Toy model of evolving quantum cosmology with dark energy

Abstract

Qualitatively we assume that, at any stage of cosmic evolution, 1) Planck scale Hubble parameter plays a crucial role in cosmic evolution. 2) Ratio of critical energy density to thermal energy density plays a crucial role in understanding 'matter' density and 'dark matter' density. 3) Space-time curvature follows, $GM_t \cong R_t c^2$ where M_t and R_t represent the cosmic mass and radius respectively. With further research, a unified model of 'evolving quantum cosmology' with dark energy can be developed.

Keywords: Big bang; Planck scale Hubble parameter, Quantum cosmology; Mach's principle; Holographic principle; Observational cosmology; Dark energy;

1 Introduction

Photons and black holes can be considered as the best candidates of quantum gravitational objects. It is true that, without the existence of universe, there is no independent existence to any photon or any black hole. Now the fundamental question to be answered is: Is our universe a quantum gravitational object or something else? Physicists expressed several opinions with many possible solutions [1-5] and references therein. We could also express different unified views in this direction [6-15] and readers are strongly encouraged to go through. In an optimistic approach, some of the modern cosmologists believe that, during cosmic evolution, Planck scale quantum gravitational interactions might have an observable effect on the current observable cosmological phenomena. Clearly speaking, with respect to 'Quantum gravity' and Planck scale early universal laboratory, current universe can be considered as a low energy scale laboratory. If one is willing to consider the current observable universe as a low energy scale laboratory, currently believed cosmic microwave back ground temperature can be considered as the low energy quantum gravitational effect. At any time in the past, i.e as the operating energy scale was assumed to be increasing; past high cosmic back ground temperature can be considered as the high energy quantum gravitational effect. Thinking in this way, starting from the Planck scale, 'quantum cosmology' can be considered as 'scale independent' and the universe can be considered as the best quantum gravitational object. In this context, we have chosen the following two quantitative relations.

1. We define the Planck scale Hubble parameter, $H_{pl} \cong \sqrt{\frac{c^5}{G\hbar}} \cong 1.854921 \times 10^{43}$ sec^{-1} and apply it to cos-

mological data fitting in the form of $\left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right]^n$ where H_t is the running Hubble parameter and n is a suitable power index.

2. According to G't Hooft, the combination of quantum mechanics and gravity requires the three dimensional world to be an image of data that can be stored on a two dimensional projection much like a holographic image [16,17]. The 'holographic principle' is a property of string theory and a supposed property of quantum gravity that states that the description of a volume of space can be thought of as encoded on a lower-dimensional boundary. Based on this concept, for the four dimensional space-time universe, its three dimensional increasing volume can be set by Mach's principle, $\frac{GM_t}{R_t c^2} \cong 1$. Clearly speaking, information of the evolving universe, can be extracted from $R_t \cong \frac{GM_t}{c^2}$. With this proposal, at any stage of cosmic evolution, a closed and massive universe can be defined. One can find interesting technical discussion on this assumption by D.W.Sciama, R.H. Dicke, C. Brans and G. J. Whitrow [18-25].

Based on these quantitative relations, we re-view the phenomena of 'inflation' [26-28], 'acceleration' and 'dark energy' [29-32]. We arranged our revised version in the following way. In section-2, we proposed our assumptions connected with big bang and Planck scale. In section-3 we proposed many possible applications pertaining to observational cosmology. In section-4, we proposed a simple derivation for obtaining the expression for 'critical density'. In section-5 we presented our concluding remarks.

Subject of cosmology is quite interesting, very complicated and quite controversial.

1. In June 2015, three professors, Jeppe Trost Nielsen, Alberto Guffanti and Subir Sarkar of Niels Bohr International Academy and Rudolf Peierls Centre for Theoretical Physics, using the JLA catalogue of 740 SN Ia processed by the SALT2 method, come to a conclusion that [33], evidence for the currently believed cosmic acceleration is only marginal and current universe seems to expand at a constant rate. This breakthrough work got published in the prestigious Nature journal's 'Scientific Reports'. In their words: "**The 'standard' model of cosmology is founded on the basis that the expansion rate of the universe is accelerating at present - as was inferred originally from the Hubble diagram of Type Ia supernovae. There exists now a much bigger database of supernovae so we can perform rigorous statistical tests to check whether these 'standardisable candles' indeed indicate cosmic acceleration. Taking account of the empirical procedure by which corrections are made to their absolute magnitudes to allow for the varying shape of the light curve and extinction by dust, we find, rather surprisingly, that the data are still quite consistent with a constant rate of expansion.**"

2. According to T. Padmanabhan [34]:"One natural - and in fact, inevitable - contribution to cosmological constant arises from the energy density of quantum vacuum fluctuations. The trouble is, we do not know how to compute the gravitational effects of quantum fluctuations of the vacuum from first principles. Naive estimates suggests that this will give $\Lambda\left(\frac{G\hbar}{c^3}\right) \approx 1$ which misses the correct result by 120 orders of magnitude! It is possible to get around this difficulty and get the correct value but only if we are prepared to make some extra assumptions. **The appearance of G and $\hbar$ together strongly suggests that the problem of dark energy needs to be addressed by quantum gravity. None of the currently popular models of quantum gravity has anything meaningful to say on this issue (let alone predict its correct value). In fact, explaining the observed value of the dark energy is the acid test for any quantum gravity model and all the models currently available flunk this test. There is no doubt that, when we eventually figure this out, it will lead to as drastic a revolution in our conceptual understanding as relativity and quantum theory did".**

3. According to Martin Bozowald[1]:

   (a) "Quantum cosmology is based on the idea that quantum physics should apply to anything in nature, including the whole universe. Quantum descriptions of all kinds of matter fields and their interactions are well known and can easily be combined into one theory - leaving aside the more complicated question of unification, which asks for a unique combination of all fields based on some fundamental principles or symmetries. Nevertheless, quantizing the whole universe is far from being straightforward because, according to general relativity, not just matter but also space and time are physical objects. They are subject to dynamical laws and have excitations (gravitational waves) that interact with each other and with matter. Quantum cosmology is therefore closely related to quantum gravity, the quantum theory of the gravitational force and space-time. Since quantum gravity remains unfinished, the theoretical basis of quantum cosmology is unclear. And to make things worse, there are several difficult conceptual problems to be overcome".

(b) "**We remain far from a proper understanding of quantum cosmology, especially when physics at the Planck scale is involved. At the same time, research on quantum cosmology has led to progress in our understanding of generally covariant quantum systems and often showed unexpected effects of quantum space-time**".

2 Workable assumptions connected with Planck scale

With the following eight simple and logical assumptions, most of the currently believed cosmological observations can be reviewed and refined at fundamental level. In developing any novel model, one should see the possibility of minimizing the number of assumptions. In this context, we would like say that, our proposed assumptions are very clear and very simple.We are trying our level best in understanding and refining their individual roles and collective role. Our proposed set of assumptions can be divided into 'quantitative' and 'qualitative' assumptions. For the time being we appeal the readers to go through the rest of the paper and evaluate their novelty with reference to:

1. Implementing Planck scale, Mach's principle and Holographic principle;
2. Developing a model of quantum cosmology;
3. Current cosmological data fitting and ability for extrapolation to past and future;
4. Compatibility with hot big bang model and dark matter;
5. Simplicity and ability for extension or modification;

2.1 Proposed set of qualitative assumptions

At any stage of cosmic evolution,

1. Planck scale Hubble parameter plays a crucial role in cosmic evolution.
2. Ratio of critical energy density to thermal energy density plays a crucial role in understanding matter density and dark matter density.
3. Space-time curvature follows $GM_t \cong R_t c^2$, where M_t and R_t represent the cosmic mass and radius respectively.

2.2 Our basic conceptual thoughts

1. At any stage of cosmic evolution, $\left(\frac{3H_t^2 c^2}{8\pi G a T_t^4}\right) \cong \gamma_t^2$.
2. H_{pl} being the Planck scale Hubble parameter, we imagine that, at the Planck scale, $\left(\frac{3H_{pl}^2 c^2}{8\pi G a T_{pl}^4}\right) \cong \gamma_{pl}^2 \cong 1$.
3. If magnitude of H_{pl} is $\approx 10^{43}$, to a great surprise, we noticed that, $\gamma_0 \cong \sqrt{\left(\frac{3H_0^2 c^2}{8\pi G a T_0^4}\right)} \cong \left[1 + \ln\left(\frac{H_{pl}}{H_0}\right)\right] \cong 141$.
4. Based on this observation, for various decreasing values of $\gamma_t \cong \left[1 + \ln\left(\frac{H_{pl}}{H_t}\right)\right]$ in between 141 and 1, corresponding cosmic Hubble parameters and cosmic temperatures can be estimated.
5. With reference to current cosmological data,

(a) Both, matter density and dark matter density are approximately proportional to $\sqrt{\left(\frac{3H_0^2c^2}{8\pi G}\right)(aT_0^4)} \cong \frac{1}{\gamma_0}\left(\frac{3H_0^2c^2}{8\pi G}\right)$.

(b) Proportionality constant for matter density seems to be $\left(\frac{1+\sqrt{\gamma_0}}{2}\right)$.

(c) Proportionality constant for dark matter density seems to be $\left(\frac{1+\sqrt{\gamma_0}}{2}\right)^2$.

(d) Matter density seems to be approximately equal to $\left(\frac{1+\sqrt{\gamma_0}}{2}\right)\left[\frac{1}{\gamma_0}\left(\frac{3H_0^2c^2}{8\pi G}\right)\right]$.

(e) Dark matter density seems to be approximately equal to $\left(\frac{1+\sqrt{\gamma_0}}{2}\right)^2\left[\frac{1}{\gamma_0}\left(\frac{3H_0^2c^2}{8\pi G}\right)\right]$

6. Guessing that, $\gamma_{pl} \cong 1$, we noticed that, at the Planck scale, both, matter density and dark matter density seem to be equal to Planck scale critical density. It seems to be violating the currently believed Friedmann's cosmic 'density sum rule'.

7. To sustain the density sum rule for $1 \geq \gamma_t \leq 141$, we consider $\left[\frac{1}{1+\gamma_t}\left(\frac{3H_t^2c^2}{8\pi G}\right)\right]$ in place of $\left[\frac{1}{\gamma_t}\left(\frac{3H_t^2c^2}{8\pi G}\right)\right]$. If one is willing to consider this adjustment, at the Planck scale, both, matter density and dark matter density seem to be equal to $\frac{1}{2}$ of the Planck scale critical density.

8. For various increasing values of γ_t in between 1 and 141, it is noticed that, sum of matter density and dark matter density seems to be gradually decreasing and is always less than unity. With reference to cosmic 'density sun rule', one can identify [critical density-(matter density + dark matter density)] with 'dark energy'.

9. At the Planck scale, dark energy content is 'zero'. During cosmic evolution, dark energy content attains increasing values.

2.3 Proposed set of quantitative assumptions

Quantitatively, above set of qualitative assumptions can be fine-tuned with respect to current cosmological observational data and past & future cosmological predictions. In this paper, we choose the following set of assumptions. With further study, quantitatively, these set of assumptions can be modified according to one's own choice and selection.

1. Hubble parameter associated with Planck scale is, $H_{pl} \cong \sqrt{\frac{c^5}{G\hbar}} \cong 1.854921 \times 10^{43}\ \text{sec}^{-1}$.

2. With reference to the Planck scale Hubble parameter, H_{pl}:

   (a) Ratio of thermal energy density to critical energy density is, $(\Omega_T)_t \cong \left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right]^{-2} \cong \gamma_t^{-2}$.

   (b) Ratio of matter density to critical density is, $(\Omega_M)_t \cong \left(\frac{1}{1+\gamma_t}\right)\left(\frac{1+\sqrt{\gamma_t}}{2}\right)$

   (c) Ratio of dark matter density to critical density is, $(\Omega_{DM})_t \cong \left(\frac{1}{1+\gamma_t}\right)\left(\frac{1+\sqrt{\gamma_t}}{2}\right)^2$

   (d) Ratio of dark matter density to matter density is, $\frac{(\Omega_{DM})_t}{(\Omega_M)_t} \cong \left(\frac{1+\sqrt{\gamma_t}}{2}\right)$

3. Space-time curvature follows $GM_t \cong R_tc^2$, where M_t and R_t represent the cosmic mass and radius respectively.

2.4 Implications of our proposed set of assumptions

1. **About the cosmological constant problem**: With reference to assumption-1, ratio of Planck scale critical density to current critical density is, $\left(\frac{3H_{pl}^2c^2}{8\pi G}\right) \div \left(\frac{3H_0^2c^2}{8\pi G}\right) \cong 6.686 \times 10^{121}$. We wish to appeal that, our assumption-1 can be considered as a characteristic tool for constructing a model of 'quantum gravity'.

2. **About the horizon problem**: The 'horizon problem' or 'homogeneity problem' is a problem with the standard cosmological model of the hot Big Bang which was identified in the late 1960s, primarily by Charles W. Misner. It points out that different regions of the universe have not 'contacted' each other because of the great distances between them, but nevertheless they have the same temperature and other physical properties. If one is willing to consider the concept of 'matter causes the space-time to curve', 'horizon problem' can be understood. According to hot big bang model, during its evolution, as universe is expanding, thermal radiation temperature decreases and matter content increases. As matter content increases, based on Mach's principle, i.e. (with assumption 3), at any stage of evolution, it is possible to have an increasing radius of curvature,$R_t \cong \frac{GM_t}{c^2}$. Clearly speaking, for the current case, as there exists no matter outside of $R_0 \cong \frac{GM_0}{c^2}$, there is no scope for 'causal disconnection'.

3. **About cosmic inflation**: Mainstream cosmologists believe that the superluminal expansion period of the universe (called "cosmic inflation") ended by 10^{-32} seconds (a tiny fraction of a second) after the hot big bang [19-21]. Since that time, they believe, expansion initially decelerated (from gravity) and then, after about 6 billion years, began very slowly to accelerate (from dark energy). Many cosmologists proposed different starting mechanisms for initiating and fine tuning the believed 'inflation'. In this context, we would like to stress the fact that, with $(\Omega_M)_0 \cong \left(\frac{1}{1+\gamma_0}\right)\left(\frac{1+\sqrt{\gamma_0}}{2}\right)$ and $R_0 \cong \sqrt{\frac{2}{(\Omega_M)_0}}\left(\frac{c}{H_0}\right)$, estimated current cosmic radius is 93 billion light years and is just twice of the modern estimate! Clearly speaking, considering our proposed assumptions, currently believed cosmic inflation can be reviewed in a very simplified approach.

4. **About the equality of 'mass density' and 'visible matter density'**: It may be noted that, it is possible to show that, at any stage of cosmic evolution, $\left(\frac{3M_t}{4\pi R_t^3}\right) \div \left(\frac{3H_t^2}{8\pi G}\right) \cong (\Omega_M)_t$.

5. **About the current cosmic rotational kinetic energy**: With reference to $(\Omega_M)_0 \cong \left(\frac{1}{1+\gamma_0}\right)\left(\frac{1+\sqrt{\gamma_0}}{2}\right)$, $R_0 \cong \sqrt{\frac{2}{(\Omega_M)_0}}\left(\frac{c}{H_0}\right)$ and $M_0 \cong \frac{c^2R_0}{G} \cong \sqrt{\frac{2}{(\Omega_M)_0}}\left(\frac{c^3}{GH_0}\right)$ and by imagining the numerical equality of current angular velocity and current Hubble parameter, it is possible to show that, magnitude of current cosmic rotational kinetic energy (about the point of big bang) is equal to the magnitude of current dark energy. It may be an accidental coincidence also. See application-6 of subsection-3.6

3 Various applications of $\left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right]$ in cosmology

3.1 Application-1: Relation between cosmic thermal energy density and critical energy density

Let us assume that, during cosmic evolution, at any time, thermal energy density is proportional to the critical energy density.

$$aT_4^4 \propto \left(\frac{3H_t^2c^2}{8\pi G}\right) \tag{1}$$

With reference to the Planck scale and by considering the proportionality factor as $\gamma_t \cong \left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right]$, it is possible to define that,

$$aT_t^4 \cong \gamma_t^{-2}\left(\frac{3H_t^2c^2}{8\pi G}\right)$$
$$\rightarrow T_t \cong \gamma_t^{-\frac{1}{2}}\left(\frac{3H_t^2c^2}{8\pi Ga}\right)^{\frac{1}{4}} \tag{2}$$

$$\sqrt{\frac{3H_t^2c^2}{8\pi GaT_t^4}} \cong \frac{1}{\sqrt{(\Omega_T)_t}} \cong \gamma_t \tag{3}$$

T_0 being the current cosmic temperature,

$$aT_0^4 \cong \gamma_0^{-2}\left(\frac{3H_0^2c^2}{8\pi G}\right)$$
$$\rightarrow T_0 \cong \gamma_0^{-\frac{1}{2}}\left(\frac{3H_0^2c^2}{8\pi Ga}\right)^{\frac{1}{4}} \tag{4}$$

$$\sqrt{\frac{3H_0^2c^2}{8\pi GaT_0^4}} \cong \left[1+\ln\left(\frac{H_{pl}}{H_0}\right)\right] \cong \frac{1}{\sqrt{(\Omega_T)_0}} \cong \gamma_0 \tag{5}$$

With trial-error, it is noticed that, at $H_0 \cong 70\text{km/sec/Mpc} \cong 2.268529\times10^{-18}\text{sec}^{-1}$,

$$\gamma_0 \cong \frac{1}{\sqrt{(\Omega_T)_0}} \cong \left[1+\ln\left(\frac{H_{pl}}{H_0}\right)\right] \cong 141.2556$$
$$\text{and } T_0 \cong 2.72075K \tag{6}$$

As per the 2015 Planck data [30], the current value of CMBR temperature is reported to be:

1. $Planck\,TT + low\,P + BAO : (2.722 \pm 0.027)$K
2. $Planck\,TT, TE, EE + low\,P + BAO : (2.718 \pm 0.021)$K

As per the 2015 Planck data [30], the current value of the Hubble parameter is reported to be:

1. $Planck\,TT + low\,P : (67.31 \pm 0.96)$ km/sec/Mpc.
2. $Planck\,TE + low\,P : (67.73 \pm 0.92)$ km/sec/Mpc.
3. $Planck\,TT, TE, EE + low\,P : (67.77 \pm 0.66)$ km/sec/Mpc.

This fitted value of $H_0 \cong 70$ km/sec/Mpc can be compared with that of very recent reference [30]. As per reference [31], best value of $H_0 \cong (73.24 \pm 1.74)$ km/sec/Mpc.Clearly speaking, our fit of H_0 seems to lie in between the values recommended in reference [30] and reference [31] respectively. i.e. $(67.7 \pm 0.66) \geq H_0 \leq (73.24 \pm 1.74)$ km/sec/Mpc.

It may be noted that, in a quantum gravitational approach, relation (2) can be expressed in the following general form.

$$T_t \cong \left[\frac{19.5058817}{\gamma_t^{\frac{1}{2}}\Omega_M^{\frac{1}{4}}}\right]\left[\frac{\hbar c^3}{8\pi k_B G\sqrt{M_t M_{pl}}}\right] \tag{7}$$

$M_t \cong \sqrt{\frac{2}{\Omega_M}}\left(\frac{c^3}{GH_t}\right)$. Here in this relation, we try to highlight the expression, $\left[\frac{\hbar c^3}{8\pi k_B G\sqrt{M_t M_{pl}}}\right]$. Qualitatively this expression is similar to Hawking's black hole temperature formula [35] and needs further study. In our earlier publications [6-15], we proposed that,

$$T_t \cong \frac{\hbar c^3}{k_B G\sqrt{M_t M_{pl}}} \tag{8}$$

where $M_t \cong \frac{c^3}{2GH_t}$ is the mass of universe.

3.2 Application-2: To estimate the current cosmic matter density, dark matter density and dark energy density

With reference to the proposed assumptions, current matter density can be fitted by the following relation.

$$(\Omega_M)_0 \cong \left(\frac{1}{1+\gamma_0}\right)\left(\frac{1+\sqrt{\gamma_0}}{2}\right) \cong 0.0452884 \tag{9}$$

With reference to the proposed assumptions, current dark matter density can be fitted by the following relation.

$$(\Omega_{DM})_0 \cong \left(\frac{1}{1+\gamma_0}\right)\left(\frac{1+\sqrt{\gamma_0}}{2}\right)^2 \cong 0.291774 \tag{10}$$

Ratio of current matter density to dark matter density is can be expressed by the following relation.

$$\left(\frac{(\Omega_{DM})_0}{(\Omega_M)_0}\right) \cong \left(\frac{1+\sqrt{\gamma_0}}{2}\right) \cong 6.44257 \tag{11}$$

At any stage of cosmic evolution,

$$\begin{aligned} (\Omega_M)_t &\cong \left(\frac{1}{1+\gamma_t}\right)\left(\frac{1+\sqrt{\gamma_t}}{2}\right) \\ (\Omega_{DM})_t &\cong \left(\frac{1}{1+\gamma_t}\right)\left(\frac{1+\sqrt{\gamma_t}}{2}\right)^2 \\ \left(\frac{(\Omega_{DM})_t}{(\Omega_M)_t}\right) &\cong \left(\frac{1+\sqrt{\gamma_t}}{2}\right) \end{aligned} \tag{12}$$

With reference to the currently believed 'flat model concepts' and Friedmann's cosmic 'density sum rule',

$$\begin{aligned} (\Omega_{DE})_0 &\cong 1 - [(\Omega_M)_0 + (\Omega_{DM})_0] \\ &\cong 1 - (0.0452884 + 0.291774)) \cong 0.6629376 \end{aligned} \tag{13}$$

At any time in the past,

$$(\Omega_{DE})_t \cong 1 - [(\Omega_M)_t + (\Omega_{DM})_t] \tag{14}$$

Tab. 1: To understand the past cosmic density breakup

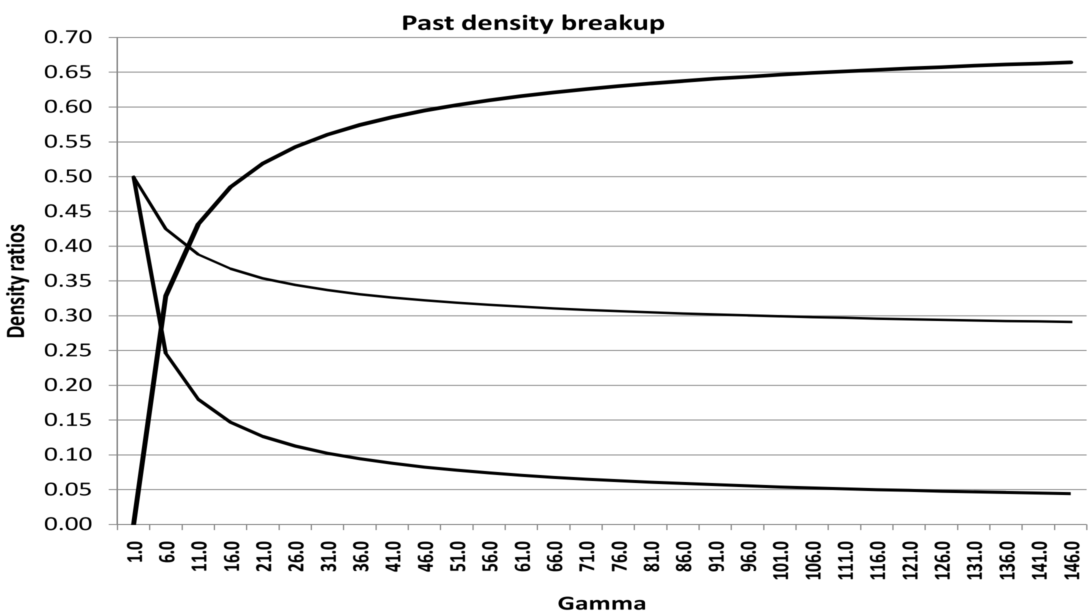
γ	$(\Omega_M)_t$	$(\Omega_{DM})_t$	$(\Omega_M)_t + (\Omega_{DM})_t$	$(\Omega_{DE})_t$
1.0	0.5000	0.5000	1.0000	0.0000
6.0	0.2464	0.4250	0.6714	0.3286
11.0	0.1799	0.3882	0.5681	0.4319
16.0	0.1471	0.3676	0.5147	0.4853
21.0	0.1269	0.3541	0.4810	0.5190
26.0	0.1129	0.3444	0.4574	0.5426
31.0	0.1026	0.3370	0.4396	0.5604
36.0	0.0946	0.3311	0.4257	0.5743
41.0	0.0881	0.3262	0.4144	0.5856
46.0	0.0828	0.3222	0.4049	0.5951
51.0	0.0783	0.3187	0.3970	0.6030
56.0	0.0744	0.3156	0.3901	0.6099
61.0	0.0711	0.3130	0.3840	0.6160
66.0	0.0681	0.3106	0.3787	0.6213
71.0	0.0655	0.3085	0.3740	0.6260
76.0	0.0631	0.3066	0.3697	0.6303
81.0	0.0610	0.3049	0.3659	0.6341
86.0	0.0590	0.3033	0.3623	0.6377
91.0	0.0573	0.3018	0.3591	0.6409
96.0	0.0557	0.3005	0.3562	0.6438
101.0	0.0542	0.2993	0.3534	0.6466
106.0	0.0528	0.2981	0.3509	0.6491
111.0	0.0515	0.2970	0.3485	0.6515

Continued on next page

γ	$(\Omega_M)_t$	$(\Omega_{DM})_t$	$(\Omega_M)_t + (\Omega_{DM})_t$	$(\Omega_{DE})_t$
116.0	0.0503	0.2960	0.3463	0.6537
121.0	0.0492	0.2951	0.3443	0.6557
126.0	0.0481	0.2942	0.3423	0.6577
131.0	0.0471	0.2934	0.3405	0.6595
136.0	0.0462	0.2926	0.3388	0.6612
141.0	0.0453	0.2918	0.3371	0.6629
146.0	0.0445	0.2911	0.3356	0.6644

See the following table 1 for past cosmic density breakup. Interesting point to be noted is that, at the Planck scale, $(\Omega_M)_{pl} \cong (\Omega_{DM})_{pl} \cong \frac{1}{2}$ and $(\Omega_{DE})_{pl} \cong 0$. See the following figure-1. Bottom curve represents the track of $(\Omega_M)_t$, middle curve represent the track of $(\Omega_{DM})_t$ and top curve represents the track of $(\Omega_{DE})_t$.

Fig. 1: **Past cosmic density breakup**



3.3 Application-3: To estimate the current cosmic radius and to estimate the current cosmic mass

According to modern cosmological observations, the commoving distance from Earth to the edge of the observable universe is about 14.26 Gpc (46.5 Gly = 4.40×10^{26} meters) in any direction. The observable universe is thus a sphere with a diameter of about 28.5 Gpc = 93 Gly = 8.80×10^{26} m). Readers are suggested to see the valuable scientific information available in Wikipedia web site on 'Observational cosmology'.

According to Mihran Vardanyan et al [32], "Bayesian model averaging is a procedure to obtain parameter constraints that account for the uncertainty about the correct cosmological model. We use recent cosmological observations and Bayesian model averaging to derive tight limits on the curvature parameter, as well as robust lower bounds on the curvature radius of the Universe and its minimum size, while allowing for the possibility of an evolving dark energy component. Because flat models are favored by Bayesian model selection, we find that model-averaged constraints on the curvature and size of the Universe can be considerably stronger than non model-averaged ones. For the most conservative prior choice (based on inflationary considerations), our procedure improves on non model-averaged constraints on the curvature by a factor of 2. The curvature scale of the Universe is conservatively constrained to be R_c>42 Gpc (99 %), corresponding to a lower limit to the number of Hubble spheres in the Universe NU >251 (99%)".

With reference to our proposed assumptions, current cosmic radius (including observable and non-observable) can be estimated in the following way.

From the beginning of Planck scale which is assumed to be associated with big bang, cosmic radius can be estimated as follows:

$$R_t \cong \sqrt{\frac{2}{(\Omega_M)_t}}\left(\frac{c}{H_t}\right) \tag{15}$$

For the current case,

$$R_0 \cong \sqrt{\frac{2}{(\Omega_M)_0}}\left(\frac{c}{H_0}\right) \cong 8.782 \times 10^{26} meters \tag{16}$$

From our estimate, current distance (observable and non-observable) about the point of big bang is 92.826 Gly=28.474 Gpc. Clearly speaking, current universe seems to constitute 293 Hubble spheres [32].This is really a very interesting coincidence and needs further study at fundamental level. Our estimate seems to be just 2 times higher than modern estimation. With further research and analysis and by understanding the galactic red shifts, discrepancy can be reviewed and resolved. Diameter of current (observable and non-observable) cosmic sphere about the point of big bang is 185.65Gly/56.95Gpc. See table 2.

Tab. 2: To fit the current cosmic radius

Estimating method	Cosmic distance from and about the reference point
Modern estimate (Observable)	46.5Gly/14.26Gpc (About Earth)
Our estimate (Observable+Non-observable)	92.826 Gly=28.474 Gpc (About point of BigBang)

$$M_t \cong \frac{c^2 R_t}{G} \cong \sqrt{\frac{2}{(\Omega_M)_t}}\left(\frac{c^3}{GH_t}\right) \tag{17}$$

For the current case,

$$M_0 \cong \frac{c^2 R_0}{G} \cong \sqrt{\frac{2}{(\Omega_M)_0}}\left(\frac{c^3}{GH_0}\right) \cong 1.182615 \times 10^{54}\, kg \tag{18}$$

3.4 Application-4: To interpret the cosmic expansion velocity and age

Based on the estimated cosmic matter density, it is possible to interpret the cosmic expansion velocity in the following way.At any stage of cosmic evolution,

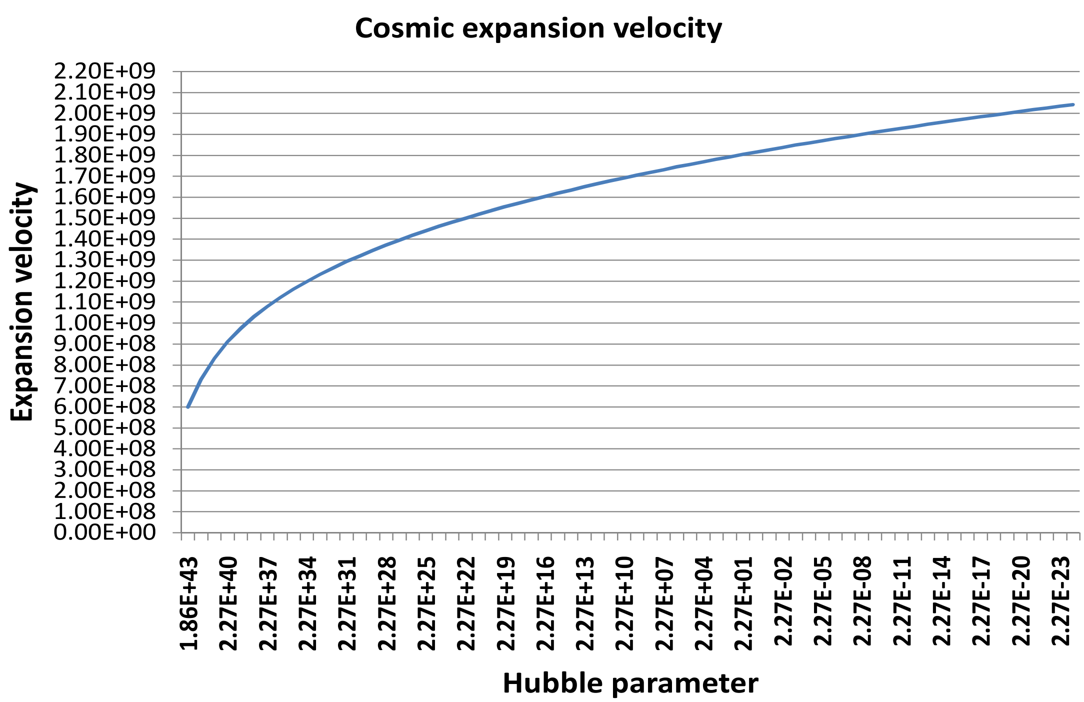
$$\begin{aligned} \frac{V_t}{c} &\cong \sqrt{\frac{2}{(\Omega_M)_t}} \\ \rightarrow V_t &\cong \sqrt{\frac{2}{(\Omega_M)_t}}c \end{aligned} \tag{19}$$

For the current case,

$$\frac{V_0}{c} \cong \sqrt{\frac{2}{(\Omega_M)_0}} \cong 6.64529$$
$$\to V_0 \cong \sqrt{\frac{2}{(\Omega_M)_0}}c \cong 1.9922 \times 10^9\, m.sec^{-1} \quad (20)$$

See the following figure 2 for the increasing cosmic expansion velocity. Based on the estimated cosmic matter

Fig. 2: **Increasing cosmic expansion velocity**



density, cosmic age can be expressed by the following relation. At any stage of cosmic evolution,

$$t \cong \frac{R_t}{V_t} \cong \frac{1}{H_t} \quad (21)$$

For the current case,

$$t_0 \cong \frac{R_0}{V_0} \cong \frac{1}{H_0} \cong 4.408 \times 10^{17} sec \quad (22)$$

See table-3 for various estimated cosmic physical parameters.

Tab. 3: Estimated cosmic physical parameters

H_t (1/sec)	$\left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right]$ Ratio	Temperature (K)	Radius (m)	Mass (kg)	Velocity (m/sec)	Age (sec)
1.855E+43	1.000e+00	9.247e+31	3.232e-35	4.353e-08	5.996e+08	5.391e-44
2.269E+42	3.101e+00	1.836e+31	3.221e-34	4.338e-07	7.308e+08	4.408e-43
2.269E+41	5.404e+00	4.399e+30	3.668e-33	4.940e-06	8.321e+08	4.408e-42
2.269E+40	7.706e+00	1.165e+30	4.013e-32	5.405e-05	9.104e+08	4.408e-41
2.269E+39	1.001e+01	3.232e+29	4.298e-31	5.788e-04	9.750e+08	4.408e-40
2.269E+38	1.231e+01	9.216e+28	4.541e-30	6.116e-03	1.030e+09	4.408e-39
2.269E+37	1.461e+01	2.675e+28	4.756e-29	6.404e-02	1.079e+09	4.408e-38
2.269E+36	1.692e+01	7.862e+27	4.948e-28	6.663e-01	1.122e+09	4.408e-37
2.269E+35	1.922e+01	2.333e+27	5.122e-27	6.898e+00	1.162e+09	4.408e-36
2.269E+34	2.152e+01	6.970e+26	5.282e-26	7.113e+01	1.198e+09	4.408e-35
2.269E+33	2.382e+01	2.095e+26	5.430e-25	7.313e+02	1.232e+09	4.408e-34
2.269E+32	2.613e+01	6.326e+25	5.568e-24	7.499e+03	1.263e+09	4.408e-33
2.269E+31	2.843e+01	1.918e+25	5.698e-23	7.674e+04	1.293e+09	4.408e-32
2.269E+30	3.073e+01	5.833e+24	5.820e-22	7.838e+05	1.320e+09	4.408e-31
2.269E+29	3.303e+01	1.779e+24	5.936e-21	7.994e+06	1.347e+09	4.408e-30
2.269E+28	3.534e+01	5.440e+23	6.046e-20	8.142e+07	1.372e+09	4.408e-29
2.269E+27	3.764e+01	1.667e+23	6.151e-19	8.283e+08	1.395e+09	4.408e-28
2.269E+26	3.994e+01	5.117e+22	6.251e-18	8.418e+09	1.418e+09	4.408e-27
2.269E+25	4.225e+01	1.573e+22	6.347e-17	8.547e+10	1.440e+09	4.408e-26
2.269E+24	4.455e+01	4.845e+21	6.439e-16	8.671e+11	1.461e+09	4.408e-25
2.269E+23	4.685e+01	1.494e+21	6.528e-15	8.791e+12	1.481e+09	4.408e-24
2.269E+22	4.915e+01	4.612e+20	6.613e-14	8.906e+13	1.500e+09	4.408e-23
2.269E+21	5.146e+01	1.426e+20	6.696e-13	9.017e+14	1.519e+09	4.408e-22
2.269E+20	5.376e+01	4.410e+19	6.776e-12	9.125e+15	1.537e+09	4.408e-21
2.269E+19	5.606e+01	1.366e+19	6.853e-11	9.229e+16	1.555e+09	4.408e-20
2.269E+18	5.836e+01	4.233e+18	6.928e-10	9.330e+17	1.572e+09	4.408e-19
2.269E+17	6.067e+01	1.313e+18	7.001e-09	9.428e+18	1.588e+09	4.408e-18
2.269E+16	6.297e+01	4.075e+17	7.072e-08	9.524e+19	1.604e+09	4.408e-17
2.269E+15	6.527e+01	1.266e+17	7.141e-07	9.616e+20	1.620e+09	4.408e-16
2.269E+14	6.757e+01	3.934e+16	7.208e-06	9.707e+21	1.635e+09	4.408e-15
2.269E+13	6.988e+01	1.223e+16	7.273e-05	9.795e+22	1.650e+09	4.408e-14
2.269E+12	7.218e+01	3.806e+15	7.337e-04	9.881e+23	1.664e+09	4.408e-13
2.269E+11	7.448e+01	1.185e+15	7.400e-03	9.965e+24	1.679e+09	4.408e-12
2.269E+10	7.678e+01	3.690e+14	7.460e-02	1.005e+26	1.692e+09	4.408e-11
2.269E+09	7.909e+01	1.150e+14	7.520e-01	1.013e+27	1.706e+09	4.408e-10
2.269E+08	8.139e+01	3.584e+13	7.578e+00	1.021e+28	1.719e+09	4.408e-09
2.269E+07	8.369e+01	1.118e+13	7.635e+01	1.028e+29	1.732e+09	4.408e-08
2.269E+06	8.599e+01	3.487e+12	7.691e+02	1.036e+30	1.745e+09	4.408e-07
2.269E+05	8.830e+01	1.088e+12	7.746e+03	1.043e+31	1.757e+09	4.408e-06
2.269E+04	9.060e+01	3.397e+11	7.800e+04	1.050e+32	1.769e+09	4.408e-05
2.269E+03	9.290e+01	1.061e+11	7.852e+05	1.057e+33	1.781e+09	4.408e-04
2.269E+02	9.520e+01	3.314e+10	7.904e+06	1.064e+34	1.793e+09	4.408e-03
2.269E+01	9.751e+01	1.036e+10	7.955e+07	1.071e+35	1.805e+09	4.408e-02
2.269E+00	9.981e+01	3.237e+09	8.005e+08	1.078e+36	1.816e+09	4.408e-01
2.269E-01	1.021e+02	1.012e+09	8.054e+09	1.085e+37	1.827e+09	4.408e+00

Continued on next page

H_t (1/sec)	$\left[1+\ln\left(\frac{H_{pl}}{H_t}\right)\right]$ Ratio	Temperature (K)	Radius (m)	Mass (kg)	Velocity (m/sec)	Age (sec)
2.269E-02	1.044e+02	3.165e+08	8.102e+10	1.091e+38	1.838e+09	4.408e+01
2.269E-03	1.067e+02	9.899e+07	8.149e+11	1.097e+39	1.849e+09	4.408e+02
2.269E-04	1.090e+02	3.097e+07	8.196e+12	1.104e+40	1.859e+09	4.408e+03
2.269E-05	1.113e+02	9.692e+06	8.242e+13	1.110e+41	1.870e+09	4.408e+04
2.269E-06	1.136e+02	3.034e+06	8.287e+14	1.116e+42	1.880e+09	4.408e+05
2.269E-07	1.159e+02	9.497e+05	8.332e+15	1.122e+43	1.890e+09	4.408e+06
2.269E-08	1.182e+02	2.974e+05	8.376e+16	1.128e+44	1.900e+09	4.408e+07
2.269E-09	1.205e+02	9.314e+04	8.419e+17	1.134e+45	1.910e+09	4.408e+08
2.269E-10	1.228e+02	2.918e+04	8.461e+18	1.139e+46	1.919e+09	4.408e+09
2.269E-11	1.251e+02	9.141e+03	8.503e+19	1.145e+47	1.929e+09	4.408e+10
2.269E-12	1.274e+02	2.864e+03	8.545e+20	1.151e+48	1.938e+09	4.408e+11
2.269E-13	1.297e+02	8.977e+02	8.586e+21	1.156e+49	1.948e+09	4.408e+12
2.269E-14	1.320e+02	2.814e+02	8.626e+22	1.162e+50	1.957e+09	4.408e+13
2.269E-15	1.343e+02	8.822e+01	8.666e+23	1.167e+51	1.966e+09	4.408e+14
2.269E-16	1.367e+02	2.766e+01	8.705e+24	1.172e+52	1.975e+09	4.408e+15
2.269E-17	1.390e+02	8.675e+00	8.744e+25	1.178e+53	1.984e+09	4.408e+16
2.269E-18	1.413e+02	2.721e+00	8.782e+26	1.183e+54	1.992e+09	4.408e+17
2.269E-19	1.436e+02	8.535e-01	8.820e+27	1.188e+55	2.001e+09	4.408e+18
2.269E-20	1.459e+02	2.677e-01	8.857e+28	1.193e+56	2.009e+09	4.408e+19

3.5 Application-5: To estimate the galactic receding speeds and galactic distances in the current expanding universe

Based on relations (15) and (16), within the current radius of 92.826 Gly=28.474 Gpc, from and about the point of big bang, galactic receding speeds can be approximated by the following relation.

$$(v_g)_0 \cong \left(\frac{d_g}{R_0}\right) V_0 \cong \left(\frac{(d_g)_0}{R_0}\right) 6.64529c \tag{23}$$

where $(d_g)_0$ is the current galactic distance from the point of big bang and $(v_g)_0$ is the current galactic receding speed. Based on this relation (23), within the current boundary of 92.826 Gly=28.474 Gpc, galactic distances corresponding to assumed galactic receding speeds can be expressed in the following way. See table-4.

$$\begin{aligned}(d_g)_0 &\cong \left(\frac{(v_g)_0}{V_0}\right) R_0 \cong \left(\frac{(v_g)_0}{6.64529c}\right) 8.782\times 10^{26}\text{ meters}\\ &\rightarrow (d_g)_0 \cong \frac{(v_g)_0}{H_0} \text{ where } v_g \leq 6.64529c\end{aligned} \tag{24}$$

$$\begin{aligned}(d_g)_0 &\cong \left(\frac{(v_g)_0}{V_0}\right) R_0 \cong \left(\frac{(v_g)_0}{6.64529c}\right) 8.782\times 10^{26}\text{ meters}\\ &\rightarrow (d_g)_0 \cong \frac{(v_g)_0}{H_0} \text{ where } v_g \leq 6.64529c\end{aligned} \tag{25}$$

From and about the point of big bang, by co-relating the 'actual' galactic distances and 'actual' galactic receding speeds with observed galactic red shifts, further research can be carried out.

Tab. 4: Galactic receding speeds and distances from and about the point of big bang

Galactic receding speed	Galactic distance in meters, Giga Light years and Giga parsec		
m/sec	m	Gly	Gpc
0.1c	1.32E+25	1.40	0.43
0.2c	2.64E+25	2.79	0.86
0.3c	3.96E+25	4.19	1.29
0.4c	5.29E+25	5.59	1.71
0.5c	6.61E+25	6.98	2.14
0.6c	7.93E+25	8.38	2.57
0.7c	9.25E+25	9.78	3.00
0.8c	1.06E+26	11.17	3.43
0.9c	1.19E+26	12.57	3.86
1.0c	1.32E+26	13.97	4.28
1.1c	1.45E+26	15.37	4.71
1.2c	1.59E+26	16.76	5.14
1.3c	1.72E+26	18.16	5.57
1.4c	1.85E+26	19.56	6.00
1.5c	1.98E+26	20.95	6.43
1.6c	2.11E+26	22.35	6.86
1.7c	2.25E+26	23.75	7.28
1.8c	2.38E+26	25.14	7.71
1.9c	2.51E+26	26.54	8.14
2.0c	2.64E+26	27.94	8.57

3.6 Application-6: To estimate the current cosmic rotational kinetic energy

For a moment if one is willing to assume that, at any stage of cosmic evolution, magnitude of angular velocity is equal to the Hubble parameter, it is possible to show that, magnitude of current cosmic rotational kinetic energy density is equal to the fitted current dark energy. It can be understood in the following way. From classical mechanics, rotational kinetic energy of any spherical body is given by,

$$K_{rot} \cong \frac{1}{2} I \omega^2 \tag{26}$$

where, I is the moment of inertia of the rotating body and ω is the angular velocity. Based on this relation, current cosmic rotational kinetic energy can be expressed by the following relation.

$$(K_{rot})_0 \cong \frac{1}{2} I_0 \omega_0^2 \tag{27}$$

As current 'mass density' is very small in magnitude, current observable universe can be considered as a thin spherical shell and hence its corresponding current moment of inertia can be expressed by the following relation.

$$I_0 \cong \frac{2}{3} M_0 R_0^2 \tag{28}$$

From the above two relations, current cosmic rotational kinetic energy can be expressed by the following simple relation.

$$(K_{rot})_0 \cong \frac{1}{3} M_0 R_0^2 \omega_0^2 \cong \frac{1}{3} M_0 R_0^2 H_0^2 \cong 1.5646 \times 10^{72}\ J \tag{29}$$

$$\left[(K_{rot})_0 \div \left(\frac{4\pi R_0^3}{3} \right) \right] \div \left[0.6667 \left(\frac{3H_0^2 c^2}{8\pi G} \right) \right] \cong 0.98 \tag{30}$$

where $\left[0.68\left(\frac{3H_0^2c^2}{8\pi G}\right)\right]$ is the currently believed dark energy density [23]. With reference to critical energy density, current rotational kinetic energy density can be expressed by the following relation.

$$\left[(K_{rot})_0 \div \left(\frac{4\pi R_0^3}{3}\right)\right] \cong \frac{M_0\omega_0^2}{4\pi R_0} \cong \frac{M_0 H_0^2}{4\pi R_0} \tag{31}$$

With respect to Hubble parameter,

$$\left[(K_{rot})_0 \div \left(\frac{4\pi R_0^3}{3}\right)\right] \cong \frac{\omega_0^2}{4\pi R_0}\left(\frac{c^2R_0}{G}\right) \cong \frac{H_0^2}{4\pi R_0}\left(\frac{c^2R_0}{G}\right) \cong \frac{H_0^2c^2}{4\pi G} \tag{32}$$

Ratio of current cosmic rotational kinetic energy density and critical energy density is equal to $\frac{2}{3} \cong 0.6667$. It can be expressed in the following way.

$$\left[(K_{rot})_0 \div \left(\frac{4\pi R_0^3}{3}\right)\right] \div \left[\frac{3H_0^2c^2}{8\pi G}\right] \cong \left[\frac{H_0^2c^2}{4\pi G}\right] \div \left[\frac{3H_0^2c^2}{8\pi G}\right] \cong \frac{2}{3} \tag{33}$$

If one is willing to consider this coincidence as 'heuristic', it is possible to say that, there exists a characteristic relation between currently believed 'dark energy' and 'estimated cosmic rotational kinetic energy' and needs further study at fundamental level.

4 Understanding cosmic critical density in relation to cosmic angular velocity

Given that some degree of rotation is observed in nearly every highly studied cosmic object, and that the universe in our model is assumed to be bounded and finite, then the possibility of cosmic rotation must be seriously considered. In this section, we try to derive the expression for 'critical density' in a very simple way.

Consider a cosmological body of mass M and radius R rotating with angular velocity ω_e and linear velocity v_e in such a way that a free or loosely bound particle of mass m lying on its equator has a kinetic energy equal to gravitational potential energy as,

$$\frac{1}{2}mv_e^2 \cong \frac{GMm}{R} \tag{34}$$

Then,

$$R\omega_e \cong v_e \cong \sqrt{\frac{2GM}{R}} \text{ and } \omega_e \cong \frac{v_e}{R} \cong \sqrt{\frac{2GM}{R^3}} \tag{35}$$

Therefore, the linear velocity of the body's rotation is equal to the particle's escape velocity. If, $M \cong \frac{4\pi}{3}R^3\rho_e$, then:

$$\omega_e \cong \frac{v_e}{R} \cong \sqrt{\frac{8\pi G\rho_e}{3}} \text{ Or } \omega_e^2 \cong \frac{8\pi G\rho_e}{3} \tag{36}$$

$$Density, \rho_e \cong \frac{3\omega_e^2}{8\pi G} \tag{37}$$

$$\text{Density} \propto (\text{angular velocity})^2 \tag{38}$$

Thus, density is proportional to the squared angular velocity. And, since our model also equates the magnitude of cosmic angular velocity with Hubble's parameter,

$$\rho_c \cong \frac{3H_t^2}{8\pi G} \tag{39}$$

The above relationships appear to be a natural consequence of a rotating cosmic model. Cosmic models that depend on this critical density may substitute cosmic angular velocity for Hubble's parameter and vice versa. At any stage of cosmic evolution,

1. If observable mass density is less than critical density, there is a scope for believing in cosmic expansion.

2. If observable mass density is greater than critical density, there is a scope for believing in cosmic collapse.
3. If observable mass density approaches critical density, there is a scope for believing in cosmic collapse after a long span of time.
4. In our case, estimated current mass density is 22.08 times less than the current critical density and supports current cosmic expansion.

5 Discussion and conclusion

Points to be noted in this toy model are:

1. We have successfully implemented the Planck scale in current cosmological observations.
2. We have perfectly connected the current Hubble parameter and current cosmic temperature.
3. We have successfully implemented Mach‘s principle and Holographic principle in modern cosmological observations.
4. We have estimated matter density and dark matter density with reference to the ratio of critical energy density to thermal energy density and by following the ‘density sum rule’, we have fitted the currently dark energy density.
5. We have estimated current cosmic radius, mass, velocity and age with reference to the current mass density.
6. We have estimated the current cosmic rotational kinetic energy and fitted with the currently believed dark energy.

Proceeding further, with the proposed set of assumptions,

1. Extrapolation to past and future is very easy.
2. With minor changes and with further study, a unified model of evolving quantum cosmology can be developed.
3. With further study and observations, actual galactic distances, actual galactic receding speeds and observed galactic redshifts can be studied in a unified approach.

In any model of cosmology, fundamental questions to be solved are: 1) Why do ‘dark matter’ and ‘visible matter’ have their measured values of $\approx 33\%$ of critical energy? 2) Why do ‘dark energy‘ has its measured values of $\approx 68\%$ of critical energy? 3) How to estimate their past and future magnitudes? These are the puzzling questions raised by the Royal Swedish Academy of Sciences [29] in 2011. In the conclusion part, Royal Swedish Academy of Sciences quoted like this: “The study of distant supernovae constitutes a crucial contribution to cosmology. Together with galaxy clustering and the CMB anisotropy measurements, it allows precise determination of cosmological parameters. The observations present us with a challenge, however: What is the source of the dark energy that drives the accelerating expansion of the Universe? Or is our understanding of gravity as described by general relativity insufficient? Or was Einstein‘s “mistake” of introducing the cosmological constant one more stroke of his genius? Many new experimental efforts are under-way to help shed light on these questions”.

In this context, we appeal that, our set of assumptions can be some consideration and with further research, their scope and workability can be scrutinized and validated.

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