Finding on the Similarity between the Two Empirical Formulas: Temperature Dependence of Volumetric Expansion of Gas and Temperature Dependence of Resistivity of Conductor

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ABSTRACT

This paper finds and examines the similarity between the temperature dependence of volumetric expansion of gases and the temperature dependence of resistance of conductors. In the 1780s, Jacques Charles came to know that the volume of the gas was proportionally increasing as temperature increased. He also stated that the rate of volume expansion was not dependent on the kinds of gases. In the early 19th century, Georg Ohm discovered the electric resistance. It was known that the electric resistivity of conductors changed with temperature. At the room temperature, the measured temperature coefficients of resistance for silver, copper, aluminum and gold are 0.0038, 0.0039, 0.0039 and 0.0034 respectively. When the temperature coefficient, 1/273, in Charles's law is expressed in a decimal, it indicates 0.0037. The finding of the similarity between the temperature dependence of volumetric expansion of gases and the temperature dependence of resistance of conductors is as follows: first, they have a linear relationship with respect to the temperature change; second, temperature coefficients are very close to each other; third, they are not dependent on the kinds of materials, lastly, the empirical formulas are also convertible each other mathematically. In this study, I here find that there is a similarity between the two empirical formulas. The temperature dependence of atomic vibration is suggested as the cause of the similarity. In addition, it is suggested that the volumetric expansion of gas could be related with the atomic vibration. This finding of similarity will be helpful for our understanding in the features of intrinsic behaviors of a gas molecular motion.

Keywords: atomic vibration; temperature coefficient of resistance; Charles's law

1. INTRODUCTION

Charles's law, or the temperature dependence of volumetric expansion of gases (TDVEG), states that the volume of a gas is directly proportional to the temperature with constant pressure[1,2]. The obvious fact in the empirical gas law is that the increase of temperature gives a rise to the increase of the speed of the molecules in a gas. Although it is not yet obvious how temperature increases the kinetic energy of gases, we know that temperature affects the amplitude of atomic vibration[3-11]. Interestingly, there is another empirical formula that is similarly dependent on temperature and atomic vibration. That is the formula of temperature dependence of resistivity of conductor (TDRC).

TDRC and TDVEG have a very similar temperature coefficient. The temperature coefficients of resistance of the good conductors such as silver, copper and aluminum are very close to 1/273 [see

Table 1, [12-20], which is the temperature coefficient of the volumetric expansion of gas in Charles's law formula. This similarity implies that there must be a common natural phenomenon between the two empirical formulas. In this study, I want to examine the common features between the two empirical formulas and investigate the reason of this similarity.

This paper is organized as follows. First, the inherent features of the Charles's law have been analyzed. Second, general descriptions of TDRC have been presented. Third, both temperature coefficients of the resistance of the conductors and the volumetric expansion of gas have been compared and an analysis of this similarity is presented. Lastly, the formula of the temperature dependence of resistivity of conductor has been converted into Charles's law formula mathematically. Later, the mechanism through which gas molecules get kinetic energy from temperature is discussed through the finding of this similarity.

2. SIMILARITY BETWEEN THE FORMULAS OF THE TEMPERATURE DEPENDENCE OF VOLUMETRIC EXPANSION OF GAS (TDVEG) AND THE TEMPERATURE DEPENDENCE OF RESISTIVITY OF CONDUCTOR (TDRC)

2.1 Temperature Dependence of Volumetric Expansion of Gas (TDVEG)

To see Charles's law expressed by the equation (2.1), the most important feature is that the increase of temperature causes the volume expansion of gases. The temperature coefficient is constant whatever the gas is [1, 2].

$$V_T = V_0[1 + \frac{1}{273}(T - T_0)] \tag{2.1}$$

Here, V_T is the volume of gas at temperature T ℃, V_0 is the volume at 0 ℃, 1/273 is the temperature coefficient of volumetric expansion of gas. Charles's law is not a theoretically developed formula. It is not known so far how temperature gives a rise to volume expansion of gases.

To see Avogadro's law, it states that there are the same numbers of molecules in the equal volume of gases under the same pressure and temperature whatever the gas is. Therefore, another important feature of the gas laws is that they are not dependent on the kinds of gases.

Let's look at the temperature dependent atomic vibration to understand these two important features of the gas laws. Every gas molecule is composed of atoms, and the amplitude of atomic vibration increases with temperature [3-11]. However, when temperature reaches 0 K, the volume of a gas disappears according to Charles's law. It means that the speed of molecules in a gas is zero with no atomic vibration. Therefore, considering Charles's law in relation to the atomic vibration, it can be assumed that the atomic vibration gives a rise to the kinetic energy of a gas. Under this assumption, Charles's law can be reinterpreted as follows: first, the increased temperature increases the amplitude of atomic vibration proportionally. Then, the increased amplitude of atomic vibration increases the speed of molecules in gases. Thus, temperature increase contributes the linear volume expansion of gases. Second, every gas molecule is composed of atoms and the atomic vibration is majorly dependent on temperature. Thus, the temperature coefficient of gases, which is based on the atomic vibration, is constant whatever the gas is.

2.2 Temperature Dependence of Resistivity of Conductor (TDRC)

Electrical resistivity of a material shows how strongly the material opposes the flow of free electrons passing through it. If there is no free electron, the material will be a nonconductor. On the other hand, the conductors have a lot of free electrons. But they have different specific resistivity because of their different numbers of free electrons and different crystal lattice structures. Also, the electrical resistivity of conductors changes with temperature. It is well known that the increased amplitude of atomic vibration increases the collision of electrons with the atoms which makes up the crystal lattice[3]. In other words, the temperature dependence of resistivity of conductor is directly related to the atomic vibration [21, 22]. In general, resistance is proportional to the temperature if the temperature does not vary too much. The well-known formula of the temperature dependence of resistivity of conductor is as follows [23]:

$$R_T = R_0[1+\alpha(T - T_0)] \tag{2.2}$$

Here, R_T is resistance at T ℃, R_0 is resistance at T_0 ℃, α is temperature coefficient of resistance at T_0 ℃. Let's look at the temperature coefficients of r esistance for the good conductors having a low specific resistivity in Table 1 [12-20].

Table 1. The measured temperature coefficient of resistance for the good conductors.

Material	Specific resistivity($\Omega \cdot m$)*	Temperature coefficient of resistance(1/℃)**, α	Reference
Ag	1.59×10^{-8}	0.0038	[12,13,15]
Cu	1.70×10^{-8}	0.0039~0.0040	[18,16]
Au	2.44×10^{-8}	0.0034~0.0037	[13,15]
Al	2.82×10^{-8}	0.0037~0.0043	[13,16]
W	5.6×10^{-8}	0.0045~0.0048	[13,16]
Zn	5.9×10^{-8}	0.0037~0.0038	[14]
Mo	6.0×10^{-8}	0.0044	[14]
Co	6.3×10^{-8}	0.0066	[17]
Ni	7.0×10^{-8}	0.006~0.0068	[13]
Fe	1.0×10^{-7}	0.005	[13,20]
Pt	1.1×10^{-7}	0.0039	[14,15]
V	1.9×10^{-7}	0.0039	[19]
Pb	2.2×10^{-7}	0.0039	[13,20]

*(* All values at 20 ℃, ** the exact value dependents on the purity of material as well as the temperature.)*

The temperature coefficient α is typically 0.003 to 0.006 for metals for room temperature.

2.3 The Similarities Between the Two Empirical Formulas

It is shown that the volume and the resistance are linearly dependent on the change of temperature in the two equations (2.1) and (2.2). It is also meaningful to make a comparison between the temperature coefficients in the two formulas.

Temperature coefficient of resistance (1/℃)

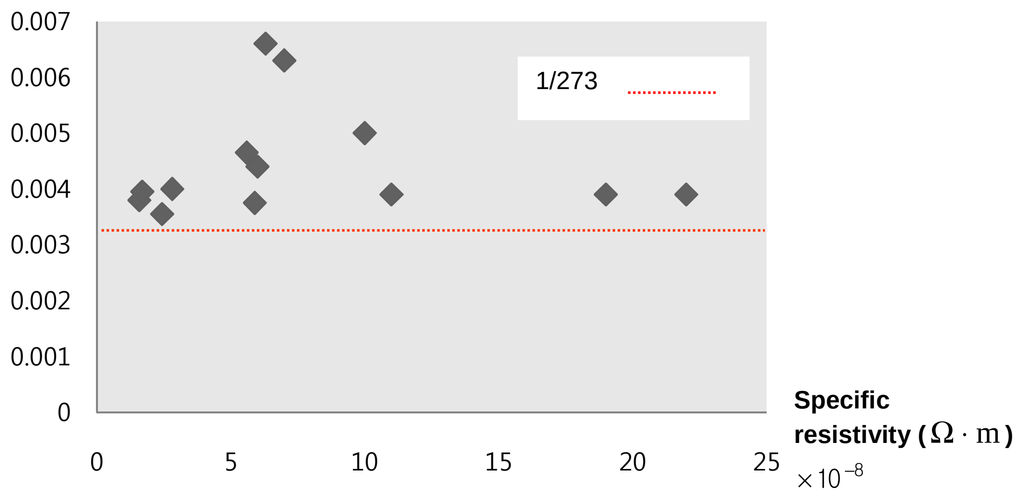

Fig. 1. The scatter diagram of the temperature coefficients of resistance of the good conductors in Table 1 compared with the temperature coefficient, 1/273, in Charles's law.

In Table 1, the average temperature coefficient of resistance of the good conductors is 0.0045, which is about 21% larger than the value of 1/273. However, most of the good conductors have temperature coefficients of resistance which are very close to the value of 1/273. In summary, there are shared features in the two empirical formulas. First, they are linearly dependent on the temperature changes. Second, the values of the temperature coefficients are very close to each other. Third, they are not dependent on the kinds of material. These common features are remarkable considering that the volume expansion of gases and the resistance of conductors are very different fields of physics.

To examine the common features between them, atomic vibration can be considered to be a determinant factor, because all of the gases or the conductors are composed of atoms. In addition, the mean square amplitude of vibration of atoms is proportional to the temperature [3-11]. We know that temperature dependence of resistivity is caused by the atomic vibration [3, 21, 22]. In Charles's law, the volume of a gas becomes zero at the temperature of 0 K. At this temperature, atoms do not vibrate anymore. This is the evidence for that there must be a relationship between the atomic vibration and the volume expansion of a gas. Therefore, the similarity shown in the two empirical formulas can be explained by the temperature dependence of atomic vibration. The common features between the two empirical formulas are summarized in Table 2.

Table 2. The similarity between TDVEG and TDRC

	TDVEG	**TDRC**
Formula	$V_T = V_0[1 + \frac{1}{273}(T - T_0)]$	$R_T = R_0[1 + \alpha(T - T_0)]$
Object	Volume of gas	Resistance of conductor
Temperature coefficient	1/273	α ($\cong$ 1/273)
Material	Negligible	Week

dependency		(for the good conductors)
Cause	Atomic vibration (inferred) (Evidence: When atomic vibration stops at 0K, the volume of gas becomes zero) .	Atomic vibration

2.4 Conversion between the two empirical formulas

If the two formulas are equally temperature dependent and related with atomic vibration, are they convertible to each other? Note that resistance is linearly proportional to the amplitude of atomic vibration and the amplitude of vibration is proportional to the temperature. Then, the general relationship between resistance and temperature for the conductors can be expressed by:

$$R = k_1 T \tag{2.3}$$

Also, temperature has a relationship with the volume of gas by the ideal gas law. By applying the ideal gas law to the equation (2.3), it can be rewritten as:

$$R_T = k_1 \frac{PV_T}{k_2} \tag{2.4}$$

Finally, by using equation (2.4), equation (1) can be rewritten and summarized by:

$$k_1 \frac{PV_T}{k_2} = k_1 \frac{PV_0}{k_2}[1+\alpha(T - T_0)]$$

$$V_T = V_0[1+\alpha(T - T_0)] \tag{2.5}$$

Here, k_1, k_2 are proportional constants, P is pressure, V_T is the volume of gas at temperature T ℃, V_0 is the volume at 0 ℃, α is the temperature coefficient of volumetric expansion of gas in Charles's law. The converted equation (2.5) is mathematically isomorphic to Charles's law formula shown in equation (2.1).

3. DISCUSSIONS

The good conductors have been compared in this study because their resistance changes are the most sensitive to atomic vibration. Not good conductor such as nickel-chrome or Manganin is not sensitive to the change of temperature and it has a very small value of temperature of coefficient of resistance.

The similarity found in this study indicates that the two empirical formulas might be affected by a common feature of natural phenomena. If the common feature is the atomic vibration, the suggested assumption that the atomic vibration gives a rise to the kinetic energy of a gas is valid. How the atomic vibration is related with the volume expansion of a gas is not well investigated. Nonetheless, there is a research that the asymmetric repulsive forces on the nucleus by the atomic vibration give speed to the gas [24], although more evidences are needed to prove the suggested assumption.

4. CONCLUSIONS

In this study, I find a similarity between the two empirical formulas, the TDVEG and TDRC; first, the two formulas have a linear relationship with temperature change, second, the temperature coefficients are very close to each other, third, they both do not vary depending on the kind of materials, lastly, they are mathematically convertible. This finding is remarkable because the volume expansion of gases and the resistance of conductors are very different fields of physics. The temperature dependent atomic vibration as the cause of this similarity has been suggested. Although it is not well known in Charles's law how the volume expansion of a gas is related with atomic vibration, it is widely known that temperature dependence of resistance of the conductor is directly related with atomic vibration. Thus, the finding of this similarity shows that the kinetic energy of a gas in Charles's law could be related with the atomic vibration. This is helpful to understand the features of intrinsic behaviors of a gas molecular motion for future research.

COMPETING INTERESTS

No competing interests.

AUTHOR'S CONTRIBUTIONS

The sole author analyzed, interpreted and prepared the manuscript.

REFERENCES

[1] Krimsley VS, Introductory Chemistry (Brooks/Cole, Boston); 1995.

[2] Zumdahl SS and Zumdahl SA, Chemistry (Cengage Learning, Boston); 2013.

[3] Gilmore C, Materials Science and Engineering Properties, SI Edition (Cengage Learning, Boston); 2014.

[4] Mittemeijer EJ, Fundamentals of Materials Science: The Microstructure–Property Relationship Using Metal as Models Systems (Springer Science & Business Media, Berlin); 2010.

[5] Carter GF, Paul DE and Editors, Materials Science and Engineering (ASM International, Almere);1991.

[6] Lonsdale K, Acta Cryst. 1948; 1: 142-149.

[7] Jiang Q, Ao ZM and Zheng WT, Chemical Physics Letters 2007; 438: 102-104

[8] Kellett EA, Jackets BP, Richards BP, Carbon 1964; 2(2): 175-183.

[9] http://web.mit.edu/mbuehler/www/SIMS/Thermal%20Expansion.html

[10] Ziman, JM, Principles of the Theory of Solids, 2nd Ed. (Cambridge University Press, Cambridge); 1972.

[11] Perepechko I, Low-Temperature Properties of Polymers (Pergamon press, Oxford, New York); 1977.

[12] Smith WF and Hashemi J, Foundations of Material Science and Engineering McGrawHill SciTech 4th (McGraw-Hill, New York); 2003.

[13] Serway RA and Jewett JW, Principles of Physics: A Calculus-Based Text, Volume 2 (Cengage Learning, Boston); 2014.

[14]http://www.allaboutcircuits.com/textbook/direct-current/chpt-12/temperature-coefficient-resistance/

[15] http://www.electrical4u.com/temperature-coefficient-of-resistance/

[16] http://hyperphysics.phy-astr.gsu.edu/hbase/tables/rstiv.html

[17] http://www.radio-electronics.com/info/formulae/resistance/resistance-temperature-coefficient.php

[18] Giancoli DC, Physics, 4th Ed (Prentice Hall, Upper Saddle River); 1995.

[19] Dorf RC, The Electrical Engineering Handbook,Second Edition (CRC Press, Boca Raton); 1997.

[20] Chakrabarti, Basic Electrical Engineering (Tata McGraw-Hill Education, Pennsylvania Plaza); 2009.

[21] Murray G, White CV and Weise W, Introduction to Engineering Materials: Behavior: Properties, and Selection (Marcel Dekker, New York); 1993.

[22] Kazuyuki O and Yoshihide N, The society of Applied Physics 1971; 10(1): 99-108.
[23] Singh KC, Basic Physics (PHI Learning Pvt. Ltd., Delhi); 2009.
[24] Sim CG, Int. J. of Phys. Sci. 2015; 10(22): 576-583.