candidates of quantum gravitational objects. It is true that, without the existence of universe, there is no independent existence to any photon or any black hole. Now the fundamental question to be answered is: Is our universe a quantum gravitational object or something else? Physicists expressed several opinions with many possible solutions [1-5] and references therein. We could also express different unified views in this direction [6-15] and readers are strongly encouraged to go through. In an optimistic approach, some of the modern cosmologists believe that, during cosmic evolution, Planck scale quantum gravitational interactions might have an observable effect on the current observable cosmological phenomena. Clearly speaking, with respect to 'Quantum gravity' and Planck scale early universal laboratory, current universe can be considered as a low energy scale laboratory. If one is willing to consider the current observable universe as a low energy scale laboratory, currently believed cosmic microwave back ground temperature can be considered as the low energy quantum gravitational effect. At any time in the past, i.e as the operating energy scale was assumed to be increasing; past high cosmic back ground temperature can be considered as the high energy quantum gravitational effect. Thinking in this way, starting from the Planck scale, 'quantum cosmology' can be considered as 'scale independent'. If one is willing to consider the observable evolving universe as an evolving quantum gravitational object, there is a scope for initiating a toy model of quantum cosmology. To proceed further, we have chosen the following two quantitative relations.

1. We define the Planck scale Hubble parameter, $H_{pl} \cong \sqrt{\frac{c^5}{G\hbar}} \approx 1.86 \times 10^{43}\ \text{sec}^{-1}$ and apply it to cosmological data fitting in the form of, $\gamma_t \cong \left[1 + \ln\left(\frac{H_{pl}}{H_t}\right)\right]$ where H_t is the running Hubble parameter.

2. According to G't Hooft, the combination of quantum mechanics and gravity requires the three dimensional world to be an image of data that can be stored on a two dimensional projection much like a holographic image [16,17]. The 'holographic principle' is a property of string theory and a supposed property of quantum gravity that states that the description of a volume of space can be thought of as encoded on a lower-dimensional boundary. Based on this concept, for the four dimensional space-time universe, its three dimensional increasing volume can be set by Mach's principle, $\frac{GM_t}{R_t c^2} \cong 1$. Clearly speaking, information of the evolving universe, can be extracted from $R_t \cong \frac{GM_t}{c^2}$. With this proposal, at any stage of cosmic evolution, a closed and massive universe can be defined. One can find interesting technical discussion on this assumption by D.W.Sciama, R.H. Dicke, C. Brans and G. J. Whitrow [18-25].

Based on these quantitative relations, we re-view the phenomena of 'inflation' [26-28], 'acceleration' and 'dark energy' [29-31]. We arranged our revised version in the following way. In section-2, we proposed our assumptions connected with big bang and Planck scale. In section-3 we proposed many possible applications of the proposed new number γ_t pertaining to observational cosmology. In section-4 we presented our concluding remarks.

1.2 Important points pertaining to modern cosmological observations

Subject of cosmology is quite interesting, very complicated and quite controversial.

1. In June 2015, three professors, J. T. Nielsen, Alberto Guffanti and Subir Sarkar of Niels Bohr International Academy and Rudolf Peierls Centre for Theoretical Physics, using the JLA catalogue of 740 SN Ia processed by the SALT2 method, come to a conclusion that [32], evidence for the currently believed cosmic acceleration is only marginal and current universe seems to expand at a constant rate. This breakthrough work got published in the prestigious Nature journal's 'Scientific Reports'. In their words: "The 'standard' model of cosmology is founded on the basis that the expansion rate of the universe is accelerating at present - as was inferred originally from the Hubble diagram of Type Ia supernovae. There exists now a much bigger database of supernovae so we can perform rigorous statistical tests to check whether these 'standardisable candles' indeed indicate cosmic acceleration. Taking account of the empirical procedure by which corrections are made to their absolute magnitudes to allow for the varying shape of the light curve and extinction by dust, we find, rather surprisingly, that the data are still quite consistent with a constant rate of expansion."

2. According to T. Padmanabhan [33]:"One natural - and in fact, inevitable - contribution to cosmological constant arises from the energy density of quantum vacuum fluctuations. The trouble is, we do not know how to compute the gravitational effects of quantum fluctuations of the vacuum from first principles. Naive estimates suggests that this will give $\Lambda\left(\frac{G\hbar}{c^3}\right) \approx 1$ which misses the correct result by 120 orders of magnitude! It is possible to get around this difficulty and get the correct value but only if we are prepared to make some extra assumptions. The appearance of G and $\hbar$ together strongly suggests that the problem of dark energy needs to be addressed by quantum gravity. None of the currently popular models of quantum gravity has anything meaningful to say on this issue (let alone predict its correct value). In fact, explaining the observed value of the dark energy is the acid test for any quantum gravity model and all the models currently available flunk this test. There is no doubt that, when we eventually figure this out, it will lead to as drastic a revolution in our conceptual understanding as relativity and quantum theory did".

3. According to Martin Bozowald[1]:

    (a) "Quantum cosmology is based on the idea that quantum physics should apply to anything in nature, including the whole universe. Quantum descriptions of all kinds of matter fields and their interactions are well known and can easily be combined into one theory - leaving aside the more complicated question of unification, which asks for a unique combination of all fields based on some fundamental principles or symmetries. Nevertheless, quantizing the whole universe is far from being straightforward because, according to general relativity, not just matter but also space and time are physical objects. They are subject to dynamical laws and have excitations (gravitational waves) that interact with each other and with matter. Quantum cosmology is therefore closely related to quantum gravity, the quantum theory of the gravitational force and space-time. Since quantum gravity remains unfinished, the theoretical basis of quantum cosmology is unclear. And to make things worse, there are several difficult conceptual problems to be overcome".

    (b) "We remain far from a proper understanding of quantum cosmology, especially when physics at the Planck scale is involved. At the same time, research on quantum cosmology has led to progress in our understanding of generally covariant quantum systems and often showed unexpected effects of quantum space-time".

2 Workable assumptions connected with Planck scale

With the following three simple and logical assumptions, most of the currently believed cosmological observations can be reviewed and refined at fundamental level. Our proposed set of assumptions can be divided into 'quantitative' and 'qualitative' assumptions. We appeal the readers to go through the rest of the paper and evaluate their novelty with reference to:

1. Implementing Planck scale, Mach's principle and Holographic principle;
2. Developing a model of quantum cosmology;
3. Current cosmological data fitting and ability for extrapolation to past and future;
4. Compatibility with hot big bang model and dark matter;
5. Simplicity and ability for extension or modification;

2.1 Proposed set of qualitative assumptions

At any stage of cosmic evolution,

1. Space-time curvature follows $GM_t \cong R_t c^2$, where M_t and R_t represent the ordinary cosmic mass and radius respectively.
2. Planck scale Hubble parameter plays a crucial role in cosmic evolution.
3. Ratio of cosmic mass and volume is equal to the ordinary matter density.

2.2 Our basic conceptual thoughts and numerical fits

1. H_{pl} being the Planck scale Hubble parameter, at any stage of cosmic evolution, let, $\gamma_t \cong \left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right]$ and at the Planck scale, $H_{pl} \cong H_t$ and $\gamma_{pl} \cong 1$.
2. If magnitude of H_{pl} is $\approx 10^{43}$, for the current case, we noticed that, $\gamma_0 \cong \left[1+\ln\left(\frac{H_{pl}}{H_0}\right)\right] \cong 141$.
3. Based on this observation, for various decreasing values of $\gamma_t \cong \left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right]$ in between 141 and 1, corresponding cosmic Hubble parameters and cosmic temperatures can be estimated.
4. With reference to current cosmological data,
   (a) Both, ordinary matter density and dark matter density are approximately proportional to $\frac{1}{\gamma_0}\left(\frac{3H_0^2}{8\pi G}\right)$.
   (b) Proportionality constant for current ordinary matter density seems to be $\left(\frac{1+\sqrt{\gamma_0}}{2}\right)$.
   (c) Proportionality constant for current dark matter density seems to be $\left(\frac{1+\sqrt{\gamma_0}}{2}\right)^2$.
   (d) Current ordinary matter density seems to be approximately equal to $\left(\frac{1+\sqrt{\gamma_0}}{2}\right)\left[\frac{1}{\gamma_0}\left(\frac{3H_0^2}{8\pi G}\right)\right]$.
   (e) Current dark matter density seems to be approximately equal to $\left(\frac{1+\sqrt{\gamma_0}}{2}\right)^2\left[\frac{1}{\gamma_0}\left(\frac{3H_0^2}{8\pi G}\right)\right]$
   (f) Ratio of current dark matter density and ordinary matter density is close to $\left(\frac{1+\sqrt{\gamma_0}}{2}\right)$.
5. Guessing that, $\gamma_{pl} \cong 1$, we noticed that, at the Planck scale, both, ordinary matter density and dark matter density seem to be equal to Planck scale critical density. It seems to be violating the currently believed Friedmann's cosmic 'density sum rule'.
6. To sustain the density sum rule for $1 \geq \gamma_t \leq 141$, we consider $\left[\frac{1}{(1+\gamma_t)}\left(\frac{3H_t^2}{8\pi G}\right)\right]$ in place of $\left[\frac{1}{\gamma_t}\left(\frac{3H_t^2}{8\pi G}\right)\right]$. If one is willing to consider this adjustment, at the Planck scale, both, ordinary matter density and dark matter density seem to be equal to $\frac{1}{2}$ of the Planck scale critical density.
7. For various increasing values of γ_t in between 1 and 141, it is noticed that, sum of ordinary matter density and dark matter density seems to be gradually decreasing and is always less than unity. With reference to cosmic 'density sum rule', one can identify [critical density-(ordinary matter density + dark matter density)] with 'dark energy'. Clearly speaking, during cosmic evolution, dark energy content attains increasing values according to [critical density-(ordinary matter density + dark matter density)].
8. At the Planck scale,
   (a) $\gamma_{pl} \cong 1$ and $\left(\frac{1+\sqrt{\gamma_{pl}}}{2}\right) \cong 1$.
   (b) $(\Omega_{OM})_{pl} \cong (\Omega_{DM})_{pl} \cong \frac{1}{2}$ and $(\Omega_{DE})_{pl} \cong 0$.
   (c) Characteristic radius and ordinary mass are: $(R_t)_{pl} \cong 2\left(\frac{c}{H_{pl}}\right)$ and $(M_t)_{pl} \cong \frac{2c^3}{GH_{pl}} \cong 2\sqrt{\frac{\hbar c}{G}} \cong 2M_{pl}$ respectively.
   (d) Characteristic thermal wavelength is,$(\lambda_{max})_{pl} \cong \frac{G(2M_{pl})}{c^2} \cong 2\left(\frac{c}{H_{pl}}\right)$.
   (e) Characteristic temperature is, $T_{pl} \cong \frac{2.898\times10^{-3}K.m}{(\lambda_{max})_{pl}} \cong \left[2.898\times10^{-3}K.m \div \frac{G(2M_{pl})}{c^2}\right] \approx 9.0\times10^{31}K$.
9. If one is willing to define M_0 as the current cosmic ordinary mass and $2M_{pl} \cong 2\sqrt{\frac{\hbar c}{G}}$ as the Planck scale cosmic mass, then $\left(\frac{1+\sqrt{\gamma_0}}{2}\right)$ seems to be the ratio of current cosmic thermal wavelength $(\lambda_{max})_0$ and $\sqrt{\left(\frac{GM_0}{c^2}\right)\left(\frac{G(2M_{pl})}{c^2}\right)} \cong (R_t)_{pl}$. Clearly speaking, $\left(\frac{1+\sqrt{\gamma_0}}{2}\right) \cong \left[\frac{2.898\times10^{-3}K.m}{T_0}\right] \div \frac{G\sqrt{M_0(2M_{pl})}}{c^2}$.
   Alternatively, $(\lambda_{max})_0 \cong \left[\frac{2.898\times10^{-3}K.m}{T_0}\right] \cong \left[\frac{(\Omega_{DM})_0}{(\Omega_{OM})_0}\right]\frac{G\sqrt{M_0(2M_{pl})}}{c^2} \cong \left(\frac{1+\sqrt{\gamma_0}}{2}\right)\frac{G\sqrt{M_0(2M_{pl})}}{c^2}$

2.3 Proposed set of quantitative assumptions

Quantitatively, above set of qualitative assumptions can be fine-tuned with respect to current cosmological observational data and past and future cosmological predictions. In this paper, we choose the following set of assumptions. With further study, quantitatively, these set of assumptions can be modified according to one‘s own choice and selection.

1. Space-time curvature follows $GM_t \cong R_t c^2$, where M_t and R_t represent the ordinary cosmic mass and radius respectively.

2. Hubble parameter associated with Planck scale is, $H_{pl} \cong \sqrt{\frac{c^5}{G\hbar}} \approx 1.86 \times 10^{43}$ sec^{-1}.

3. With reference to the Planck scale Hubble parameter, H_{pl}:

   (a) It is useful to define a number, $\gamma_t \cong \left[1 + \ln\left(\frac{H_{pl}}{H_t}\right)\right]$.

   (b) Ratio of ordinary matter density to critical density is, $(\Omega_{OM})_t \cong \left(\frac{1}{1+\gamma_t}\right)\left(\frac{1+\sqrt{\gamma_t}}{2}\right)$

   (c) Ratio of dark matter density to critical density is, $(\Omega_{DM})_t \cong \left(\frac{1}{1+\gamma_t}\right)\left(\frac{1+\sqrt{\gamma_t}}{2}\right)^2$

   (d) Ratio of dark matter density to ordinary matter density is, $\frac{(\Omega_{DM})_t}{(\Omega_{OM})_t} \cong \left(\frac{1+\sqrt{\gamma_t}}{2}\right)$

   (e) Cosmic thermal wavelength is,
   $(\lambda_{max})_t \cong \left[\frac{2.898\times10^{-3} K.m}{T_t}\right] \cong \left[\frac{(\Omega_{DM})_t}{(\Omega_{OM})_t}\right]\frac{G\sqrt{M_0(2M_{pl})}}{c^2}$
   $\cong \left(\frac{1+\sqrt{\gamma_t}}{2}\right)\frac{G\sqrt{M_t(2M_{pl})}}{c^2} \cong \left(\frac{1+\sqrt{\gamma_t}}{2}\right)\sqrt{R_t\left(R_t\right)_{pl}}$.

2.4 POSSIBLE implications of our proposed set of assumptions

1. **About the equality of ‘cosmic mass density’ and ‘ordinary matter density’**: It may be noted that, at any stage of cosmic evolution, $\frac{(\text{Cosmic mass})_t}{(\text{Cosmic volume})_t} \cong \left(\frac{3M_t}{4\pi R_t^3}\right) \cong (\Omega_{OM})_t\left(\frac{3H_t^2}{8\pi G}\right) \cong$ Visible or ordinary matter density. For various values of $(\Omega_{OM})_t$, it is possible to show that, $R_t \cong \sqrt{\frac{2}{(\Omega_{OM})_t}}\left(\frac{c}{H_t}\right)$.

2. **About the horizon problem**: The ‘horizon problem‘ or ‘homogeneity problem’ is a problem with the standard cosmological model of the hot Big Bang which was identified in the late 1960s, primarily by Charles W. Misner. It points out that different regions of the universe have not ‘contacted’ each other because of the great distances between them, but nevertheless they have the same temperature and other physical properties. If one is willing to consider the concept of ‘matter causes the space-time to curve’, ‘horizon problem’ can be understood. According to hot big bang model, during its evolution, as universe is expanding, thermal radiation temperature decreases and matter content increases. As ordinary matter content increases, based on Mach‘s principle, i.e. (with assumption 1), at any stage of evolution, it is possible to have an increasing radius of curvature,$R_t \cong \frac{GM_t}{c^2}$. Clearly speaking, for the current case, as there exists no matter outside of $R_0 \cong \frac{GM_0}{c^2}$, there is no scope for ‘causal disconnection’.

3. **About the cosmological constant problem**: With reference to assumption-2, ratio of Planck scale critical density to current critical density is, $\left(\frac{3H_{pl}^2c^2}{8\pi G}\right) \div \left(\frac{3H_0^2c^2}{8\pi G}\right) \cong \left(\frac{H_{pl}}{H_t}\right)^2 \cong 6.686 \times 10^{121}$. We wish to appeal that, our assumption-2 can be considered as a characteristic tool for constructing a model of ‘quantum gravity’.

4. **About cosmic inflation**: Mainstream cosmologists believe that the superluminal expansion period of the universe (called “cosmic inflation”) ended by 10^{-32} seconds (a tiny fraction of a second) after the hot big bang [19-21]. Since that time, they believe, expansion initially decelerated (from gravity) and then, after about 6 billion years, began very slowly to accelerate (from dark energy). Many cosmologists proposed different starting mechanisms for initiating and fine tuning the believed ‘inflation’. In this context, we would like to stress the fact that, with $(\Omega_{OM})_0 \cong \left(\frac{1}{1+\gamma_0}\right)\left(\frac{1+\sqrt{\gamma_0}}{2}\right)$ and $R_0 \cong \sqrt{\frac{2}{(\Omega_{OM})_0}}\left(\frac{c}{H_0}\right)$, estimated current cosmic

radius is 92.8 billion light years or 28.5 giga parsec and is just twice of the modern estimate [34-37]! Clearly speaking, considering our proposed assumptions, currently believed cosmic inflation can be reviewed in a very simplified approach.

5. **About CMBR anisotropy**:Observed anisotropy in current CMBR temperature can be understood with the relational condition: $\left(\frac{(\Omega_{OM})_0}{(\Omega_{DM})_0}\right)_{galaxy}$ is greater than or less than $\left(\frac{1+\sqrt{\gamma_0}}{2}\right)^{-1}$.See subsection-3.3.

6. **About thermal radiation redshift**: Redshift associated with thermal radiation can be understood with the relation: $(z+1) \approx exp\left(\frac{\gamma_0-\gamma_t}{2}\right) \approx \left(\frac{T_t}{T_0}\right)$. See application-4 of subsection-3.4.

7. **About the evolving vacuum energy**: Based on the proposed set of assumptions and cosmic density break up relations, currently believed dark energy can be identified with current vacuum energy and can be expressed by the relation: $(\Omega_{DE})_0 \left(\frac{2}{(\Omega_{OM})_0}\right)^{\frac{3}{2}} \left(\frac{c^5}{2GH_0}\right)$.

2.5 To choose the value of H_0

As per the 2015 Planck data [30], the current value of the Hubble parameter is reported to be:

1. $Planck\,TT + low\,P : (67.31 \pm 0.96)$ km/sec/Mpc.
2. $Planck\,TE + low\,P : (67.73 \pm 0.92)$ km/sec/Mpc.
3. $Planck\,TT, TE, EE + low\,P : (67.77 \pm 0.66)$ km/sec/Mpc.

According to Adam G. Riess et al and advanced observational data[31], current best value of $H_0 \cong (73.24 \pm 1.74)$ km/sec/Mpc. In this paper, we choose the lower limit, $H_0 \cong (73.24 - 1.74) \cong 71.5\,\text{km/sec/Mpc} \cong 2.35 \times 10^{-18} sec^{-1}$.
Note: In the forgoing sections, we show a procedure for fitting the observed T_0 with adopted H_0.

3 Various applications of $\gamma_t \cong \left[1 + \ln\left(\frac{H_{pl}}{H_t}\right)\right]$ in cosmology

3.1 Application-1: To estimate the current cosmic ordinary matter density, dark matter density and dark energy density

Let, $\gamma_0 \cong \left[1 + \ln\left(\frac{H_{pl}}{H_0}\right)\right] \approx 141.2$.
Current ordinary matter density can be fitted by the following relation.

$$(\Omega_{OM})_0 \cong \left(\frac{1}{1+\gamma_0}\right)\left(\frac{1+\sqrt{\gamma_0}}{2}\right) \cong 0.045 \tag{1}$$

With reference to the proposed assumptions, current dark matter density can be fitted by the following relation.

$$(\Omega_{DM})_0 \cong \left(\frac{1}{1+\gamma_0}\right)\left(\frac{1+\sqrt{\gamma_0}}{2}\right)^2 \cong 0.292 \tag{2}$$

Ratio of current dark matter density to ordinary matter density can be expressed by the following relation.

$$\left(\frac{(\Omega_{DM})_0}{(\Omega_{OM})_0}\right) \cong \left(\frac{1+\sqrt{\gamma_0}}{2}\right) \cong 6.44 \tag{3}$$

With reference to the currently believed 'flat model concepts' and Friedmann's cosmic 'density sum rule',

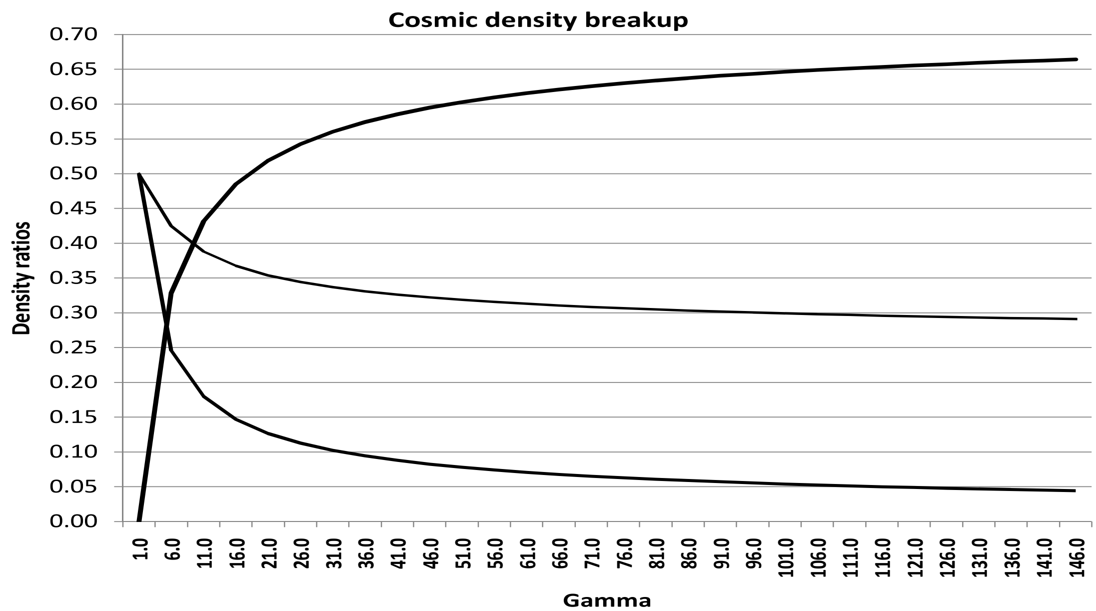
$$\begin{gathered}(\Omega_{DE})_0 \cong 1 - [(\Omega_{OM})_0 + (\Omega_{DM})_0] \\ \cong 1 - (0.045 + 0.292)) \cong 1 - 0.337 \cong 0.663\end{gathered} \tag{4}$$

At any time in the past,

$$(\Omega_{DE})_t \cong 1 - [(\Omega_{OM})_t + (\Omega_{DM})_t] \tag{5}$$

Interesting point to be noted is that, at the Planck scale, $(\Omega_{OM})_{pl} \cong (\Omega_{DM})_{pl} \cong \frac{1}{2}$ and $(\Omega_{DE})_{pl} \cong 0$. See the following figure-1. Bottom curve represents the track of $(\Omega_{OM})_t$, middle curve represent the track of $(\Omega_{DM})_t$ and top curve represents the track of $(\Omega_{DE})_t$.

Fig. 1: **Past, present and future cosmic density breakup**



3.2 Application-2: To estimate the current cosmic radius and current cosmic mass

3.2.1 About the cosmic radius:

According to modern cosmological observations, the commoving distance from Earth to the edge of the observable universe is about 14.26 Gpc (46.5 Gly = 4.40×10^{26} meters) in any direction [34-37]. The observable universe is thus a sphere with a diameter of about 28.5 Gpc = 93 Gly = 8.80×10^{26} m). Readers are suggested to see the valuable scientific information available in Wikipedia web site on 'Observational cosmology'.

According to Mihran Vardanyan et al [36], "Bayesian model averaging is a procedure to obtain parameter constraints that account for the uncertainty about the correct cosmological model. We use recent cosmological observations and Bayesian model averaging to derive tight limits on the curvature parameter, as well as robust lower bounds on the curvature radius of the Universe and its minimum size, while allowing for the possibility of an evolving dark energy component. Because flat models are favored by Bayesian model selection, we find that model-averaged constraints on the curvature and size of the Universe can be considerably stronger than non model-averaged ones. For the most conservative prior choice (based on inflationary considerations), our procedure improves on non model-averaged constraints on the curvature by a factor of 2. The curvature scale of the Universe is conservatively constrained to be R_c>42 Gpc (99 %), corresponding to a lower limit to the number of Hubble spheres in the Universe NU >251 (99%)".

With reference to our proposed assumptions, current cosmic radius (including observable and non-observable) can be estimated in the following way. If it is assumed that, $\left(\frac{3M_t}{4\pi R_t^3}\right) \cong (\Omega_{OM})_t \left(\frac{3H_t^2}{8\pi G}\right)$ and $GM_t \cong c^2 R_t$ and for various values of $(\Omega_{OM})_t$,

$$R_t \cong \sqrt{\frac{2}{(\Omega_{OM})_t}}\left(\frac{c}{H_t}\right) \tag{6}$$

For the current case,

$$R_0 \cong \sqrt{\frac{2}{(\Omega_{OM})_0}}\left(\frac{c}{H_0}\right) \cong 8.5 \times 10^{26}\, m \tag{7}$$

From our estimate, current distance (observable and non-observable) about the point of big bang is 89.6 Gly=27.5 Gpc. Clearly speaking, current universe seems to constitute 293 Hubble spheres [37].This is really a very interesting coincidence and needs further study at fundamental level. Our estimate seems to be just 2 times higher than the modern estimation. With further research and analysis and by understanding the galactic red shifts, discrepancy can be reviewed and resolved. Diameter of current (observable and non-observable) cosmic sphere about the point of big bang is 179.2Gly/54.9Gpc. See table 1. For the Planck scale,

$$(R_t)_{pl} \cong \sqrt{\frac{2}{(\Omega_{OM})_{pl}}}\left(\frac{c}{H_{pl}}\right) \cong 2\left(\frac{c}{H_{pl}}\right) \cong 2\sqrt{\frac{G\hbar}{c^3}} \tag{8}$$

Tab. 1: To fit the current cosmic radius

Estimating method	Cosmic distance from and about the reference point
Modern estimate (Observable)	46.5Gly/14.3Gpc (About Earth)
Our estimate (Observable+Non-observable)	89.6 Gly=27.5 Gpc (About point of BigBang)

3.2.2 About the ordinary cosmic mass:

With reference to the estimated cosmic radius, at any stage of cosmic evolution,

$$M_t \cong \frac{c^2 R_t}{G} \cong \sqrt{\frac{2}{(\Omega_{OM})_t}}\left(\frac{c^3}{GH_t}\right) \tag{9}$$

For the current case,

$$M_0 \cong \frac{c^2 R_0}{G} \cong \sqrt{\frac{2}{(\Omega_{OM})_0}}\left(\frac{c^3}{GH_0}\right) \cong 1.14 \times 10^{54}\, kg \tag{10}$$

For the Planck scale,

$$(M_t)_{pl} \cong \frac{c^2 (R_t)_{pl}}{G} \cong \frac{2c^3}{GH_{pl}} \cong 2\sqrt{\frac{\hbar c}{G}} \cong 2M_{pl} \tag{11}$$

3.3 Application-3: Relation between cosmic thermal wavelength and ordinary cosmic mass or cosmic radius

About thermal radiation wavelength:With reference to the observed isotropic temperature, it is very interesting to note that,

$$\begin{aligned}(\lambda_{max})_0 &\cong \left(\frac{(\Omega_{DM})_0}{(\Omega_{OM})_0}\right)\sqrt{\left(\frac{GM_0}{c^2}\right)\left(\frac{G\,(M_t)_{pl}}{c^2}\right)} \\ &\cong \left(\frac{1+\sqrt{\gamma_0}}{2}\right)\left(\frac{G\sqrt{M_0\,(M_t)_{pl}}}{c^2}\right) \\ &\cong \left(\frac{1+\sqrt{\gamma_0}}{2}\right)\sqrt{R_0\,(R_t)_{pl}} \cong 1.067\,mm\end{aligned} \tag{12}$$

$$T_0 \cong \left(\frac{2.898\times10^{-3}K.m}{(\lambda_{max})_0}\right) \cong \left(\frac{2.898\times10^{-3}K.m}{0.001067\,m}\right) \cong 2.72\,K \tag{13}$$

As per the 2015 Planck data [30], the current value of CMBR temperature is reported to be:

1. $Planck\,TT + low\,P + BAO$: (2.722 ± 0.027)K
2. $Planck\,TT, TE, EE + low\,P + BAO$: (2.718 ± 0.021)K.

With reference to Hawking's black hole temperature formula [38], current cosmic temperature can be estimated by the following relation:

$$\begin{aligned}T_0 &\cong \left(\frac{(\Omega_{OM})_0}{(\Omega_{DM})_0}\right)\left(\frac{1}{4.965}\right)\frac{hc^3}{k_BG\sqrt{M_0\,(2M_{pl})}} \\ &\cong \left(\frac{0.285}{1+\sqrt{\gamma_0}}\right)\frac{hc^3}{k_BG\sqrt{M_0M_{pl}}}.\end{aligned} \tag{14}$$

Based on this relation,

$$\frac{T_t}{T_0} \cong \left[\frac{1+\sqrt{\gamma_0}}{1+\sqrt{\gamma_t}}\right]\left[\frac{(\Omega_{OM})_t}{(\Omega_{OM})_0}\right]^{\frac{1}{4}}\sqrt{\frac{H_t}{H_0}} \tag{15}$$

It is also possible to co-relate the critical energy density and thermal energy density in the following way. At any stage of cosmic evolution,

$$\begin{aligned}\frac{3H_t^2c^2}{8\pi G\,(aT_t^4)} &\approx \left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right]^2 \approx \gamma_t^2 \\ T_0 \approx \left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right]^{-\frac{1}{2}}\left(\frac{3H_t^2c^2}{8\pi Ga}\right)^{\frac{1}{4}} &\approx (\gamma_t)^{-\frac{1}{2}}\left(\frac{3H_t^2c^2}{8\pi Ga}\right)^{\frac{1}{4}}\end{aligned} \tag{16}$$

Note: In our earlier publications [6-15], we proposed that, $T_t \cong \frac{\hbar c^3}{8\pi k_BG\sqrt{M_tM_{pl}}}$ where $M_t \cong \frac{c^3}{2GH_t}$ is the mass of universe.

3.3.1 To understand the current CMBR temperature anisotropy

Observed anisotropy in current CMBR temperature can be understood in the following way. For any galaxy,

1. $(T_0)_{galaxy}$ is on higher side, if
   (a) $\left(\frac{(\Omega_{OM})_0}{(\Omega_{DM})_0}\right)_{galaxy} > \left(\frac{1+\sqrt{\gamma_0}}{2}\right)^{-1}$
   (b) $[(\Omega_{OM})_0]_{galaxy} > \left[\left(\frac{1+\sqrt{\gamma_0}}{2}\right)\left(\frac{1}{1+\gamma_0}\right)\right]$

   (c) $[(\Omega_{DM})_0]_{galaxy} < \left[\left(\frac{1+\sqrt{\gamma_0}}{2}\right)^2\left(\frac{1}{1+\gamma_0}\right)\right]$

2. $(T_0)_{galaxy}$ is on lower side, if

   (a) $\left(\frac{(\Omega_{OM})_0}{(\Omega_{DM})_0}\right)_{galaxy} < \left[\left(\frac{1+\sqrt{\gamma_0}}{2}\right)\right]^{-1}$.

   (b) $[(\Omega_{OM})_0]_{galaxy} < \left[\left(\frac{1+\sqrt{\gamma_0}}{2}\right)\left(\frac{1}{1+\gamma_0}\right)\right]$

   (c) $[(\Omega_{DM})_0]_{galaxy} > \left[\left(\frac{1+\sqrt{\gamma_0}}{2}\right)^2\left(\frac{1}{1+\gamma_0}\right)\right]$

3.4 Application-4:To understand the CMBR redshift and scale factor

With the proposed γ_0 and γ_t it is possible to fit the CMBR redshift z and scale factor $\left[a_t = \frac{1}{z+1}\right]$ in the following way. With reference to current phase and for the past cosmic evolution,

$$\text{Let,}\,(z+1) \cong \frac{T_t}{T_0} \cong exp\left(\frac{\gamma_0-\gamma_t}{2}\right) \tag{17}$$

Based on this relation and for any value of z,

$$\begin{aligned} \gamma_t &\cong \gamma_0 - [2\ln(z+1)] \\ H_t &\cong \frac{H_{pl}}{exp\,(\gamma_t-1)} \cong (z+1)^2 H_0 \\ (\lambda_{max})_t &\cong \left(\frac{1+\sqrt{\gamma_t}}{2}\right)\sqrt{R_t\,(R_t)_{pl}} \\ T_t &\cong \frac{2.898\times10^{-3}K.m}{(\lambda_{max})_t} \end{aligned} \tag{18}$$

With reference to the currently believed CMBR redhift of 1090, obtained γ_t, Hubble parameter and temperature are: $(\gamma_t)_{z\sim1090} \cong 127.23$, $(H_t)_{z\sim1090} \cong 2.8\times10^{-12}sec^{-1}$, $(M_t)_{z\sim1090} \cong 9.33\times10^{47}\,kg$, $(\lambda_{max})_{z\sim1090} \cong 9.2\times10^{-7}\,m$ and $(T_t)_{z\sim1090} \cong 3154.0\,K$ respectively. This fit or coincidence, directly and indirectly supports the possible role of the proposed γ term. In addition, relations (17) and (18) can be applied to the past cosmic evolution also. We noticed that,

$$\begin{aligned} (z+1) &\approx \frac{T_t}{T_0} \approx exp\left(\frac{\gamma_0-\gamma_t}{2}\right) \approx \sqrt{\frac{H_t}{H_0}} \\ \to \ln(z+1) &\approx \ln\left(\frac{T_t}{T_0}\right) \approx \left(\frac{\gamma_0-\gamma_t}{2}\right) \approx \ln\sqrt{\frac{H_t}{H_0}} \end{aligned} \tag{19}$$

3.5 Application-5:To understand the redshift dependent cosmic age

With reference to the currently believed CMBR radiation and its corresponding age of 3,80,000 years and by considering the above relations (17) to (19), cosmic age can be understood by the following relation.

$$\begin{aligned} t_z &\approx \frac{(1+z)^{-\frac{3}{2}}}{H_0} \approx \left(\frac{\sqrt{z+1}}{H_t}\right) \approx \left(H_t^{0.75}H_0^{0.25}\right)^{-1} \\ &\approx \left(\frac{1}{H_t}\right)\sqrt{\frac{T_t}{T_0}} \approx \left(\frac{1}{H_t}\right)\sqrt{exp\left(\frac{\gamma_0-\gamma_t}{2}\right)} \end{aligned} \tag{20}$$

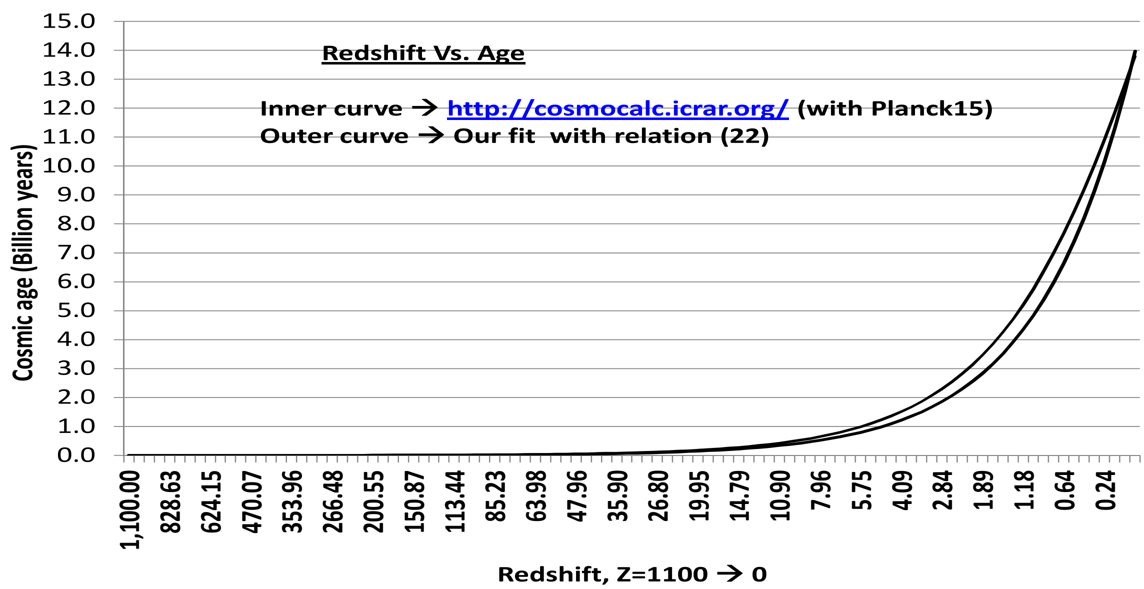
Cosmic age corresponding to a temperature of 3154 K, CMBR redshift of 1090 and Hubble parameter of $2.8\times10^{-12}sec^{-1}$ can be estimated by the following relation.

$$t_{z\sim1090} \approx \left[\frac{\sqrt{1091}}{(H_t)_{z\sim1090}}\right] \approx 3,74,264 \quad \text{years} \tag{21}$$

For the current case, $z = 0$ and $t_0 \cong \left(\frac{1}{H_0}\right)$. If one is willing to apply relation (19) to the Planck scale, redshift based cosmic age seems to begin at around 9.0×10^{-29} sec and its corresponding Hubble age is, $H_{pl}^{-1} \cong 5.4 \times 10^{-44}$ sec. It needs further study. See the following figure 2 for the estimated redshift based cosmic age. In this figure 2, inner curve refers to the data taken from http://cosmocalc.icrar.org/ with Planck15 survey within the range $z = 0$ to 1100. Outer curve is our approximate fit with the following relations.

$$\gamma_t \cong \gamma_t - [2\ln(z+1)]\,, H_t \cong \frac{H_{pl}}{exp\,(\gamma_t - 1)} \text{ and } t_z \approx \left(\frac{\sqrt{z+1}}{H_t}\right) \tag{22}$$

Fig. 2: **Redshift Vs. Cosmic age**



3.6 Application-6: To interpret the cosmic expansion velocity

Based on the estimated cosmic matter density, it is possible to interpret the cosmic expansion velocity V_t in the following way.At any stage of cosmic evolution,

$$\frac{V_t}{c} \cong \sqrt{\frac{2}{(\Omega_{OM})_t}} \rightarrow V_t \cong \sqrt{\frac{2}{(\Omega_{OM})_t}}c \tag{23}$$

For the current case,

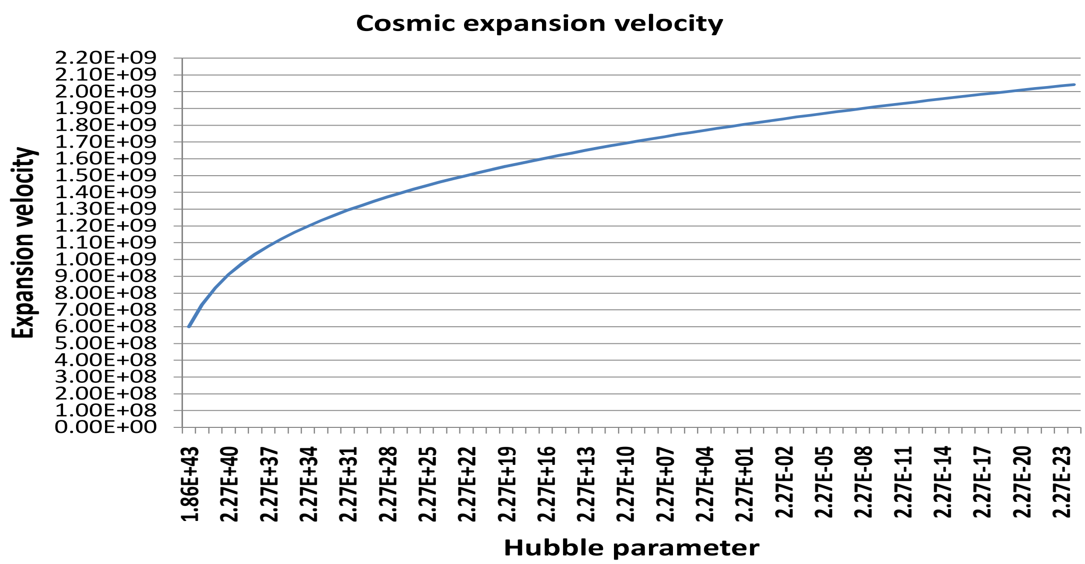
$$\begin{aligned} \frac{V_0}{c} &\cong \sqrt{\frac{2}{(\Omega_{OM})_0}} \cong 6.65 \\ \rightarrow V_0 &\cong \sqrt{\frac{2}{(\Omega_{OM})_0}}c \cong 2.0 \times 10^9\, m.sec^{-1} \end{aligned} \tag{24}$$

For the Planck scale,

$$\begin{aligned} \frac{V_{pl}}{c} &\cong \sqrt{\frac{2}{(\Omega_{OM})_{pl}}} \cong 2 \\ \rightarrow V_{pl} &\cong \sqrt{\frac{2}{(\Omega_{OM})_{pl}}} c \cong 6.0 \times 10^8 \, m.sec^{-1} \end{aligned} \tag{25}$$

See the following figure 3 for the increasing cosmic expansion velocity.

Fig. 3: **Increasing cosmic expansion velocity**



3.7 Application-7: To estimate the galactic receding speeds and galactic distances in the current expanding universe

Based on relations (6) and (7), within the current radius of 89.6 Gly=27.5 Gpc, from and about the point of big bang, galactic receding speeds can be approximated by the following relation.

$$(v_g)_0 \cong \left(\frac{(d_g)_0}{R_0}\right) V_0 \cong \left(\frac{(d_g)_0}{R_0}\right) 6.65c \tag{26}$$

where $(d_g)_0$ is the current galactic distance from the point of big bang and $(v_g)_0$ is the current galactic receding speed. Based on this relation, within the current radius of 92.8 Gly=28.5 Gpc, galactic distances corresponding to assumed galactic receding speeds can be expressed in the following way.

$$\begin{gathered} (d_g)_0 \cong \left(\frac{(v_g)_0}{V_0}\right) R_0 \cong \left(\frac{R_0}{V_0}\right) (v_g)_0 \cong \frac{(v_g)_0}{H_0} \\ \text{where } 0 \geq (v_g)_0 \leq 6.65c \end{gathered} \tag{27}$$

From and about the point of big bang, by co-relating the 'actual' galactic distances and 'actual' galactic receding speeds with observed galactic red shifts, further research can be carried out.

4 Discussion and conclusion

Points to be noted in this toy model are:

1. We have successfully implemented the Planck scale in current cosmological observations.
2. We have perfectly connected the current Hubble parameter and current cosmic temperature.
3. We have successfully implemented Mach's principle and Holographic principle in modern cosmological observations.
4. We have fitted current ordinary matter density and dark matter density with reference to current and Planck scale Hubble parameters and by following the 'density sum rule', we have fitted the current dark energy density.
5. We have estimated current cosmic radius, ordinary mass, velocity and age with reference to the current ordinary matter density.

Proceeding further, with the proposed set of assumptions,

1. Extrapolation to past and future is very easy.
2. With further study and observations, actual galactic distances, actual galactic receding speeds and observed galactic redshifts can be studied in a unified approach.
3. With minor changes and with further study, a unified model of quantum cosmology can be developed.

In any model of cosmology, fundamental questions to be solved are: 1) Why do 'dark matter' and 'ordinary matter' have their measured values of $\approx 33\%$ of critical energy? 2) Why do 'dark energy' has its measured values of $\approx 68\%$ of critical energy? 3) How to estimate their past and future magnitudes? These are the puzzling questions raised by the Royal Swedish Academy of Sciences [29] in 2011. In the conclusion part, Royal Swedish Academy of Sciences quoted like this: "The study of distant supernovae constitutes a crucial contribution to cosmology. Together with galaxy clustering and the CMB anisotropy measurements, it allows precise determination of cosmological parameters. The observations present us with a challenge, however: What is the source of the dark energy that drives the accelerating expansion of the Universe? Or is our understanding of gravity as described by general relativity insufficient? Or was Einstein's "mistake" of introducing the cosmological constant one more stroke of his genius? Many new experimental efforts are under-way to help shed light on these questions".

In this context, we appeal that, our set of assumptions can be given some consideration and with further research, their scope and workability can be scrutinized and validated.

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