Effect of Prandtl number and Inclination angle on MHD Natural Convection in Inclined Open Square Cavity

Abstract

MHD natural convection in open cavity becomes very important in many scientific and engineering problems, because it's wide range of applications in Physical science and thermal engineering problems. For example, in the design of electronic devices, solar thermal receivers, uncovered flat plate solar collectors having rows of vertical strips, geothermal reservoirs, etc. Several experimental and numerical investigation have been presented for describing the phenomenon of natural convection in open cavity for two decades. MHD natural convection and fluid flow in a two-dimensional open square cavity with inclination angle was considered in this work. The opposite wall to the opening side of the cavity was first kept to constant heat flux *q*, at the same time the surrounding fluid interacting with the aperture was maintained to an ambient temperature T_∞. The top and bottom wall was kept to low and high temperature respectively. As a result a buoyancy force is created inside the cavity due to temperature difference. Thus a natural convection is occurred inside the cavity. The governing equations for mass; momentum and energy conservation are expressed in a normalized primitive variables formulation. The streamlines and isotherms are produced, heat transfer parameter Nu are obtained for Prandtl number Pr = 0.72, 2, 5, 7 and inclination angles from 0^o, 5^o, 20^o, 35^o 50^o for fixed Hartmann number 60. The results are presented in graphical as well as tabular form. In the result it is found that heat flux is a increasing function of Prandtl number Pr while Rayleigh number is 10000 and heat flux is maximum when inclination angle is 5^o. It is observed that fluid moves counterclockwise around the cylinder. Various recirculations are formed around the cylinder and one small vortex is formed into the flow field for 50^o inclination and Pr = 0.72 near the cylinder. The almost all isotherm lines are concentrated at the right lower corner of the cavity. The present result is in a good agreement to the existent heat transfer and boundary layer theory.

Introduction

The Prandtl Number Pr is defined as the ratio of the kinematic viscosity and thermal diffusivity $Pr = \frac{\upsilon}{\alpha}$. Here υ is the kinematic viscosity and α is the thermal diffusivity. Prandtl number is named after **Ludwig Prandtl**, who introduced the concept of boundary layer in 1904 and made significant contributions to boundary layer theory.

Finite element analysis based on Galerkin weighted Residual method is used to solve the problem. Using the consistent set of boundary conditions and values of parameters it is observed that the heat transfer rate is a increasing function of Prandtl number Pr in the cavity. The results are presented in graphical as well as tabular form.

Alfven Hannes [1], (1940) is the pioneer of the field of MHD for which he received the Nobel_Prize in Physics in 1970. Ben-Nakhi, A. et al [2], (2008) studied a computational study of steady laminar natural convective fluid flow in a partially open square enclosure with a highly conductive thin fin of arbitrary length attached to the hot wall at various levels. They considered the horizontal walls and the partially open vertical adiabatic wall while the vertical wall facing the partial opening is isothermally hot. The investigation was on the flow modification due to the (i) attachment of a highly conductive thin fin of length equal to 20%, 35% or 50% of the enclosure width, attached to the hot wall at different heights and (ii) variation of the size and height of the aperture located on the vertical wall facing the hot wall. They also examine the impact of Rayleigh number ($10^4 \leq Ra \leq 10^7$) and inclination of the enclosure. To solve the problem numerically they used finite volume method of numerical techniques. They observed that the presence of the fin has counteracting effects on flow and temperature fields. These effects are dependent in a complex way, on the fin level and length, aperture altitude and size, cavity inclination angle and Rayleigh number. Furthermore, a longer fin causes higher rate of heat transfer to the fluid, although the equivalent finless cavity may have higher heat transfer rate. In general, the volumetric flow rate and the rate of heat loss from the hot surfaces are interrelated and are increasing functions of Rayleigh number. Bilgen, E.,[3], (2005) studied numerically natural convection heat transfer in differentially heated square cavities with horizontal thin fin. He found that normalized Nusselt number is an increasing function of Rayleigh number and decreasing function of fin length and relative conductivity ratio. The study related to heat absorption or rejection in the confined rectangular enclosures has been well discussed in the literature C. Taylor and P. Hood [4], Chandrasekhar [5].

Chan and Tien [6] investigated shallow open cavities and made a comparison study using a square cavity in an enlarged computational domain. In the result they observed that for a square open cavity having an isothermal vertical side facing the opening and two adjoining adiabatic horizontal sides. Satisfactory heat transfer results could be obtained, especially at high Rayleigh numbers.. Davis, S H. [7], Ostrach [12], Hossain and Wilson [8], Hossain et. al [9] studied natural convection heat transfer in rectangular enclosure. Hossain, S. A. and Alim M. A. [10], (2012) studied on natural convection heat transfer in an open rectangular cavity containing a heated circular cylinder. In the result they found that the heat transfer increases as Grashof number increases from 10^3 to 10^6.

Mohammad, A. A. [11] investigated inclined open square cavities, by considering a restricted computational domain. The gradients of both velocity components were set to zero at the opening plane in that case which were different from that of Chan and Tien [6]. In the result he found that heat transfer was not sensitive to inclination angle and the flow was unstable at high Rayleigh numbers and small inclination angles.

Reddy, J. N. [13] (2006) illustrated the Finite Element Method in a text. Roy and Basak [14] analyzed natural convection flows in a square cavity with non-uniformly heated wall(s) using finite element method. S. Pervin, R. Nasrin[15], (2011) studied MHD free convection and heat transfer for different values of Raleigh numbers Ra and Hartmann numbers Ha in a rectangular enclosure. Their results show that the flow pattern and temperature field are significantly dependent on the used parameters. Saha S. C. [16] studied thermo-magnetic convection and heat transfer of paramagnetic fluid in an open square cavity with different boundary conditions. His results show the Effects of Magnetic Rayleigh number, Prandtl number on the flow pattern and isotherm as well as on the heat absorption graphically. He found that the heat transfer rate is suppressed in decreased of the Magnetic Rayleigh number. Sarris et al [17] studied MHD natural convection in a laterally and volumetrically heated square cavity.

Objectives

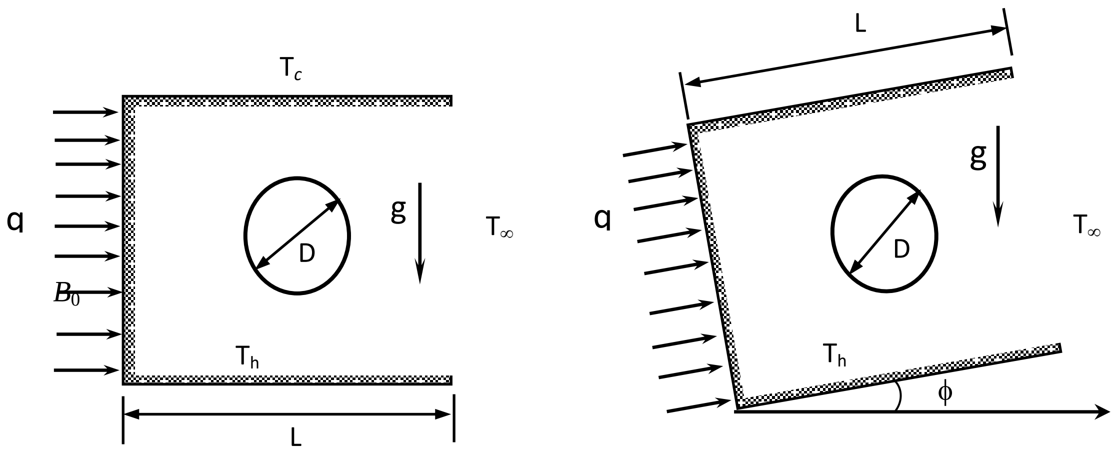
The objectives of this study are to analyze numerically the effect of Prandtl number Pr on MHD natural convection heat transfer and fluid flow inside the cavity by visualizing the fluid flow in terms of isotherms and streamlines respectively. To develop a Mathematical model regarding the effect of Prndtl number Pr on MHD natural convection flow around a heated circular cylinder at the centre of an inclined square cavity.

Research Methodology

Finite Element Methods (FEM) is a method to solve differential equations numerically which can be applied to many problems in engineering and scientific fields. Galerkin's Weighted Residual Finite Element Method is applied to descretize the non-dimensional governing equations. Triangular mesh is used to obtain the solution.

PHYSICAL FIGURE OF THE PROBLEM

In this study heat transfer and the fluid flow in a two-dimensional open rectangular cavity of length L was considered, which is shown in the following schematic diagram in Figure Fig. 1.



(a) Horizontal cavity (b) Inclined cavity

Fig. 1**:** Schematic diagram of the physical problem

Mathematical formulation

The flow inside the cavity is assumed to be two-dimensional, steady, laminar, and incompressible and the fluid properties are to be constant. The radiation effects are taken negligible and the Boussinesq approximation is used. The dimensionless governing equations describing the flow are as follows:

Continuity equation: $$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (1)$$

Momentum equations:

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\sqrt{Gr}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)+\theta\sin\phi \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\sqrt{Gr}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\theta\cos\phi+g\rho\beta(T_h-T_c)-\sigma B_o^2 v \tag{3}$$

Energy equation:

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\Pr\sqrt{Gr}}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

At Bottom wall: $U=V=0;\ \ \theta=1,\ \ 0\le X\le 1$ and Y = 0

At Top wall: $U=V=0\ ;\ \ \theta=0, 0\le X\le 1$ and $Y=1$

At the left wall : U =V =0 ; heat flux q = 150 , X = 0, 0 $\le$ Y $\le$ 1

At the right side & open side: Convective Boundary Condition (CBC), P = 0, U = V.

Used non-dimensional scales:

$$X=\frac{x}{L},\ \ Y=\frac{y}{L},\ \ U=\frac{u}{U_0},\ \ V=\frac{v}{U_0},\ \ P=\frac{P-P_\infty}{\rho\ U_0^2},\ \ \theta=\frac{T-T_\infty}{\Delta t}, \Pr=\frac{v}{\alpha},\ \ Gr=\frac{g\beta\Delta TL^3}{v^2}$$

$$\text{Ha}=\sqrt{\frac{\sigma L^3 B^2{}_0}{\mu}}\ ,\ dr=\frac{D}{L},\ \ \ \Delta t=\left(T_s-T_\infty\right)\ ,\ \Delta t=\frac{qL}{K}$$

Used Heat Transfer Characteristics

The flow field in MHD natural convection is governed by the dimensionless Rayleigh Number Ra. The Rayleigh number is defined as the product of Grashof number and Prandtl number.

i.e. Ra = Gr.Pr

Here the Grashof number represents the ratio of the buoyancy force to the viscous force acting on the fluid and the reference velocity U_o is related to the buoyancy force term and is defined as

$$U_0=\sqrt{g\beta L\left(T_w-T_\infty\right)}$$

Here, g is the gravitational acceleration, β is volumetric thermal expansion coefficient, 1/T (β = 1/T for ideal gas), T_s is temperature at the surface, T_∞ is the temperature of the fluid sufficiently far from the wall. The Nusselt number Nu is also an important non-dimensional parameter to be computed for heat transfer analysis in natural convection flow. The Nusselt number for natural convection is a function of the Grashof number only. The local Nusselt number Nu can be obtained from the temperature field by applying the following function $Nu = -\frac{1}{\theta(0,Y)}$

The overall or average Nusselt number was calculated by integrating the temperature gradient over the heated wall as follows:

$$Nu_{av} = -\int_0^1 \frac{1}{\theta(0,Y)} dy$$

Pr is a heat transfer characteristics in the flow field of natural convection. Since the dimensionless Prandtl Number Pr is the ratio of kinematic viscosity to thermal diffusivity.

Results and Discussion

A numerical study has been done on steady state MHD natural convection flow in a two-dimensional square open cavity with inclination angle. The left vertical wall of the cavity is at a constant heat flux as shown in Figure 1. A heated circular cylinder is placed at the centre of the cavity. The bottom and top walls are kept at high and cool temperature respectively. Two-dimensional Navier–Stokes equations along with the energy equations are the governing equations and are solved using Gelarkin’s Finite Element Method. The results are also presented in terms of streamlines and isotherm patterns. The effects of variations of Prandtl number and inclination angles of the cavity are predicted. The results are computed for Ha = 60; Ra = 10^4, at Pr = 0.72, 2.0, 5.0 and 7.0 while inclination angles of the cavity are $\phi = 0^o, 5^o, 20^o, 35^o, 50^o$ with hot temperatures at the bottom wall and cylinder. Other physical properties are assumed to be constant. Here the parametric analysis for a wide range of governing parameters shows consistence performance of the present numerical approach to compute as temperature profiles and streamlines. All the results of this work are computed for dr = 0.2.

Effects of Prandtl number Pr

Due to increase of inclination angle, complex flow pattern characteristics were found for various Prandtl numbers Pr. This profile of isotherms, streamlines and inclination angles 0^o, 5^o,20^o, 35^o and 50^o of the cavity are presented in Fig. 3 to 7. The isotherms are remained at the right half of the cavity and the isotherms are concentrated at right lower corner of the cavity. Recirculations are formed around the cylinder in the cavity. But one small vortex is formed in the cavity for Pr = 0.72 and angle 50^o.

Here the total heat flux decreases mainly for higher inclination angles. This is because of the opening side become the upper side and the temperature difference become less than those of previous position and buoyancy effect is less than before. It is also observe that heat flux increases for higher Prandtl numbers. The isotherms and streamlines for dr = 0.2, Pr = 0.72 ,Ha = 30, T_h = 330^{o}k, T_c =276^{o}k while inclination angles ϕ = 0^o, 5^o, 20^o 35^o, 50^o are shown in Fig. 3. In Fig. 3 heat flux is decreases for higher inclination angles. The isotherms are concentrated at the right lower corner of the cavity. The fluid flows form recirculations around the cylinder in the cavity, but one small vortex is formed near the cylinder for the inclination angle 50^o and Pr = 0.72. The total heat flux of the open cavity for Rayleigh numbers = 10^4 , inclination angles range of 0^o to 50^o and Pr = 0.72, 2.0, 5.0,7.0 are shown in the line graphs from Fig. 8 to Fig.11 respectively and these results are tabulated in the Table 1.The results show that heat flux decrease as inclination angles increase from 5^o to 50^o. The heat flux is maximum for the maximum Prandtl number Pr = 7 and angle 5^o.

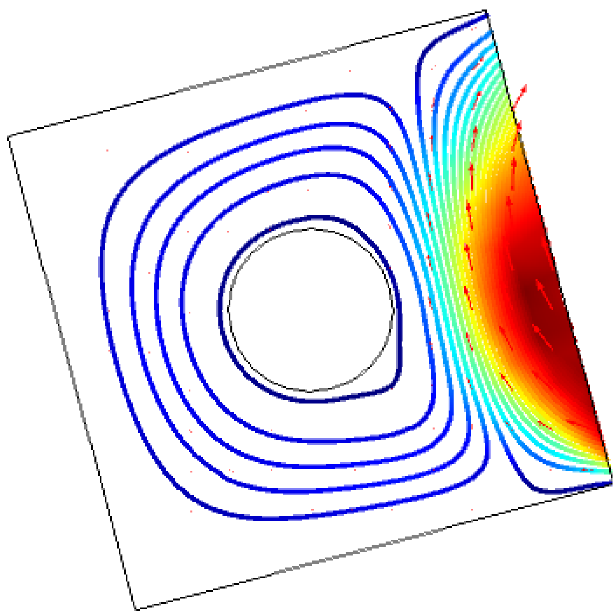


Fig. 2 Streamlines with direction around the cylinder for Pr = 1, Ha = 60, q = 150, ϕ = 15^o.

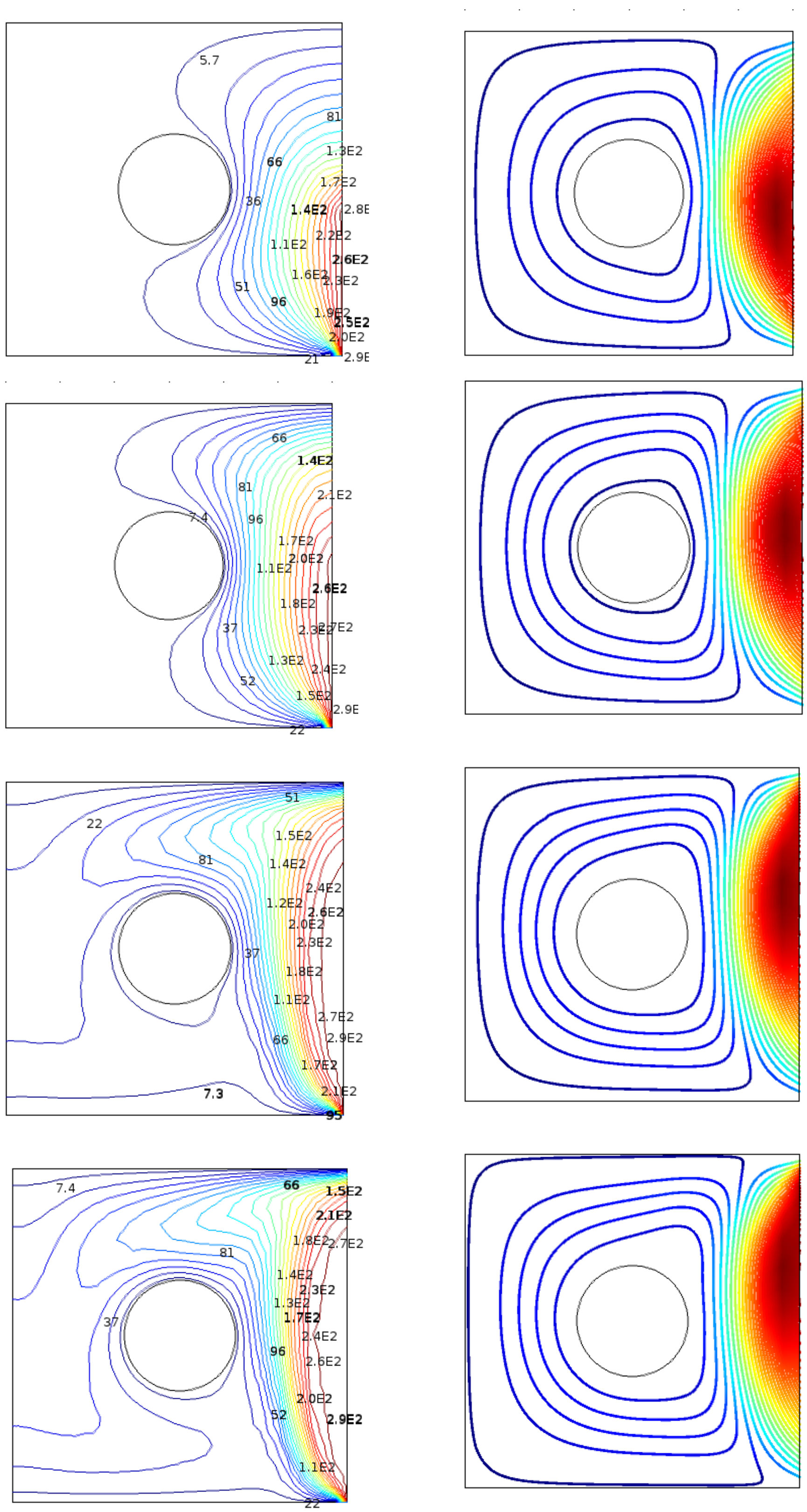
This Figure 2 shows that the fluid flow counterclockwise around the cylinder.

Pr = 0 .72

Pr = 2

Pr = 5

Pr = 7



(a) (b)

Fig.3.Isotherms (a) & Streamlines (b) in the cavity for various Pr and Ha = 60,Ra=10^5, $\phi = 0^o$.

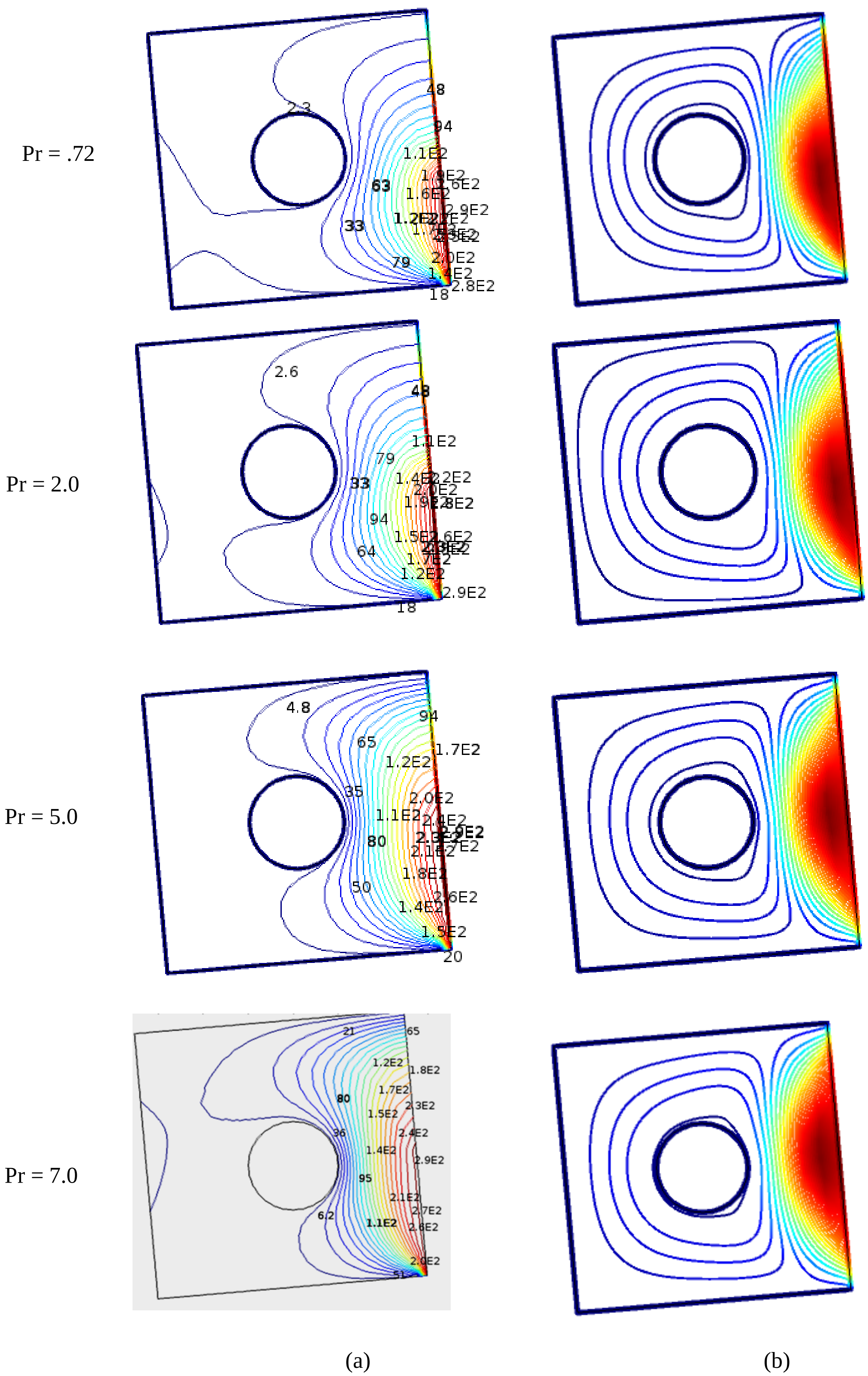
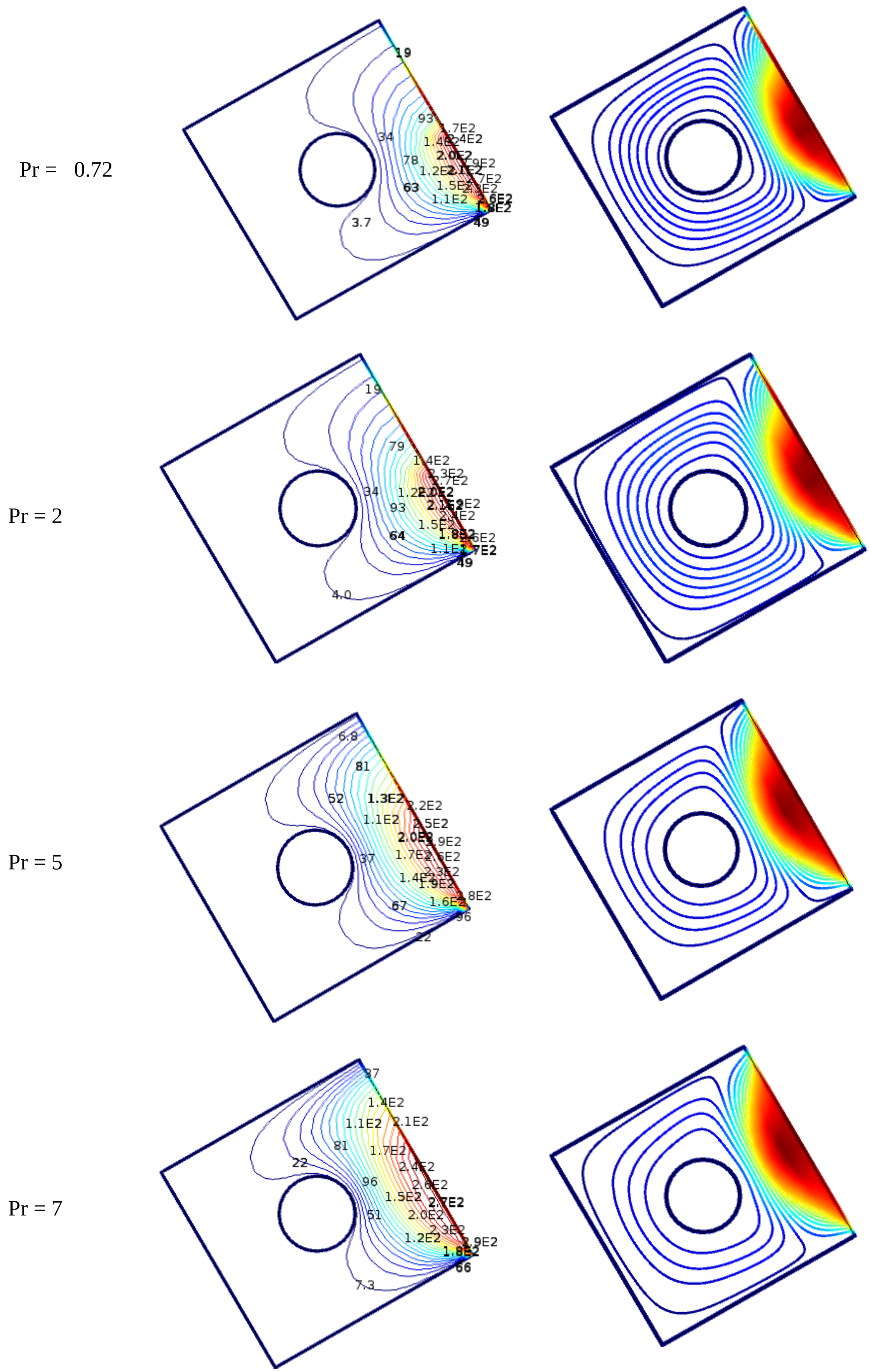
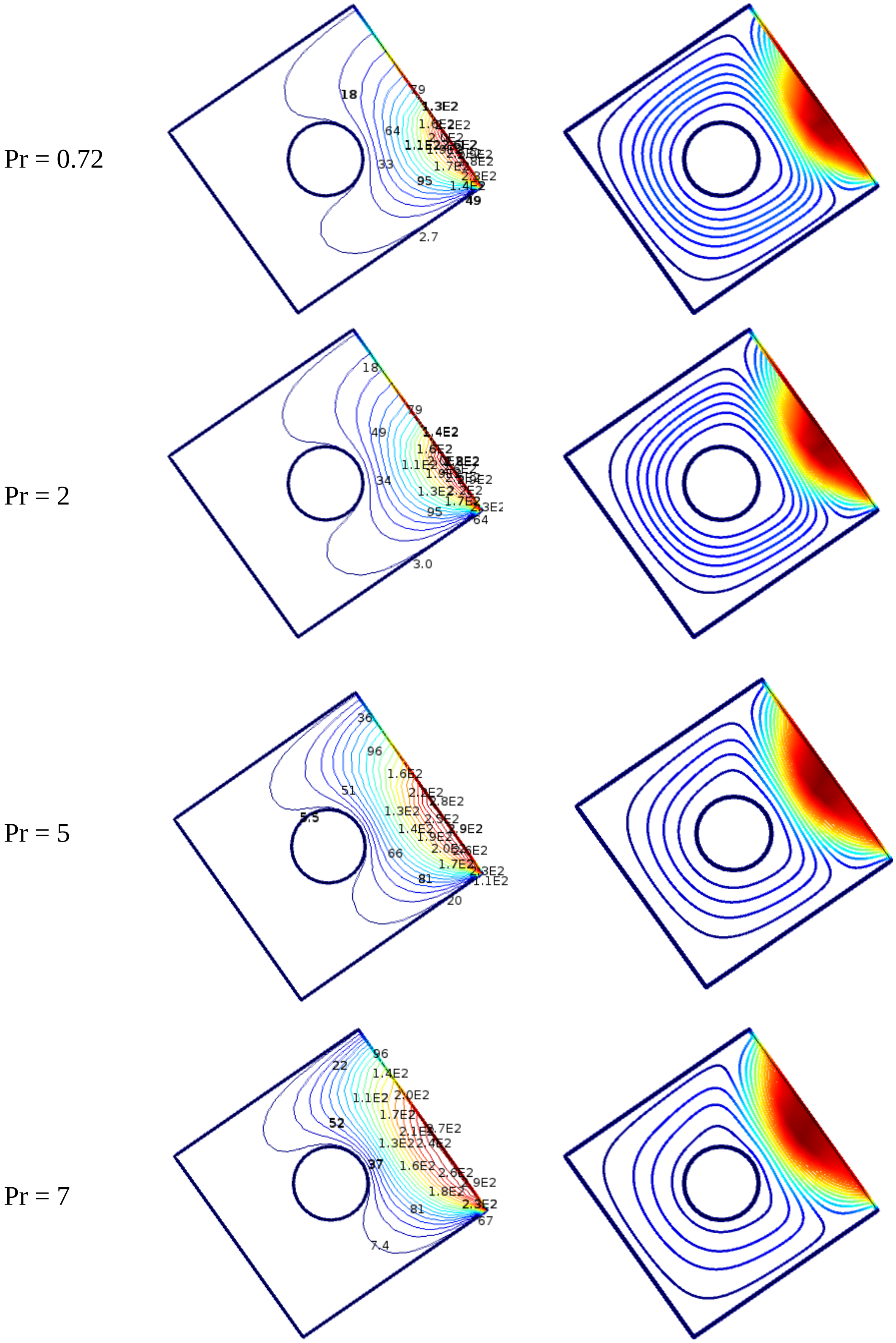


Fig.4.Isotherms (a) & Streamlines (b) for various Pr and Ra = 10^4, Ha= 60, $\phi = 5^o$ in the cavity.



(a) (b)

Fig. .5 .Isotherms (a) & Streamlines (b) in the cavity for various Pr while Ha = 60,Ra=10^4, ϕ=20^o



(a) (b)

Fig. 6. Isotherms (a) & Streamlines (b) for various Pr while Ha = 60,Ra=10^4, $\phi = 35^o$ in the cavity .

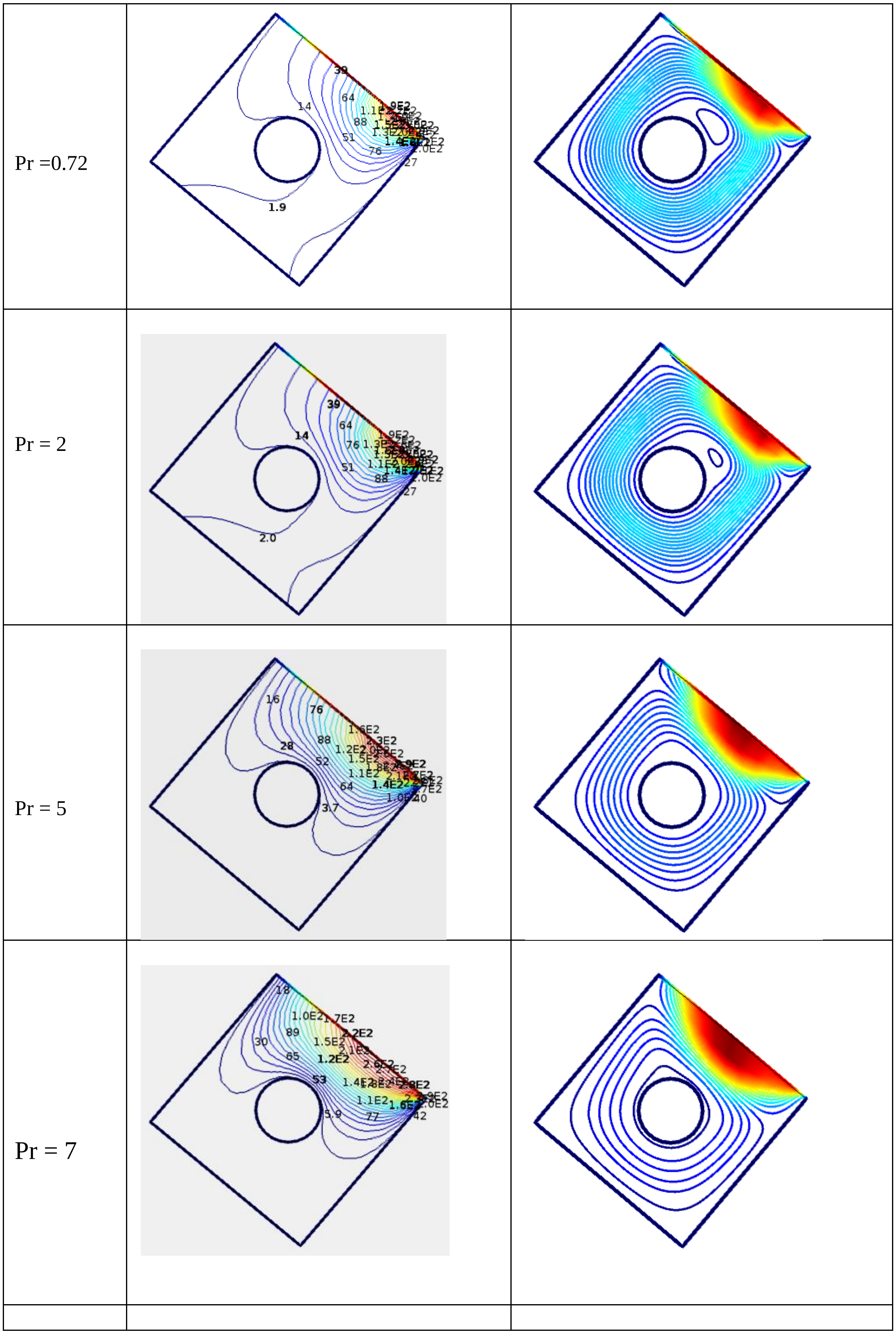


Fig. 7. Isotherms (a) & Streamlines (b) in the cavity for various Pr while Ha = 60,Ra=10^4, $\phi = 50^o$.

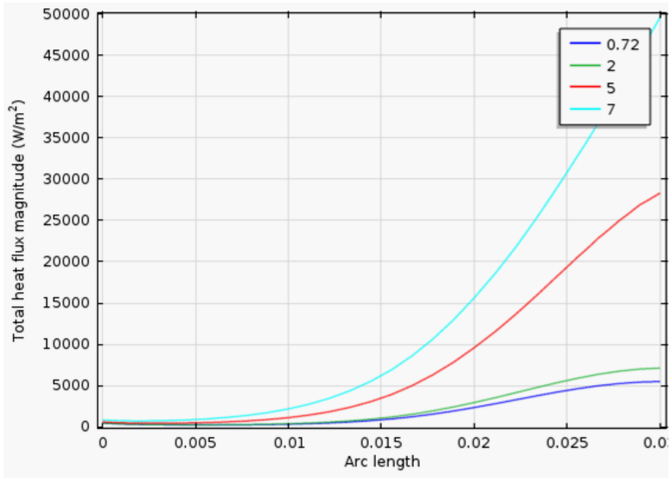


Fig. 8. Line graphs of total Heat Flux for Pr = 0 .72, 2, 5, 7, while Ha = 60, Ra=10^4, $\phi = 5^o$.

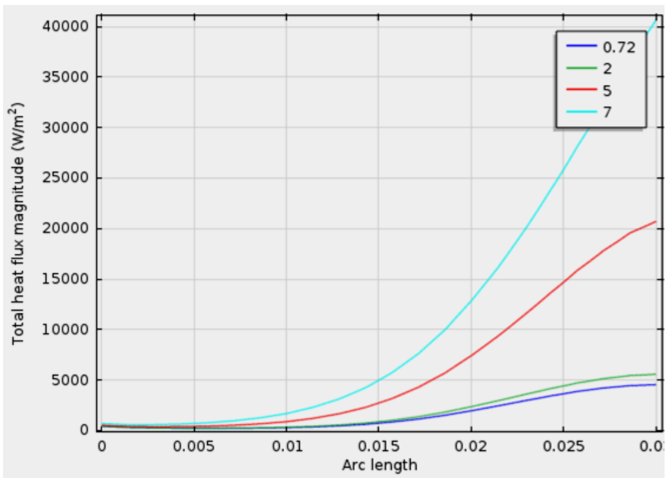


Fig. 9. Line graphs of total Heat Flux for Pr = 0 .72, 2, 5, 7, while Ha = 60, Ra=10^4, $\phi = 20^o$ in the cavity.

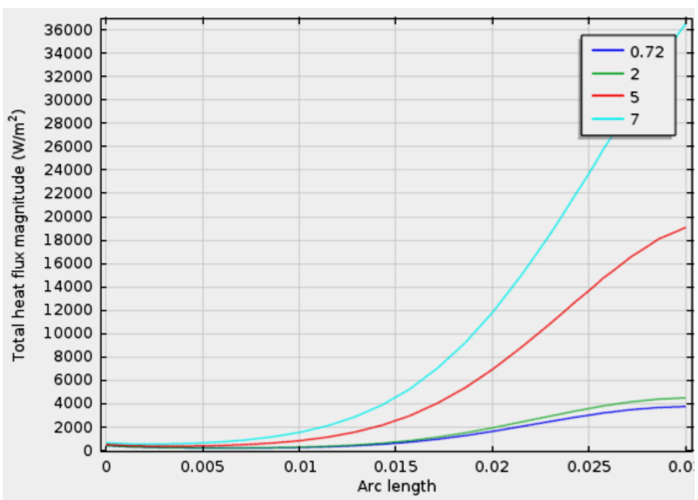


Fig. 10 Line graphs of total Heat Flux for Pr = 0 .72, 2, 5, 7, while Ha = 60, Ra=10^4, $\phi= 35^o$.

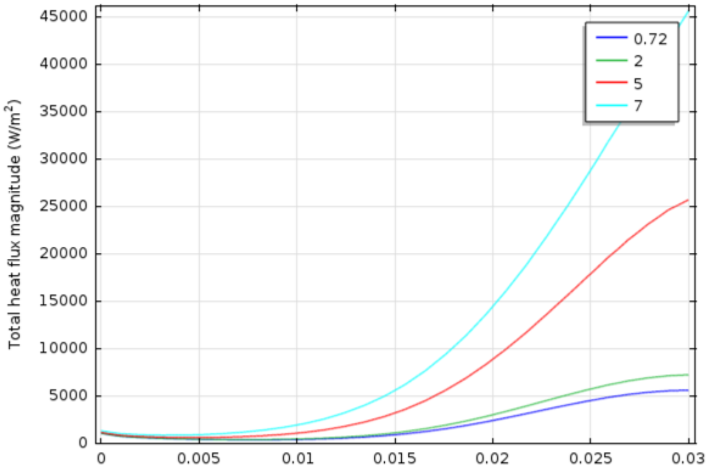


Fig.11 Line graphs of total Heat Flux for Pr = 0 .72, 2, 5, 7, while Ha = 60, Ra=10^4, ϕ= 0^o in the cavity.

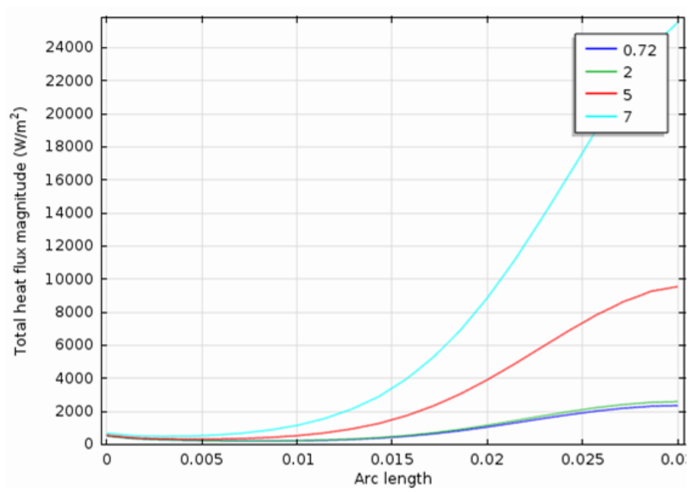


Fig. 12 Line graphs of total Heat Flux for Pr = 0 .72, 2, 5, 7, while Ha = 60, Ra=10^4, ϕ= 50^o in the cavity

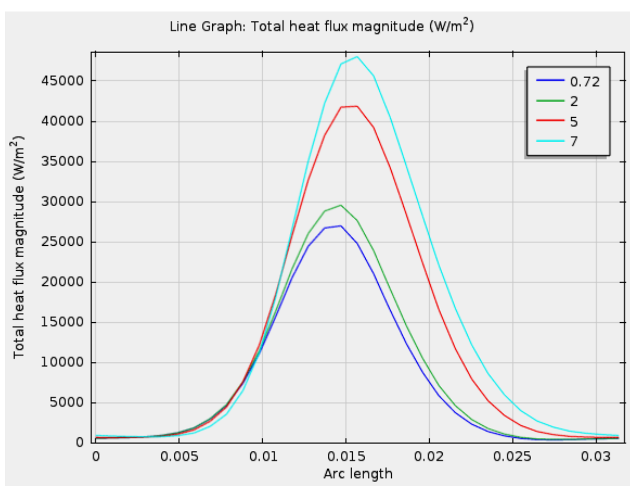


Fig.13. Line graphs of Heat Flux at the cylinder for Pr = 0 .72, 2, 5, 7, while Ha = 60, Ra=10^4, ϕ= 50^o

The effects of Prandtl number on heat flux in the cavity for various inclination angles are shown in the line graphs from Fig.8 to Fig.12 and these results are tabulated in the Table 1 below for Ha = 60.

Table 1. Heat flux for various angles and Pr while Ha = 60, Ra = 10^4

Angle = ϕ	0^o	5^o	20^o	35^o	50^o
Heat flux for Pr = 0.72	5400	5400	4900	4000	2200
Heat flux for Pr = 2.0	7500	7500	5100	4500	2300
Heat flux for Pr = 5.0	25500	28000	20100	19000	9000
Heat flux for Pr = 7.0	45000	49500	40000	36000	25500

The line graphs in Fig.8 to Fig. 12 are show that heat flux is maximum when Prandtl number Pr is maximum but heat flux is affected inversely by inclination angle of the cavity from 5^o to 50^o. It is found that heat flux is maximum at inclination angle 5^o for variation of Pr.

Conclusion

Finite element method is used to solve the present physical problem and analyze the effects of inclination angles 0^o to 50^o on heat flux and fluid flow for steady- state, incompressible , laminar and MHD free convection flow in a square open cavity containing a heated circular cylinder. The flow with Ra = 10000 in this work have been affected by the buoyancy force. Temperature fields are illustrated in the flow region. The high temperature region remains at the lower portion near the open side of the cavity and the isothermal lines are nonlinear for all inclination angles used in this study and they occupied almost half of the region of the cavity near the open side. The significant findings of this work are that for all cases of inclinations and Ra the isothermal lines concentrated to the right lower corner of the cavity and there are recirculations around the cylinder counterclockwise and one small vortex has been created in the flow field while angle 45^o and Pr = 0.72. Inclination angle inversely affects on Heat flux from angle 5^o to 50^o. That s heat flux is maximum when inclination angle is 5^o. If inclination angel increases then heat flux decreases.

References

[1] Alfven, Hannse (1942). "Existence of electromagnetic-hydrodynamic waves". Nature 150: 405–406. Bibcode:1942Natur.150..405A. doi:10.1038/150405d0.

[2] Ben-Nakhi, M. M. Iftekhari and D.I. Loveday, 'Natural Convection Heat Transfer in a partially open Square Cavity with fin attested to the wall' ASME Journal of Heat Transfer, 2008.

[3] Bilgen, E 'Natural Convection in cavities with a thin fin on the hot wall", Int. J. of Heat and Mass Transfer, Vol. 48,pp. 3493-3505.

[4] C. Taylor and P. Hood, " A Numerical solution of the Navier-Stoke's equations using finite element technique" Int J. of Computational Fluids, Vol. 1,(1973), pp. 73- 89.

[5] Chandrasekhor S. (1961), Hydrodynamics and Hydromagnetic Stability, Oxford Calderon Press, London, pp. 09-79.

[6] Chan, Y. L. and Tien, C. L. (1985), "A Numerical study of Two-dimensional Laminar natural convection in a shallow open cavity" International Journal of Heat Mass Transfer, Vol. 28, 603-612.

[7] Davis, S. H. (1967), Convection in a Box: Linear Theory, Journal of Fluid Mechanics, Vol.30. pp 465- 478

[8] Hossain, M A and Wilson, M. (2002), Natural Convection Flow in a Fluid-Saturated Porous Medium Enclosed by Non-Isothermal Walls with Heat Generation, *International Journal of Thermal Sciences*, Vol.41, 447-454.

[9] Hossain M.A., Hafiz, M. S. and Rees, D. A. S. (2005), Bouyancy and Thermocapillary Driven Convection Flow of an Electrically Conducting Fluid in an Enclosure with Heat Generation, *International Journal of Thermal Sciences*, Vol.44,676-684.

[10] Hossain, S. A. & Alim, M. A. (2012), Effects of Natural Convection from an Open Square Cavity Containing a heated Circular Cylinder, BJSIR, Dhaka, Vol. 47(1)Jan- March, 2012.19-28.

[11] Mohamad, A. A. (1995), "Natural convection in open cavities and slots",. Numerical Heat Transfer, Vol. 27.705-716..

[12] Ostrach,S. (1988) " MHD natural convection in a laterally and volumetrically heated square cavity", ASME Journal of Heat Transfer, Vol.110,1175-1190, 1988.

[13] Reddy, J. N.,(2006), An Introduction to the Finite Element Method (Third ed.), McGraw-Hill. ISBN 9780071267618

[14] Roy, S.and Basak, T. (2005) " Finite Element Analysis of Natural Convection Flows in a Square Cavity with Non-Uniformly Heated Wall(s).Int. Journal of Engineering Science, 43, 668-1453.

[15] S. Pervin, and R. Nasrin (2011), Analysis of the Flow and Heat Transfer Characteristics for MHD Free Convection in an Enclosure with a Heated Obstacle. Nonlinear Analysis, Modeling and Control, Vol. 16, No. 1. 89-99.

[16] Saha S.C, (2013) " Effect of MHD and Heat Generation on Natural Convection Flow in an open square cavity under Microgravity Condition", Int. Journal of Engineering Computations- Australia , 30, pp. 5-20, 2013.

[17] Sarris, I. E., Kakarantzas,S. C., Grecos, A. P., and Vlachos, N. S.,"MHD natural convection in a laterally and volumetrically heated square cavity", Int. J. of Heat and Mass Transfer, Vol. 48, Issue 16, pp. 3443-3453, 2005.