Original Research Article

Effects of MHD Thermal Radiation and Heat Source of viscous flow over a nonlinearly stretching sheet with Viscous Dissipation

Abstract:

This investigation is undertaken to study Effects of MHD Thermal Radiation and Heat Source of viscous flow over a nonlinearly stretching sheet with Viscous Dissipation. The governing equations of the problem are transformed into non-linear ordinary differential equations by using similarity transformations. The resulting equations are solved numerically by using an implicit finite difference method known as Keller Box method. The effect of various physical parameters on the temperature, vertical velocity field and skin - friction and they are presented graphically for different values of physical parameters involved. The Present results are comparisons have been made with previously published work and results are found to be very good agreement. Numerical results for local skin friction, local Nusselt number are tabulated for various physical parameters.

Keywords:

MHD, Viscous dissipation, nonlinear Stretching Sheet, Thermal radiation, Heat source, viscous flow and Keller Box Method

I Introduction

The study of Effects of MHD Thermal Radiation and Heat Source of viscous flow over a nonlinearly stretching sheet with Viscous Dissipation is very important since it finds many practical applications in many of the areas. In some of the industrial applications of viscous flow over a stretching sheet are aerodynamic extrusions of plastic sheets, condensation process of metallic plate in a cooling bath, extrusion of a polymer sheet from a dye process. Since, The high applicability of this problem in such industrial phenomena, it has attracted the attentions of many researchers and one of the pioneering studies has been performed by Sakiadis [1]. The study of stretching surfaces and the several combinations of additional effects on the stretching problems are important in many practical applications because the production of sheeting material comes up in a number of industrial and manufacturing Processes, it includes both metal and polymer sheets. In the manufacture of the latter, the material is in a molten phase when thrust through an extrusion die, then cools and solidifies some distance away from the die before arriving at the collecting stage.

Hydro magnetic flows and heat transfer have become more important in recent years because of its various applications in agricultural engineering and petroleum industries. In recent times, considerable attention has also been focused on new applications of magneto-hydrodynamics (MHD) and heat transfer such as metallurgical processing. Melt refining engrosses magnetic field applications to control excessive heat transfer rate. Other application of MHD heat transfer consists of MHD generators, plasma propulsion in astronautics, nuclear reactor thermal dynamics and ionized-geothermal energy systems etc. A small amount of studies have been focused on exponentially and nonlinear stretching sheet. Magyari and Kellar [2] and Elbashbeshy [3] studied heat and mass transfer in the boundary layer on an exponentially stretching sheet, whereas Vajravelu [4], Vajravelu and Cannon [5] and Cortell [6] have investigated viscous flow over a nonlinear stretching sheet. The role of thermal radiation is a major important in some industrial applications since glass production and furnace design, and also in space technology applications, such as comical flight aerodynamics rocket, propulsion systems, plasma physics and space craft reentry aerodynamics which operates at high temperatures. Cortell [7] analyzed the effects of viscous dissipation and thermal radiation over a non-linear stretching sheet. Hady et al. [8] investigated the effects of heat transfer over a nonlinearly stretching sheet in the presence of thermal radiation a nanofluid. Vajravelu and Rollins [9] investigated heat transfer in an electrically conducting fluid over a stretching surface taking into account the magnetic field. Hamad and Pop [10] studied theoretically the wobbly magneto hydrodynamic flow of a nanofluid past an oscillatory moving vertical permeable semi-infinite plate in the presence of constant heat source in a rotating frame of reference. Solutions are presented for velocity and temperature fields for various values of magnetic field and suction parameters.

Effect of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet was studied by Cortell [11]. Jat and Chaudhary [12-14] studied the MHD boundary layer flow over a stretching sheet for stagnation point, heat transfer with and without viscous dissipation and Joule heating. Ellahi et al. [15-16] obtained investigative solution for MHD flow in an annulus and MHD flow in a third grade fluid with variable viscosity. Radiation effects on the MHD flow near the stagnation point of a stretching sheet was studied by Jat and Chaudhary [17]. Vyas and Rai [18] studied radiative flow with thermal conductivity over a non-isothermal stretching sheet in a porous medium. Later, Numerical study of MHD free convective flow and mass transfer over a stretching sheet considering Dufour and Soret effects in the presence of magnetic field was investigated by Ahammad and Mollah [19]. Recently, Alinejad and Samarbakhsh [20] studied viscous flow over nonlinearly stretching sheet with effect of viscous dissipation. Slip effects on ordinary viscous fluid flow have been discussed by Ellahi et al. [21], while MHD flow have been studied by Ellahi and Hammed [22]. in recent times, Ellahi [23] and Ellahi et al. [24] studied the effects of MHD and temperature dependent viscosity on the flow of non-Newtonian nanofluid in a pipe and flow through a porous medium between two coaxial cylinders with heat transfer and variable viscosity respectively. Zeeshan and Ellahi [25] obtained the Series solutions for nonlinear partial differential equations with slip boundary conditions for non-Newtonian MHD fluid in porous space.

The present paper studies Effects of MHD Thermal Radiation and Heat Source of viscous flow over a nonlinearly stretching sheet with Viscous Dissipation. This is the extension for viscous flow over a nonlinearly stretching sheet with Viscous Dissipation.

II Mathematical formulation

The governing equations of continuity equations, momentum equation, energy equation and concentration equations are given below.

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho_f}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{u}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{Q_0}{\rho c_p}(T-T_\infty) \tag{3}$$

The boundary conditions for the velocity, temperature and nanoparticles fraction are given below:

$y = 0 :$ $u_w = ax^n$, v=0, T= T_w, C= C_w, (4)

$y = \infty :$ u = 0, v = 0, T= T_∞, C= C_∞, (5)

Here, u and v are the velocity components along the x and y axes, respectively. $\alpha = k/(\rho c)_f$ is the thermal diffusivity, σ is electrical conductivity, ν is the kinematic viscosity, ρ_f is the density Of the base fluid. $\tau = (\rho C)_p / (\rho C)_f$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid, c is the volumetric volume coefficient, ρ_p is the density of the particles, and C is rescaled nanoparticle volume fraction. We assume that the variable magnetic field B(x) is of the form B(x) = $B_0 x^{(n-1)/2}$

$$\eta = y\sqrt{\frac{a(n+1)}{2v}}x^{\frac{(n-1)}{2}}, \quad u = ax^n f'(\eta), \quad v=-\sqrt{\frac{a(n+1)}{2}}x^{\frac{(n-1)}{2}}\left(f(\eta)+\frac{n-1}{n+1}\eta f'(\eta) \right),$$

$$\theta(\eta) = (T - T_\infty) \big/ (T_W - T_\infty). \tag{6}$$

Using Rosseland approximation for radiation
we can write

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6a}$$

where k^* is the absorption coefficient, σ^* is the Stefan-Boltzman constant, Assuming the temperature difference within the flow is such that T^4 may be expanded in a Taylor series about T_∞ and neglecting higher orders we get $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$.
Hence Eq. (6a), becomes

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3(\rho_{cp})_f k^*}\frac{\partial^2 T}{\partial y^2} \tag{6b}$$

Where ψ represents stream functions and is defined as $u = \frac{\partial \psi}{\partial y}$, $v = \frac{-\partial \psi}{\partial y}$ so that equation (1) is satisfied identical. The governing eqs (2) – (3) or reduced by eqs (6).

$$f''' + ff'' - \left(\frac{2n}{n+1}\right)f'^2 - Mf' = 0 \tag{7}$$

$$\left(1+\frac{4}{3}R\right)\frac{1}{pr}\theta''+f\theta'+E_c f''^2+\left(\frac{2}{n+1}\right)Q\theta=0 \quad (8)$$

The transformed boundary conditions

$$f(0)=0,\ \ f'(0)=1,\ \theta(0)=1,$$

$$f'(\infty)=0,\ \theta(\infty)=0, \quad (9)$$

Where primes denote differentiation with respect to η, the involved physical parameters are defined as:

$$\Pr=\frac{v}{a},\ M=\frac{2\sigma B_0^2}{a\rho f\,(n+1)},$$

$$Ec=\frac{u_w^2}{c_\rho(T_w-T_\infty)},\ Q=\frac{2Q_0}{(\rho C)_f\,a(n+1)}x^{\frac{n-1}{2}},\qquad R=\frac{4\sigma^* T_\infty^3}{kk^*}\ . \quad (10)$$

Here Pr, M, Ec , Q and R denote the Prandtl number, Magnetic parameter, Eckert number, Heat source/sink and Thermal radiation respectively. This boundary value problem is reduced to the classical problem of flow and heat and mass transfer due to a stretching surface in a viscous fluid when n = 1 in eqs (8) .

The quantities of practical interest, in this study, are the local skin friction C_{fx}, Nusselt number Nu_x which are defined as

$$C_{fx}=\frac{\mu_f}{\rho u_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0},\qquad Nu_x=\frac{xq_w}{k(T_w-T_\infty)}, \quad (11)$$

Where k is the thermal conductivity of the nanofluid, and q_w, q_m are the heat and mass fluxes at the surface, respectively, given by

$$q_w=-\left[\frac{\partial T}{\partial y}\right]_{y=0},\quad q_m=-D_B\left[\frac{\partial C}{\partial y}\right]_{y=0} \quad (12)$$

Substituting Eq (6) into Eqs (11) – (12), we obtain

$$\mathrm{Re}_x^{1/2}\,C_{fx}=\sqrt{\frac{n+1}{2}}f''(0),\qquad \mathrm{Re}_x^{-1/2}\,Nu_x=-\sqrt{\frac{n+1}{2}}\theta'(0),$$

Where $\mathrm{Re}_x=u_w\dfrac{x}{v}$ is the local Reynolds number.

III Results and Discussion

The reduced Eqs. (7) – (8) are nonlinear and coupled, and thus their exact analytical solutions are not possible. They can be solved numerically using Keller Box for different values of parameters such as magnetic parameter, Prandtl, Eckert Heat source/sink and Thermal radiation. The effects of the emerging parameters on the dimensionless velocity, temperature, skin friction, the rates of heat and mass transfer are investigated.

The important steps in using the Keller Box method are:
1) Reducing higher order ODEs (systems of ODEs) in to system of first order ODEs;
2) Writing the systems of first order ODEs into difference equations using central differencing scheme;
3) Linearizing the difference equations using Newton's method and writing it in vector form;
4) Solving the system of equations using block eliminations method.

In order to solve the above differential equations numerically, we adopt Matlab software which is very efficient in using the well known Keller Box method.

To validate the present solution, comparisons have been made with previously published data in the literature for $-\theta'(0)$ and $-\phi'(0)$ in Table 1, and they are found to be in an excellent.

Table 1, it is clear that the Nusselt number is a decreasing function of M, R, whereas. An increase in the Heat Source Q means that the fluid becomes more viscous. R, Q an increase Thermal radiation and Heat Source/sink that the fluid becomes more viscous. Therefore, causes an increase in the rate of mass transfer. As expected, increasing the Lewis number reduces the rate of heat transfer. The values of the skin friction coefficient can be observed in an increasing manner for various values of M in Table 1

Table 1: Resulting Table of Magnetic field (M), Heat Source (Q) and Thermal Radiation (R) effects in $-f''(0)$ (Skin Friction) and $-\theta'(0)$ (Nusselt Number) for the values of Pr=6.2, Ec=0.1 and n=2.

M	**Q**	**R**	$-f''(0)$ **(Skin Friction)**	$-\theta'(0)$ **(Nusselt Number)**
0.0	0.0	0.0	1.1014	1.5298
0.1			1.1462	1.5171
0.2			1.1893	1.5011
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0.0	0.1	0.0	1.1014	1.5326
	0.2			1.5352
	0.3			1.5375
	0.5			1.5410
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0.0	0.0	0.1	1.1014	1.9254
		0.2		1.8412
		0.3		1.7296
		0.5		1.5272

The effect of the magnetic parameter M is shown in Fig. 1. It is observed that the tangential velocity of the fluid increases with the increase of the magnetic field number values. The increase in the tangential velocity as the magnetic parameter M decreases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in Fig. 1

For different values of the magnetic parameter M the temperature profiles are plotted in Fig. 2. It is obvious that an increase in the Magnetic parameter M results in an increase in the temperature within the boundary layer.

Figure 3 depicts the variation of temperature with respect to heat source/sink. It is observed that the increase in heat source/sink parameter increases the temperature profile. This is due to the fact that heat source can add more heat to the stretching sheet which increases its temperature. This increases the thermal boundary layer thickness . In addition to this the velocity approaches to zero as the distance from the sheet increases.

The influence of M, nonlinear stretching parameter n on dimensionless, and nanoparticle concentrations are shown in Fig. 4. The increase in M will enhance the dimensionless concentration profiles can be observed in Fig. 4. Lorentz forces are resistive in nature opposing the fluid motion and hence the result is the production of heat. In such scenarios, in the magnetic field, the thermal boundary layer and nanoparticle volume fraction boundary layers thickness will increase. The nonlinear stretching parameter is slightly negligible for the variation of nanoparticle concentration. Note that these parameters are negligible for both positive and negative values of n.

Figures 5 show the behavior of temperature for different values Prandtl number. The numerical results show that the effect of decreasing values of Prandtl number results in a decreasing velocity. It is observed that an increase in the Prandtl number results a increase of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to decrease in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of Pr . Hence in the case of smaller Prandtl number as the thermal boundary layer is thicker and the rate of heat transfer is increased.

For different values of the Eckert number Ec the temperature profiles are plotted in Fig. 6 It is obvious that an increase in the Eckert number Ec results in an increase in the temperature within the boundary layer.

The effect of the non linear stretching parameter n is shown in Fig. 7. It is observed that the tangential velocity of the fluid increases with the decrease of the magnetic field number values. The increase in the tangential velocity as the non linear stretching parameter n decreases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the non linear stretching parameter n is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in Fig. 7.

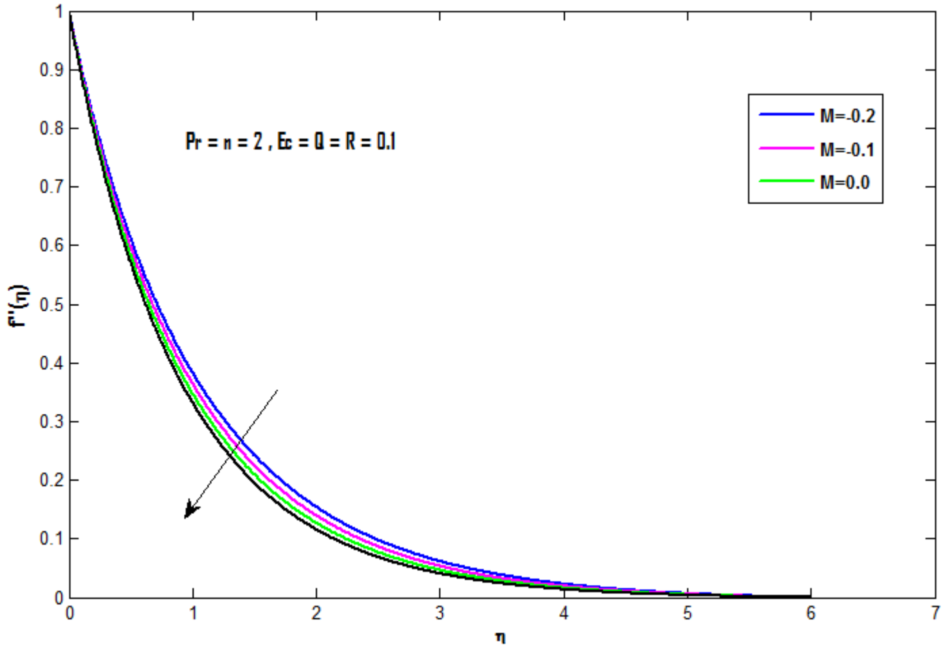

Figure 1 – Velocity profile for M

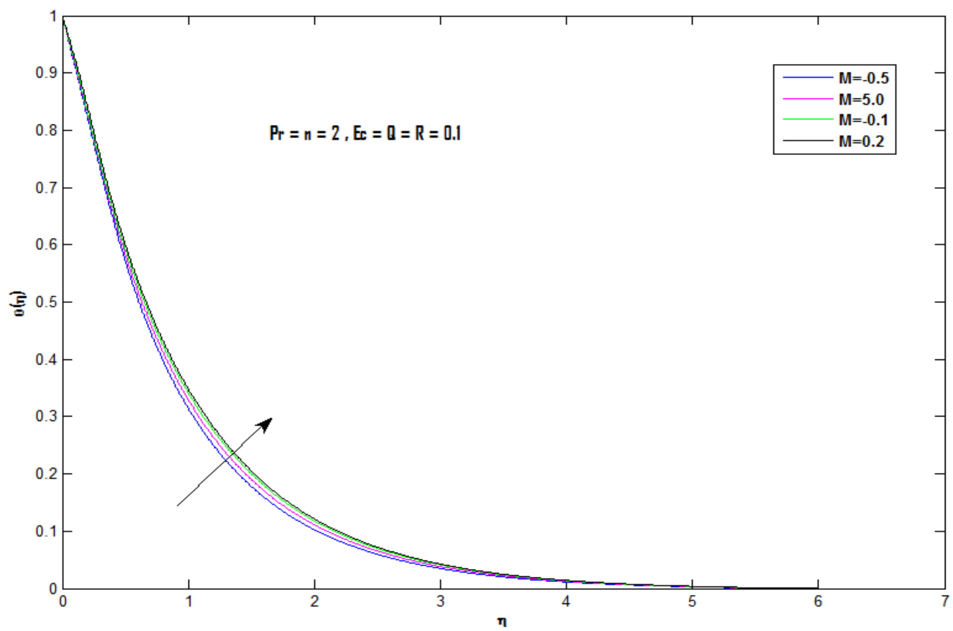

Figure 2 – Temperature profile for M

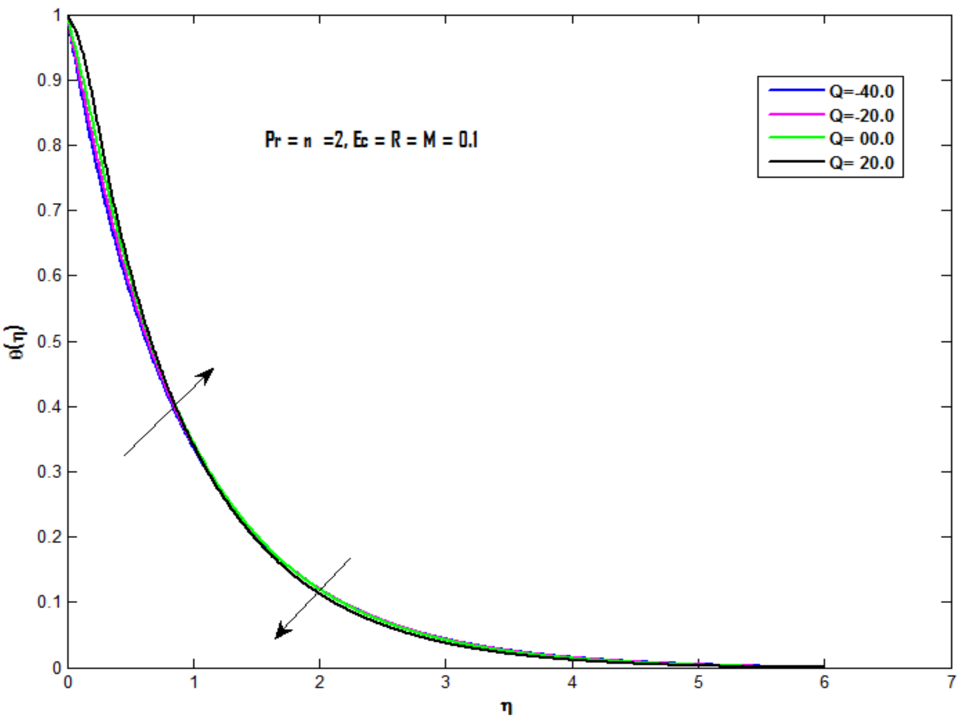

Figure 3 – Temperature profiles for Q

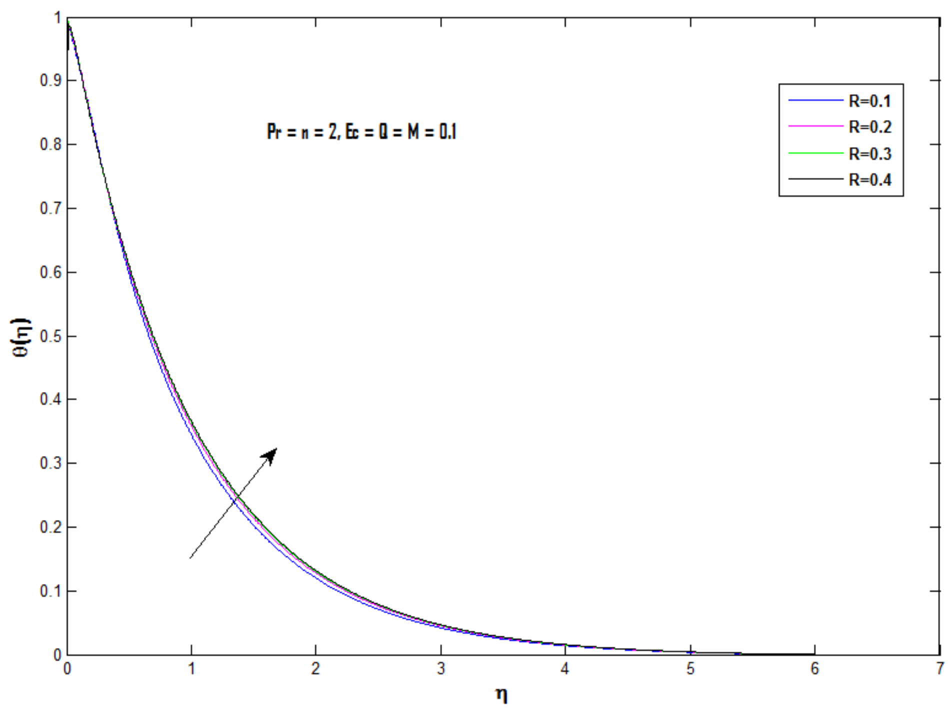

Figure 4 – Temperature profiles for R

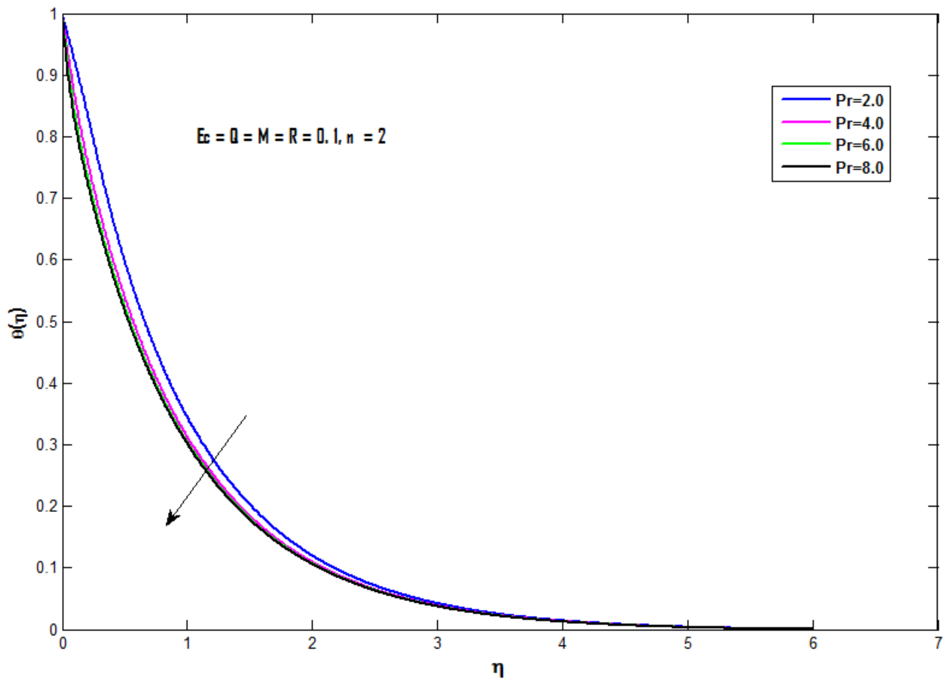

Figure 5 – Temperature profiles for Pr

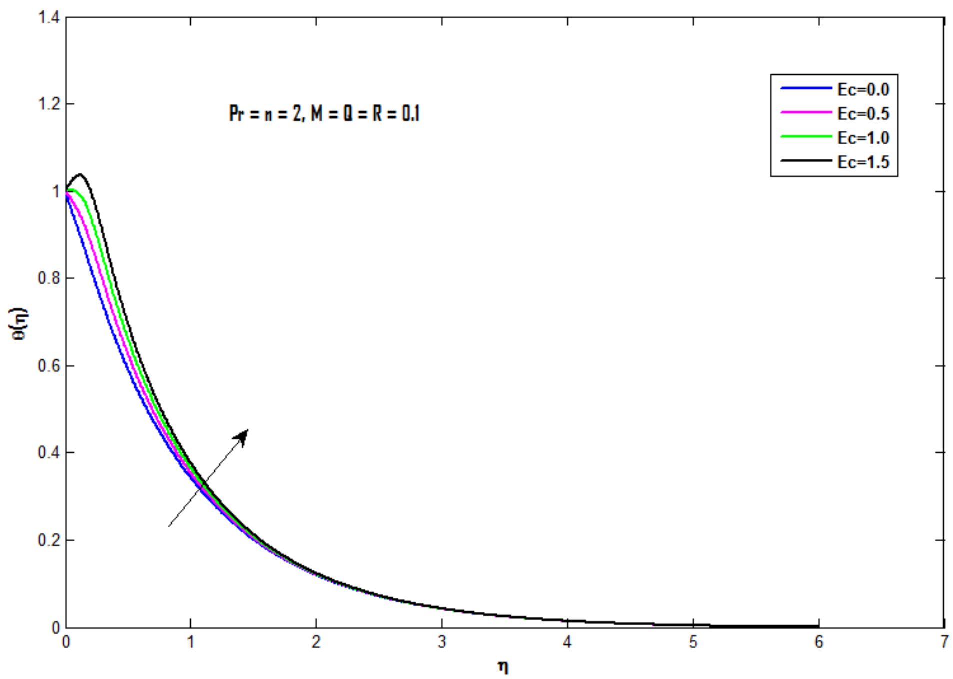

Figure 6 – Temperature profiles for Ec

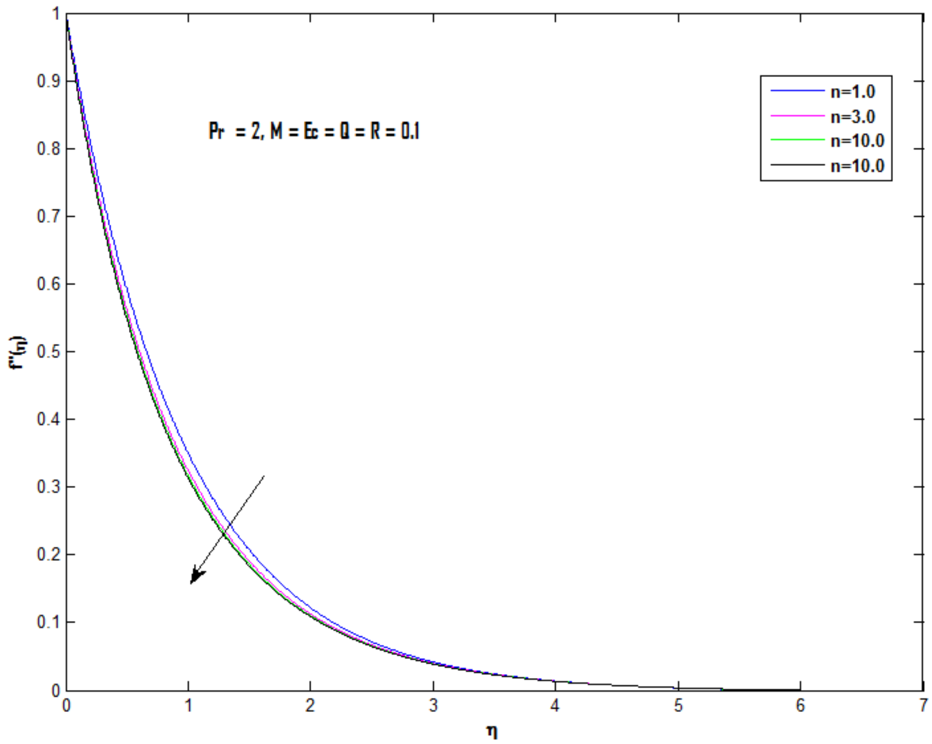

Figure 7 – Velocity profile for n

IV Conclusion

The basic governing equations are transformed into coupled nonlinear ordinary differential equations. Keller Box method is used to perform the numerical computations. The effects of nonlinear stretching parameter, Prandtl number, radiation parameter and heat source/sink parameters on the heat transfer characteristics are examined. Finally, got an excellent agreement with the previous paper. Briefly the above discussions can be summarized as follows.

1. The Local Nusselt number decreased with the increase in Magnetic parameter and Radiation parameter.It is also noticed that the nonlinear stretching parameter is to suppress the velocity field.
2. Local Nusselt number increased with the increase in Heat Source.
3. The velocity of the fluid is found to be increased with the increase in M where as the temperature is increased in this case.
4. The increasing effect of the Prandtl number decreases the temperature.
5. The increase in the Heat source increases the temperature of the fluid.
6. The increasing effect of the Radiation parameter increases the temperature.
7. The increasing effect of the Eckert number increases the temperature.

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