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1st Review

Original Research Paper

Multifractal Analysis of Sunspot Numbers During The Solar Cycle 23 and Ascending Phase of Cycle 24

ABSTRACT

Multifractal analysis based approaches have been used to study the complex dynamical fluctuations in sunspot number data including Solar Cycle 23 and ascending phase of current Solar Cycle 24. To reveal the characteristic of monthly sunspot number are analyzed by singularity spectrum and multi resolution wavelet analysis. In this work, the multifractality in sunspot number generate turbulence with the typical characteristics of the anomalous process governing the magnetosphere and interior of Sun. The result shows that singularities spectrum of sunspot data was well Gaussian shape spectrum which suggested the multifractal characteristic of data. Thus it was conclude that multifractal character provides a local and adaptive description of the cyclic components of sunspot number time series. These are non-stationary and nonlinear processes.

Keywords: Sunspot number, Magnetic field, Multifractal analysis and Wavelet transform technique.

1. INTRODUCTION

The significant feature of the Sun's outer regions is the existence of a reasonably strong magnetic field which governs the all solar activities inside the Sun and its atmosphere. According to the lowest order of approximation, the sun's magnetic field is dipolar and axisymmetric in nature and at a typical point on the solar surface its value is few gauss. However significant variation in this value and there are localized regions (sunspots) in which the field can much higher [1]. In fact, sometimes the magnetic field value is so strong which result solar flares. Solar flares cause geomagnetic storms; which affect the earth in various significant ways. The symmetry of the twisted magnetic lines as the origin of sunspots, it is generally seen in pairs or in groups of pairs at both sides of the solar equator. As the sunspot cycle progresses, spots appear closer to sun's equator giving rise to the so called "butterfly diagram" in the time latitude distribution [2]. The twisted magnetic fields above sunspots are sites where solar flares are observed. It has been found that chromospheric flares show a very close statistical relationship with sunspots [1]. The number of sunspots continuously changing in time in a random fashion and constitutes a typically random time series.

Sunspots and related activities have been analyzed by various methods, including correlation analysis [3]. Chaos analysis [4, 5] and multifractal analysis [6-8]. Multifractal theory provides an elegant statistical characterization of many complex dynamical variations in nature and engineering [9,10]. It is conceivable that it may enrich characterization of the sun's magnetic activity and its dynamical modeling [11]. In recent times, a particularly noteworthy work has been reported by [7]. The relative fraction of small scale fluctuation in the magnetic field contributes significantly more prior to flaring [12]. It was found that active regions reach a critical state of intermittency prior to flaring [6]. The multifractal scaling behaviors reported by [7] are valid for timescales up to more than 50 years. [13] found analytical connection between multifractal formalism and the set of 3D equations that govern the small-scale and large-scale magnetic structure on the Sun and also based on fractal geometry and turbulence in magnetized plasmas [14,15]. Recently, [16] and [14] achieved a contrary conclusion that studies in multifractals do not provide a predictive ability for the onset of solar flares.

This paper shows the presence of multifractality in sunspot number during the Solar Cycle 23 and 24. This paper proceeds as follows. In Section 2, we introduced the dataset of sunspot numbers for the analysis.

Co - Change the title. Please include "wavelet"
Cl - Rewrite Abstract.

In section 3, which include the DWT, wavelet based multifractal formalism and partition function we described and discussed the results. Finally, in Section 4, we present the conclusion of the analysis.

2. DATA SET

In this analysis we used the monthly counts of sunspot number for the multifractal analysis for the time span of 1996 to 2013. This period include complete Solar Cycles of 23 and ascending phase of current Solar Cycle 24. The data set available online and downloaded from <http://www.ngdc.noaa.gov/stp/space-weather/solar-data/solar-indices/>.

3. THEORETICAL BACKGROUND

3.1. WAVELET TRANSFORMS

Wavelet transform is ideal for the analysis of real world signals that contain sharp changes and localized discontinuities [17]. The wavelet transform use different window sizes, which are able to compress and stretch wavelets in different scales or widths; these are then used to decompose a time series [18] and decompose a one-dimensional signal into two-dimensional time-frequency domains at the same time [19]. Wavelet transform can be performed using two approaches: Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT). The CWT operates on smooth continuous functions and can detect and decompose signals on all scales. The scales and locations used in DWT are normally based on a dyadic arrangement (i.e. integer powers of two) [20]. DWT is especially useful for time series containing sharp jumps or shifts [21].

3.2 TIME SERIES DECOMPOSITION VIA THE DISCRETE WAVELET TRANSFORM (DWT)

The DWT is usually based on the dyadic calculation of position and scale of a signal [20]. The DWT is excellent for denoising the signals [22]. The DWT of a vector is the outcome of a linear transformation resulting in a new vector that has equal dimensions to those of the initial vector [23]. This transformation is the decomposition process. In this study all the time series (for the Solar Cycles 23 and ascending phase of current Cycle 24) were decomposed using the Daubechies (db2) and Coifman (coif5) wavelets, splitting the signal into its approximation and detail components and displayed in the figures (Figures 1 to 4). The discretization of wavelet functions is accomplished using a logarithmic uniform spacing that has a coarser resolution at higher scales [24, 25]. In this DWT- based study of sunspot data the Daubechies (db) and Coifman (coif) wavelets were used. Daubechies wavelets provide compact support [26], indicating that the wavelets have non-zero basis functions over a finite interval, as well as full scaling and translational orthonormality properties [27]. These features are very important for localizing events in the time-dependent signals [27].

3.3 WAVELET BASED MULTIFRACTAL FORMALISM

The wavelet transform not only locates isolated anomalous events, but can also characterize more complex multifractal sunspot data having non isolated singularities. Multifractal objects cannot be completely described using a single fractal dimension (mono fractals). They have an infinite number of dimension measures associated with them. The wavelet transform takes advantage of multifractal self-similarities, in order to compute the distribution of their singularities. This singularity spectrum is used to analyze multifractal properties. The concepts of fractals and multifractals and their relevance to the real world data were introduced by the [28].

In real world data, understanding of pattern of the singularities, (i.e. the graph of points at which time series changes abruptly) is quite a challenging task. The time series of sunspot numbers usually depict fractal or multifractal features. Time series are commonly called self- affine functions as their graphs are self-affine sets that are similar to themselves when transformed by anisotropic dilations.

Mathematically, if $f(x)$ is a self- affine function representing the sunspot number then

For $x_0 \in R, \exists H \in R$ such that for any $\lambda > 0$,

$$f(x_0 + \lambda x) - f(x_0) \cong \lambda^H (f(x_0 + x) - f(x_0)), \dots \dots \dots (1)$$

The exponent H here is called roughness or Hurst exponent. Note that if $H < 1$, then f is not differentiable and smaller the exponent H , the more singular is f .

For sunspot number $f(x)$, a function $h(x)$, the Holder function of f , which measures the regularity of f at which point x is associated.

The point wise Holder h of f at point x_0 is defined as:

$$h(x_0) = \limsup \{h: \exists c > 0, |f(x) - f(x_0)| \leq c|x - x_0|^h, |x - x_0| < \rho\} \dots \dots (2)$$

(Here h is an integer and f is non- differential).

One may also define local exponent $h_1(x_0)$ as:

$$h_1(x_0) = \limsup \{h: \exists c > 0, |f(x) - f(y)| \leq c|x - y|^h, |x - x_0| < \rho, |y - x_0| < \rho\} \dots \dots \dots (3)$$

Where h and h_1 are different in general. For example for

$$f(x) = |x|^h \sin \frac{1}{|x|^\beta}, h(0) = 0, \text{ while } h_1(0) = \frac{h}{1+\beta} \dots \dots \dots (4)$$

They have quite different properties. For instance h_1 is stable through differentiation ($h_1(f', x_0) = h_1(f, x_0) - 1$), whereas h is not. The smaller $h(x)$ is, the more irregular the function f is at x .

3.4 PARTITION FUNCTION

It is not possible to calculate the Pointwise Lipschitz (Holder) regularity of a multifractal because its singularities are not isolated, and the finite numerical resolution sufficient to discriminate them. To overcome this difficulty [29] have introduced the concept of wavelet transform modulus maxima using a global partition function [30]. Let Ψ be a wavelet with n vanishing moments. Mallat has shown that if f has a pointwise Holder (Lipschitz) regularity $\alpha_v \leq n$ at v then the wavelet transform $T_\Psi f(a, b)$ has a sequence of modulus maxima that converges towards v at fine scales. The set of maxima at the scale a can thus be interpreted as a covering of the singular support of f with wavelets of scale a . At these maxima locations

$$|T_\Psi f(a, b)| \approx a^{\alpha_v + 1/2} \dots \dots \dots (5)$$

Let $\{u_p(a)\}_{p \in \mathbb{Z}}$ be the position of all local maxima of $|T_\Psi f(a, b)| \approx a^{\alpha_v + 1/2}$ at a fixed scale a . The partition function Z measures the sum at a power q of all these wavelet modulus maxima:

$$Z(q, a) = \sum_p |T_\Psi f(a, u_p)|^q \dots \dots \dots (6)$$

For each $q \in R$, the scaling exponent $\tau(q)$ measures the asymptotic decay of $Z(q, a)$ at fine scale a :

$$\tau(q) = \liminf \frac{\log Z(q, a)}{\log a} \dots \dots \dots (7)$$

This typically means that $Z(q, a) \sim a^{\tau(q)}$.

3.5 RESULTS AND ANALYSIS

In this study we have analyzed the multifractal characteristics of sunspot number during the Solar Cycle 23 and current Cycle 24 using wavelet based multifractal techniques. Sunspot numbers (SNs), are widely and frequently used in astronomy and space science to reflect long term variations of solar activity, which has served as the primary time series to define solar activity [31-34].

3.6 WAVELET ANALYSIS OF SUNSPOT NUMBERS

Wavelet analysis is a tool for analyzing localized variations of power within a time series [35]. In this section, a given data set is divided into components with different scales, which allows the investigation of each component with a resolution matched to its scale. This property is especially useful for signals that are non-stationary, short-lived transient components and have features at different scales or have singularities [36]. In the present analysis both Daubechies and Coifmann wavelets are taken as mother wavelet to decompose sunspot number and Morlet wavelet used for Multifractal spectral analysis.

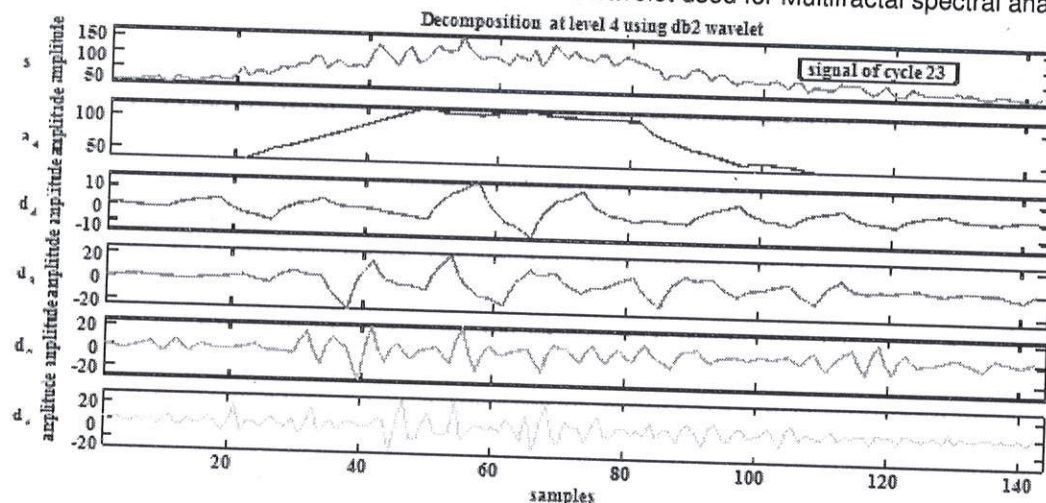


Fig. 1. DWT decompositions of Solar Cycle 23 using Daubechies2 wavelet. Anomalous variation should be found between 45 to 80 no. of samples (i.e. at the maximum phase of cycle).

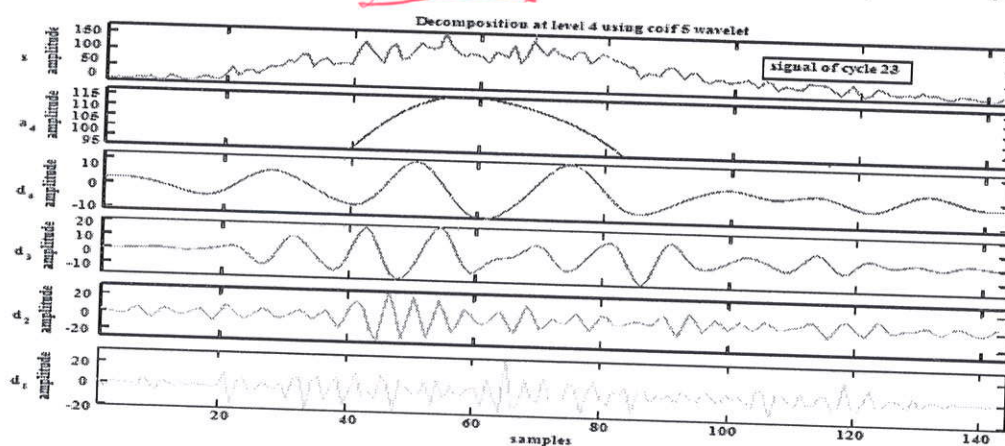


Fig. 2. DWT decompositions of Solar Cycle 23 using Coifman5 wavelet. Anomalous sharp variation should be found at 60 no. of samples (i.e. at the maximum phase of cycle).

[3] - Please further elaborate Fig 1 & Fig 2. why we get two difference maximum phase of cycle?

cp - which one correct? or more reliable?

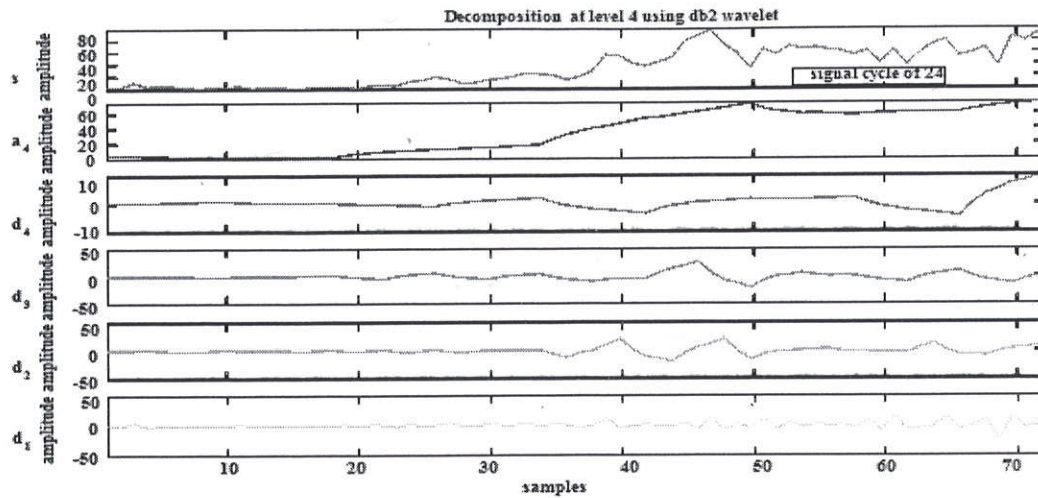


Fig. 3. DWT decompositions of Solar Cycle 24 using Daubechies2 wavelet. Anomalous variation showing from 45 to 70 no. of samples (i.e. at the maximum phase of cycle).

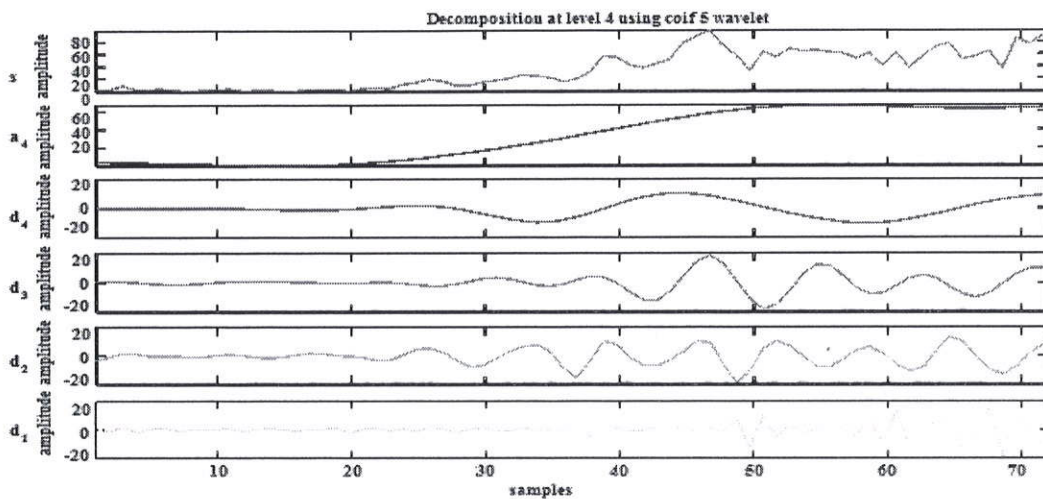


Fig. 4. DWT decompositions of Solar Cycle 24 using Coifman5 wavelet. Anomalous sharp variation should be found between 50 to 70 no. of samples (i.e. at the maximum phase of cycle).

In this section, DWT of the sunspot numbers time series is carried out in terms of approximations and details coefficients. Daubechies wavelets have fractal structure and they include both highly localized wavelets and highly smooth wavelets. The original signal (s) is decomposed into two orthonormal components, frequency (details) and approximations component. The approximations represent the long term of data which is almost identified to original signal and the detail coefficients represent the short period fluctuations in given period range. For the wavelet decomposition analysis the data of sunspot numbers is used. The decomposition analysis of Solar Cycles 23 and 24 dataset is shown in Figures 1 to 4. In these figures, the X-axis represents the time period of the data under consideration. In all figures first panel represents the observed variation of sunspot number time series. The second one gives the approximation " a_4 " which approximation part corresponds to the amplitude of the sunspot data for respective wavelets used at level 4. Approximation coefficient " a_4 " of all the figures separate the short term anomalous variation from the long term variations.

C5 - Please further elaborate Fig 3 & Fig 4,

C6 - what actually is the interpretation to all four figures? why diff wavelet we get diff. location of maximum phase? which one is correct?

On the other hand, other four parts d_1, d_2, d_3 and d_4 represent detail coefficients of the sunspot data. The detail coefficients reveal that the field strength changes between positive and negative values. This indicates the existence of a strong and variable magnetic field on the Sun as sunspots. The detail coefficients of sunspot numbers show that high frequency components during the initial and main phase of the solar cycle and their time evolution. However, it also displayed the result of cycles having high amplitudes in the period/ sampling range and they are existing for higher levels of decompositions. Here it is found that very high frequency components present only during the main phase of the respective solar cycles and they are very strong in amplitude and stable for higher level of decomposition. Even though there are some strong fluctuations in the main phase and recovery phase but they are not as persistent as that present during the initial phase and main phase of the cycles. From figures it was noticed that this multi-resolution analysis using wavelet detail and approximation coefficients allow analysis of local features of sunspot number time series to be studied in time domain and transient signals to be represented effectively. This is an important point since the sunspot number data could include some storms. The frequency of geomagnetic storm depends on increases or decreases with the sunspot cycle.

3.7 MULTIFRACTAL ANALYSIS OF SUNSPOT NUMBERS

In this section, we have done multifractal analysis of sunspot numbers time series as fractal dimensions are best known part of the fractal analysis.

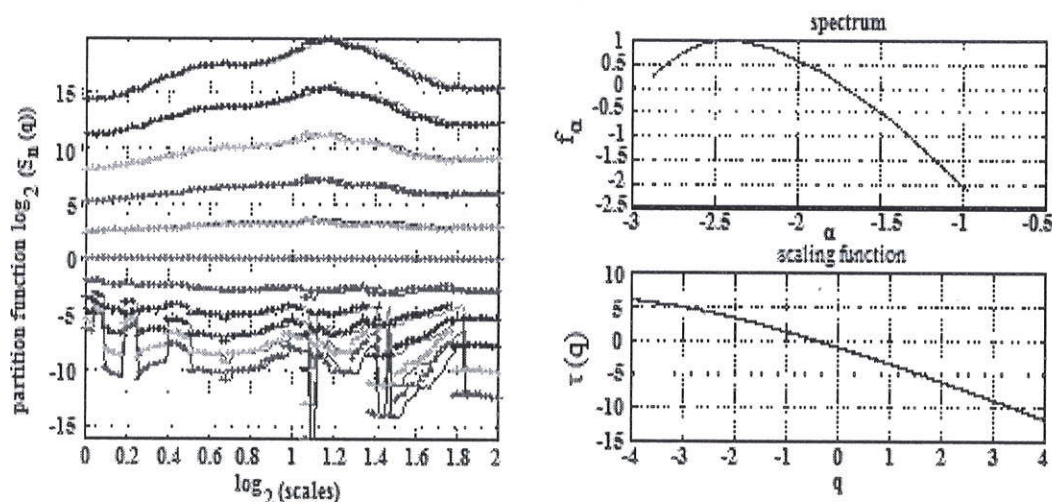


Fig. 5. Multifractal analysis of Solar Cycle 23 using CWT (Morlet wavelet of size 8 and 128 voices, LS regression and local maxima). In figure Left panel show the Legendre spectra and Right (upper) panel gives the singularity spectrum and Right (lower) panel for scaling function of the considered time series.

C7 - present the result as Table Indicate the freq/level.

C8 - Explain why' 6 spectrum & SF are diff. for Fig 5 & Fig 6 (cycle 23 vs cycle 24)

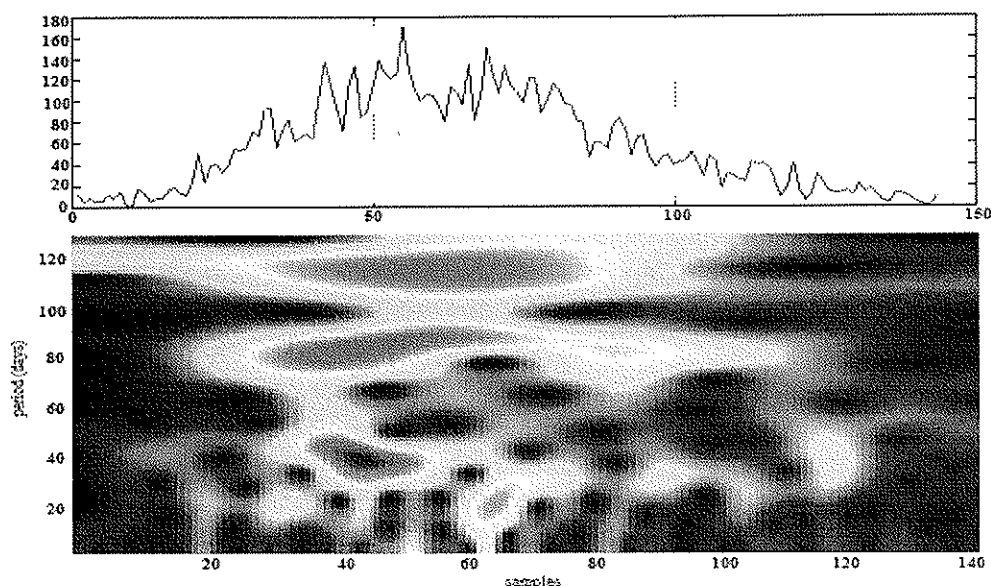


Fig. 6. CWT of Solar Cycle 23 using a Morlet Wavelet of size 8 and 128 voices. In figure X axis represents the number of samples and Y axis represents scale. The upper panel shows the raw data of the time series and lower panel its continuous wavelet transform (CWT).

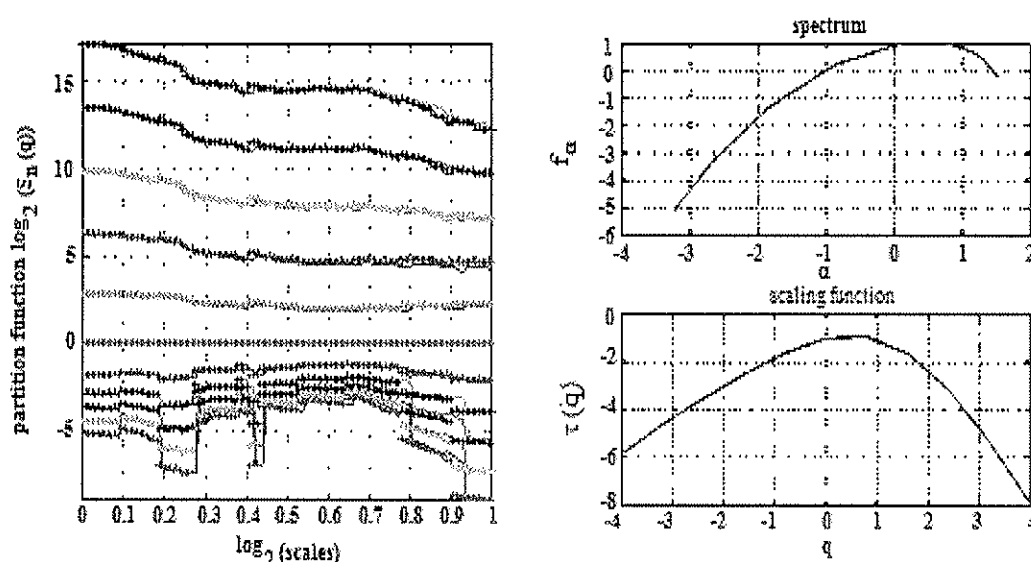


Fig. 7. Multifractal analysis of Solar Cycle 24 using CWT (Morlet wavelet of size 8 and 128 voices, LS regression and local maxima). In figure Left panel show the Legendre spectra and Right (upper) panel gives the singularity spectrum and Right (lower) panel for scaling function of the considered time series.

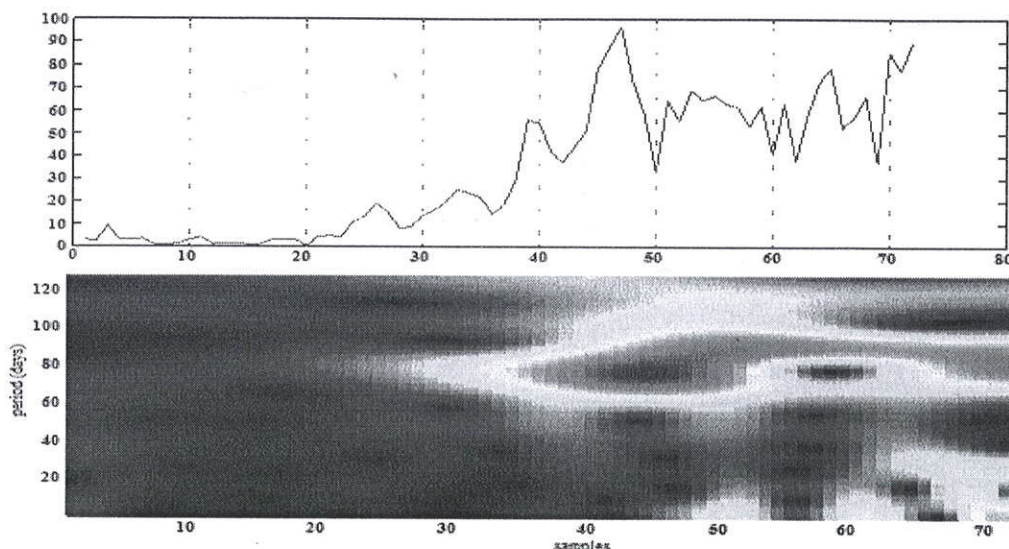


Fig. 8. CWT of Solar Cycle 24 using a Morlet Wavelet of size 8 and 128 voices. In figure X axis represents the number of samples and Y axis represents scale. The upper panel shows the raw data of the time series and lower panel its continuous wavelet transform (CWT).

We performed the multifractal analysis, calculating the Legendre spectra by means of the software FRACLAB, developed at INRIA and available online at <http://www-rocq.inria.fr>. Since the sunspot data have some missing values we take the longest segments of sunspot data without any missing values. The order of the magnitude of the length of each segment is about 120, thus permitting to obtain reliable estimates of the singularity spectrum and multifractal parameters. Figures 5 and 7 (Right panel) shows the Legendre spectra for the selected segments for both solar cycle. All the spectra present the typical single-humped shape that characterizes multifractal nature of sunspot number. The spectra of the segments for each sunspot time series, calculated for different time intervals, are not identical, although they are very similar to each other. Nonlinear fluctuations are possibly due to the presence of multifractal processes. The smaller values of α correspond to the burst of events, while higher values of α correspond to events occurring sparsely [37]. The spectrum gives geometrical information pertaining to the dimension of sets of points in a signal having a given Holder exponent. This is the most precise spectrum from a mathematical point of view, but is also difficult one to estimate. Large deviation spectrum yields statistical information, related to the probability of finding a point with a given Holder exponent in the signal. More precisely, it measures how this probability behaves with the change in resolution.

Legendre spectrum is a concave approximation to the large deviation spectrum. Its main purpose is to yield much more robust estimates, though at the expense of a loss of information. It could be based on box method or CWT techniques. In the sequel we show some sample results for the spectra computed with the Legendre technique. Figures 6 and 8 show the results of the CWT (Morlet wavelet) based estimation of the Legendre spectrum which represents an approximation of the spectrum for two different Solar Cycles. Each of the Figures 6 and 8 consist of two parts. The first part on the top of each of these figures represents the signal or raw data. The second part of these figures shows the analyzed pattern with the application of Morlet wavelet of size 8 and 128 voices of different Solar Cycles. From the figures 9 and 10 it is cleared the exponents that arise sharply in the multifractal analysis of dataset to assess the regularity of a sunspot time series. The Holder exponent is used for this study that characterizes the regularity of the measure (function) of the magnetic field strength of sunspot under consideration at either pointwise regularity or local regularity. The non-parametric point wise Holder regularity approach (in Figures 9 and 10) used with CWT with Morlet wavelet for the Solar Cycle 23 and ascending phase of Cycle 24.

C9 - Can calculate the α Holder regularity?

C10 - Please justify the finding with statistical technique. * (compulsory)

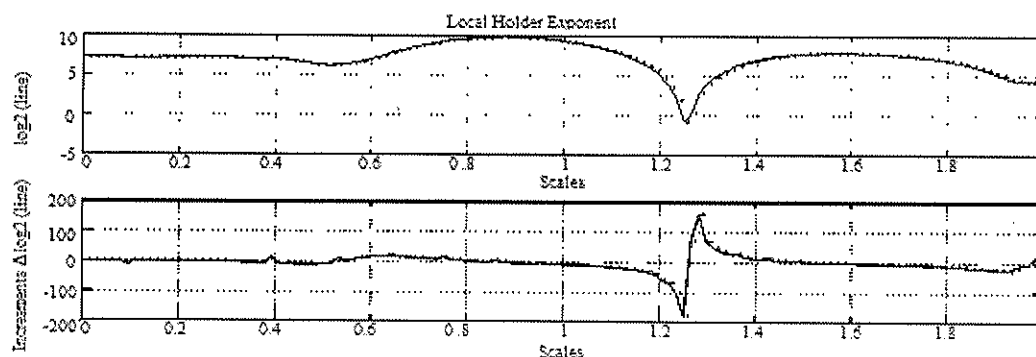


Fig. 9. Non-parametric point wise Holder regularity estimation using CWT with Morlet wavelet for the Solar Cycle 23.

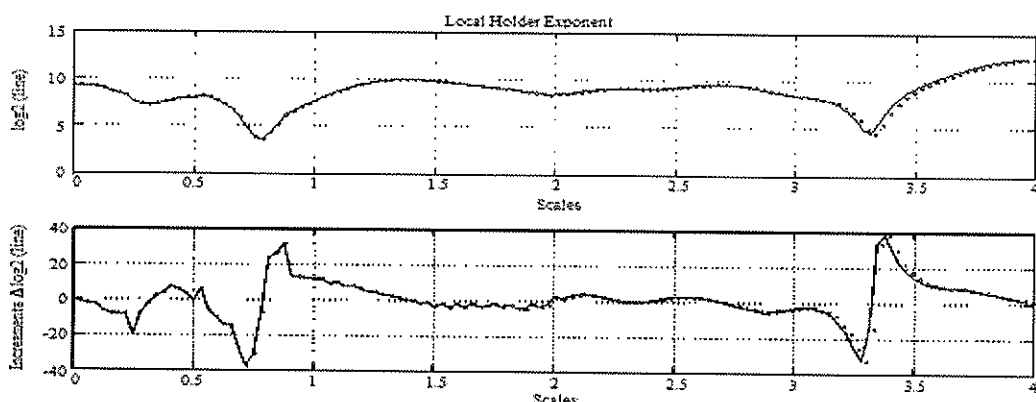


Fig. 10. Non-parametric point wise Holder regularity estimation using CWT with Morlet wavelet for the Solar Cycle 24.

4. CONCLUSION

The multifractal analysis exhibits much potential in providing a suitable, and physically motivated, proxy for the complexity of solar active regions. The Wavelet transform has been used in a variety of fields and was found advantageous compared to the convention tools for time series analysis such as Fourier transform. Wavelet transformation refers by the decomposition of the time series into many interrelated components. Wavelet analysis is particularly suited to decomposition and detect the multiple signal (time series) and follow the time evolution of the frequency distribution of a given time series [38-43]. This property is important predominantly when one considers non-stationary processes. In addition, wavelet analysis is a powerful multistage resolution technique to deeply understanding the complex features of real world processes different kinds of multifractality, long-range dependence (LRD), non-stationary, oscillatory behaviour and trends. Wavelet-based estimators used successfully for estimating scaling behaviour and applied to time series [42]. Wavelet transform techniques employed in the present analysis help to decompose the time series of sunspot numbers into different scales. Figures 1 to 4 provides decomposition of sunspot number data for the period of 1996 to 2013 at different levels through applications of db2 and coif5 wavelets. Wavelet toolbox of MATLAB has been used. Multifractal analysis provided both a local and a global description of the singularities of a signal. The wavelet spectrum displayed in Figures 6 and 8 reveals the time at which large variations and hence important in the investigation of multifractality. Various phenomena related to sun and space weather conditions show fractal and multifractal behavior when plotted against time. Multifractals have infinite number of

dimensions associated with them. Solar activity data which are multifractal, are singular (i.e. changes abruptly) at almost every point. There exist three main multifractal spectra, viz. the Hausdroff, large deviation and Legendre spectra. Basically, any of these three spectra provides information as to which singularities occur in the data, and which are dominant: a spectrum is a one dimensional curve where abscissa represents the Holder exponents actually present in the data, and ordinates are related to the amount of points where we encounter a given singularity. Wavelet analysis provided visualization of the sunspot number data at different levels.

The results of this work provide a general view of the pattern of sunspot number data of different Solar Cycles 23 and ascending phase of Solar Cycle 24. However, recent studies have shown that non-gaussian fluctuation is responsible for the presence of extreme events in space plasmas. Recently, using a non-extensive approach, [44] found out the emergence of two distinct phases: (i) the phase where the intense magnetic storms cause a higher degree of magnetic field organization, and (ii) the phase which characterizes the normal periods with lower magnetic field coherence. The phase (i) may be associated with the presence of different kinds of large scale coherent structures, also pointed out by [45]. However, from the analysis it is found that Solar Cycle 23 commonly shows the multifractality nature around the initial, maximum and declining phase of cycle rather than Solar Cycle 24 it is not effective during the initial phase of cycle. Solar Cycle 24 has more fractal nature at the maximum phase and we wait for more result till complete the ongoing cycle.

CONSENT

"All authors declare that 'written informed consent was obtained from the patient (or other approved parties) for publication of this case report and accompanying images. A copy of the written consent is available for review by the Editorial office/Chief Editor/Editorial Board members of this journal."

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C11 - relate this in Results section

C12 - Can we find the relationship between details coeff and singularity/ maximum phase?

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