

Radiation effects on heat and mass transfer of MHD fluid over a permeable stretching sheet with Buoyancy forces

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ABSTRACT:

This paper investigates the influence of Buoyancy effects and thermal radiation of the nonlinear MHD flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting and Boussinesq fluid along a permeable vertical stretching sheet. A magnetic field is applied transversely to the direction of the flow. The basic equations governing the flow, heat transfer, and concentration are reduced to a set of non linear ordinary differential equations by using appropriate transformation variables. The non linear ordinary differential equations are first linearised using Quasi-linearization and solved numerically by an implicit finite difference scheme. Then the system of algebraic equations is solved by using Gauss-Seidal iterative method. The effects of physical parameters on the velocity, temperature, and concentration profiles are illustrated graphically. Velocity, Temperature and Concentration profiles drawn for different controlling parameters reveal that the flow field is influenced appreciably by the presence of buoyancy effects, thermal radiation, and magnetic field.

Keywords: Buoyancy parameters; Thermal radiation; Suction parameter; Magnetic field; Finite difference scheme

INTRODUCTION:

The analysis of the flow field in a boundary-layer near a stretching sheet is an important part in fluid dynamics and heat transfer occurring in a number of engineering processes such as extrusion of plastic and rubber sheets, polymer processing and metallurgy [1]. The interaction of moving fluids with magnetic fields provides a rich variety of phenomena associated with mechanical energy conversion such as metals processing, heating and flow control, or power generation from two-phase mixtures [2], but the major use of MHD is in plasma physics. The study of magneto hydrodynamics is also stimulus due to its vast application to the delineation of space and astrophysical plasmas.

Considering suction/injection effects, Erickson et al. [3] studied heat and mass transfer over a moving surface. Pal [4] illustrated the impact of different parameters on velocity, temperature and concentration profiles for both assisting and opposing flows in stagnation-point flow over a stretching surface with thermal radiation numerically. In recent years, MHD flow problems have become more important industrially. Indeed, MHD laminar boundary layer behavior over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. In his pioneering work, Sakiadis [5] developed the flow field due to a flat surface, which is moving with a constant velocity in a quiescent fluid.

In recent years, MHD flow problems have become important industrially. Indeed, MHD laminar boundary layer behavior over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. The study of hydrodynamic flow and heat transfer over a stretching sheet may find its applications in polymer technology related to the stretching of plastic sheets. Also, many metallurgical processes involve the cooling of continuous strips or filaments by drawing then cooling of continuous strips or filaments by

drawing them through a quiescent fluid and while drawing these strips are sometimes stretched. The rate of cooling can be controlled by drawing such strips in an electrically conducting fluid subjected to a magnetic field in order to get the final products of desired characteristics as such a process greatly depends on the rate of cooling. For the non-Newtonian power-law fluids, the hydrodynamic problem of the MHD boundary layer flow over a continuously moving surface has been dealt by Andersson et al [6], Cortell [7] and Mahmoud and Mahmoud [8]. All the above mentioned investigators confined their analyses to MHD flow and heat transfer over a linear stretching sheet.

Further investigations on boundary layer flow and heat transfer of viscous fluids over a flat sheet are very important for development in many manufacturing processes, such as polymer extrusion, drawing of copper wires, continuous stretching of plastic films and artificial fibers, hot rolling, wire drawing, glass-fiber, metal extrusion, and metal spinning. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products (Magyari & Keller [9]). Among these studies, Sakiadis [10] initiated the study of the boundary layer flow over a stretched surface moving with a constant velocity and formulated a boundary-layer equation for two-dimensional and axisymmetric flows.

Bujurke et. al. [11] made an investigation on the heat transfer analysis in a second order fluid flow past a stretching surface with heat transfer. Datta et. al. [12] has studied the distribution of temperature in a continuous stretching sheet with uniform wall heat flux. Further flow and heat transfer from a linearly stretching sheet gained more importance due to practical applications in industrial processes. Abel and Veena [13] have analyzed visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet.

Understanding MHD is strongly related to the comprehension of physical effects which take place in MHD. When a conductor moves into a magnetic field, electric current is induced in the conductor and creates its own magnetic field (Lenz's law). Since the induced magnetic field tends to eliminate the original and external supported field, the magnetic field lines will be excluded from the conductor. Conversely, when the magnetic field influences the conductor to move it out of the field, the induced field amplifies the applied field. The net result of this process is that the lines of force appear to be dragged accompanied by the conductor. In this paper the conductor is the fluid with complex motions. To understand the second key effect which is dynamical we should know that when currents are induced by a motion of a conducting fluid through a magnetic field, a Lorentz force acts on the fluid and modifies its motion. In MHD, the motion modifies the field and vice versa. This makes the theory highly non-linear [14, 15]. Sparrow and Abraham [16] studied Universal solutions for the streamwise variation of the temperature of moving sheet in the presences of moving fluid. Numerical simulation of the radiative heating of a moving sheet was analysed by Boetcher et al [17]. Sparrow and Abraham [18] studied New buoyancy model replacing the standard Pseudo-density difference for internal natural convection in gases.

The main goal of this paper is to find the numerical solutions using numerical techniques for the velocity, temperature and concentration distributions to study the steady magneto hydrodynamic fluid flow over a stretching sheet in the presence of buoyancy forces. The effects of different involved parameters such as magnetic field parameter, suction parameter, Prandtl number, buoyancy parameters due to temperature and concentration effects, Schmidt number, Biot number and radiation parameter on the fluid velocity, temperature and concentration distributions are plotted and discussed

MATHEMATICAL FORMULATION:

In this paper we examine heat and mass transfer in a steady laminar two-dimensional boundary-layer flow of an incompressible electrically conducting fluid over a permeable stretching sheet (Fig. 1). It is assumed that the stretching velocity is in the form of $u_w(x) = c(x)^{1/3}$ (two equal and opposite forces are applied along the x – axis with the fixed origin) where c is a constant and the surface of the sheet is heated by convection from a hot fluid at temperature h_f . The flow will be induced through the stretching sheet. A magnetic field of non-uniform strength $B(x) = B_0(x)^{-1/3}$ is applied normal to the sheet. The left surface of the sheet is heated by convective heat transfer and all of the fluid properties are considered as constant properties except density. The boundary-layer governing equations and boundary conditions with the Boussinesq and the boundary-layer approximations are as contained [19]. The induced magnetic

field is neglected in comparison with the applied magnetic field and the viscous dissipation is small. The governing equations are

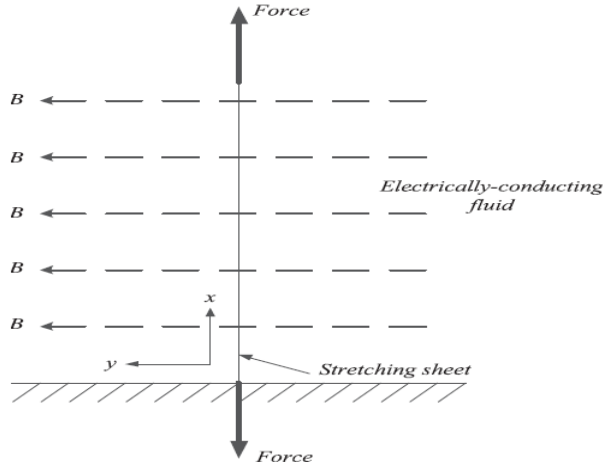


Figure 1 The schematic diagram of the stretching sheet problem.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad -- (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho} \mu + g(\beta_T(T - T_\infty) + \beta_C(C - C_\infty)), \quad -- (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_\infty^3}{3\rho C_p K_1} \frac{\partial^2 T}{\partial y^2}, \quad -- (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}, \quad -- (4)$$

where u and v are velocity components in the direction of x and y along and normal to the surface respectively (as shown in fig 1). ν is kinematic viscosity, σ is the electric conductivity, ρ is the fluid density, $B(x)$ is non-uniform magnetic field parameter, g is the acceleration due to gravity, β_T is coefficient of thermal expansion, β_C is coefficient of thermal expansion with concentration, T is the fluid temperature, C is fluid concentration, α is the thermal diffusivity, C_p is specific heat at constant pressure, σ^* is the Stephan-Boltz-man constant, K_1 is the Rosseland mean absorption coefficient and D is the coefficient of mass diffusivity. It should be made clear that the last term in Eqn. (3) refers to the radiation parameter. In this case, the Rosseland approximation has been supposed and the radiative heat flux is given by $q_r = -\frac{4\sigma^*}{3k_1} \frac{\partial T^4}{\partial y}$. Using the Taylor series T^4 is defined as a linear function of temperature $T^4 \cong 4T_\infty^3 T - 3T_\infty^4$ [20].

The thermal radiation is quite significant and the quality of final product can be controlled by the control of cooling rate via radiation parameter. In polymer industry, the thermal radiation effect may play an important role in the control of heat transfer process which is directed in a thermally controlled environment. The desired quality of the final product can be reached by the knowledge of radiative heat transfer [21]

The corresponding boundary conditions are as follow [22]:

$$u = u_w(x), v = v_w, -k \frac{\partial T}{\partial y} = h_f(x)(T_w - T),$$

$$C_w = C_\infty + bx \text{ at } y = 0,$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad -- (5)$$

After introducing stream function Ψ and similarity variable η , the ordinary form of equations can be derived. It is assumed that the temperature varies in the x-direction, too ($T_w = T_\infty + ax$).

$$\eta = ycx^{-1/3}v^{1/2}, \Psi = c^{1/2}x^{2/3}v^{1/2}f(\eta),$$

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \varphi(\eta) = \frac{C-C_\infty}{C_w-C_\infty} \quad -- (6)$$

$$f''' + \frac{2}{3}ff'' - \frac{1}{3}f'^2 - Mf' + (\lambda_r\theta + \lambda_c\varphi) = 0, \quad -- (7)$$

$$(1 + N_r)\theta'' + \frac{Pr}{3}(2f\theta' - 3f'\theta) = 0, \quad -- (8)$$

$$\varphi'' + \frac{1}{3}S_c(2f\varphi' - 3f'\varphi) = 0 \quad -- (9)$$

where

$Re_x = \frac{U_w x}{\nu}$ is Reynolds number

$\lambda_T = \frac{g\beta_T(T_w - T_\infty)}{c^2} x^{1/3}$ is Buoyancy parameter

$\lambda_C = \frac{g\beta_C(C_w - C_\infty)}{c^2} x^{1/3}$ is concentration Buoyancy parameter

$Pr = \frac{\mu}{\alpha}$ is Prandtl number

$S_c = \frac{\nu}{D}$ is Schmidt number

$M = \frac{\sigma\beta_0^2}{\rho c}$ is magnetic parameter

$N_r = \frac{16\sigma^* T_\infty^3}{3\rho C_p K_1}$ is Thermal Radiation parameter

The corresponding boundary conditions are as follows

$$f(\eta) = f_w, f'(\eta) = 1, \theta'(\eta) = -Bi[1 - \theta(0)],$$

$$\phi(\eta) = 1 \text{ at } \eta = 0,$$

$$f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \phi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad -- (10)$$

Where $f_w = -\frac{3x^{1/3}v_w}{2\nu^{1/2}c^{1/2}}$ is the suction/injection parameter ($f_w > 0$ for suction and $f_w < 0$ for injection) and

$Bi = \frac{\nu^{1/2}x^{1/3}h_f}{kc^{1/2}}$ is the Biot number. Since we have considered free convective flow, the velocity at the

infinity is equal to zero

To solve the system of transformed governing equations (7)-(9) with the boundary conditions (10) we first linearized equation (7) by Quasi-linearization technique

Then equation (7) transformed to

$$f''' + \frac{2}{3}f''(F) + f'(-\frac{2}{3}F - M) + (\frac{2}{3}F'')f + (\frac{1}{3}(F')^2 - \frac{2}{3}FF'' + \lambda_r\theta + \lambda_c\varphi) = 0, \quad -- (11)$$

where F is assumed to be known function and the above equation can be written as

$$A_0 f''' + A_1 f'' + A_2 f' + A_3 f + A_4 = 0, \quad -- (12)$$

where

$$A_0[i] = 1$$

$$A_1[i] = \frac{2}{3}F$$

$$A_2[i] = \left(-\frac{2}{3}F - M\right)$$

$$A_3[i] = \frac{2}{3}F''$$

$$A_4[i] = \left(\frac{1}{3}(F')^2 - \frac{2}{3}FF'' + \lambda_r \theta + \lambda_c \varphi\right)$$

Equation (8) can be expressed as

$$C_0 \theta'' + C_1 \theta' + C_2 \theta + C_3 = 0, \quad -- (13)$$

where

$$C_0[i] = 1 + N_r$$

$$C_1[i] = \frac{2}{3}P_r f$$

$$C_2[i] = -P_r f'$$

And equation (9) can be expressed as

$$E_0 \varphi'' + E_1 \varphi' + E_2 \varphi = 0, \quad -- (14)$$

where

$$E_0[i] = 1$$

$$E_1[i] = -\frac{2}{3}S_c f$$

$$E_2[i] = S_c(f')$$

Using implicit finite difference formulae, the equations (12),(13) and (14) are transformed to

$$B_0[i]f[i+2] + B_1[i]f[i+1] + B_2[i]f[i] + B_3[i]f[i-1] + B_4[i] = 0, \quad -- (15)$$

where

$$B_0[i] = 2A_0[i], B_1[i] = -6A_0[i] + 2hA_1[i] + h^2A_2[i]$$

$$B_2[i] = 6A_0[i] - 4hA_1[i] + 2h^3A_3[i], B_3[i] = -2A_0[i] + 2hA_1[i] - h^2A_2[i]$$

$$B_4[i] = 2h^3A_4[i]$$

$$D_0[i]g[i+1] + D_1[i]g[i] + D_2[i]g[i-1] = 0, \quad -- (16)$$

where

$$D_0[i] = C_0[i] + hC_1[i], D_1[i] = -2C_0[i] + h^2C_2[i] - hC_1[i]$$

$$D_2[i] = C_0[i]$$

$$H_0[i]\varphi[i+1] + H_1[i]\varphi[i] + H_2[i]\varphi[i-1] = 0 \quad -- (17)$$

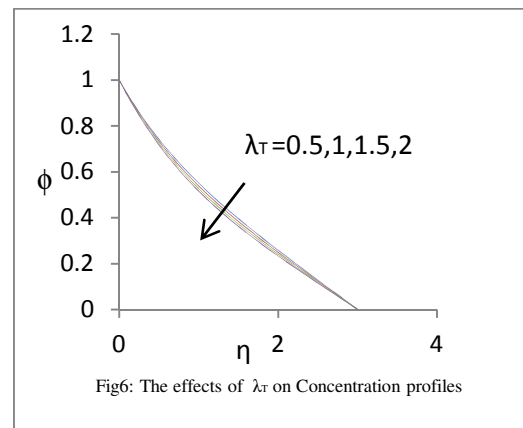
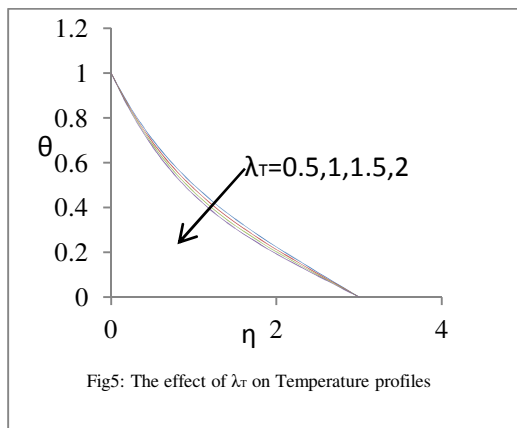
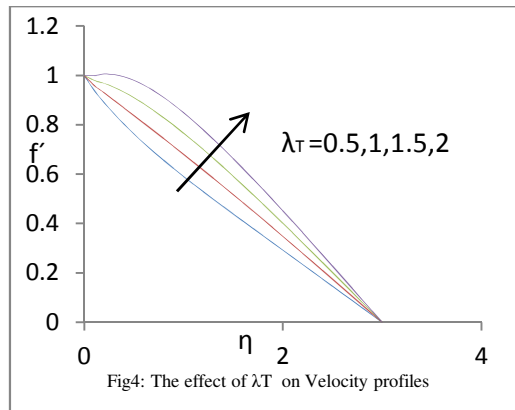
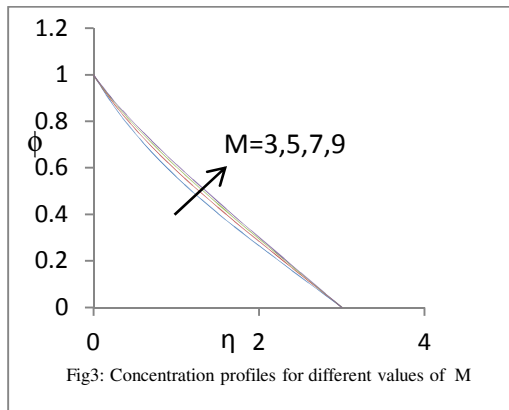
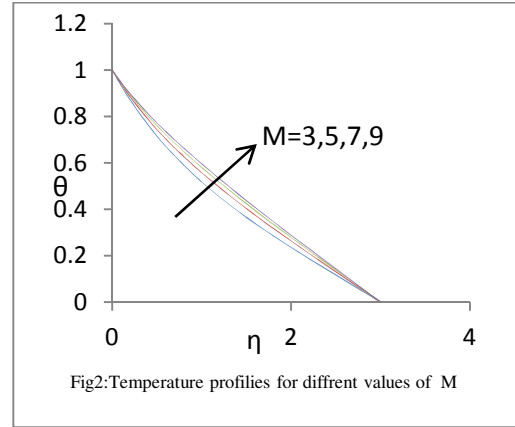
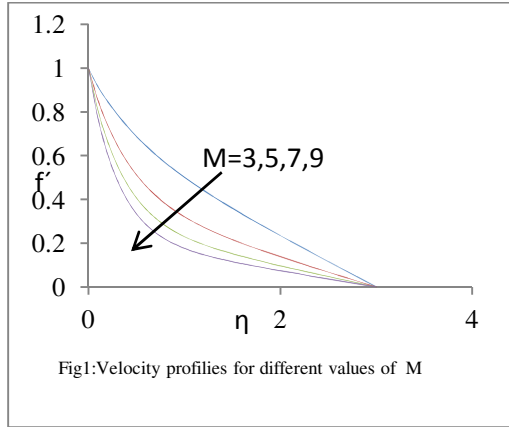
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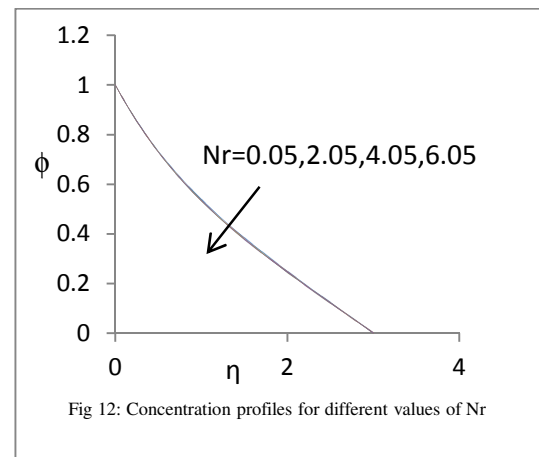
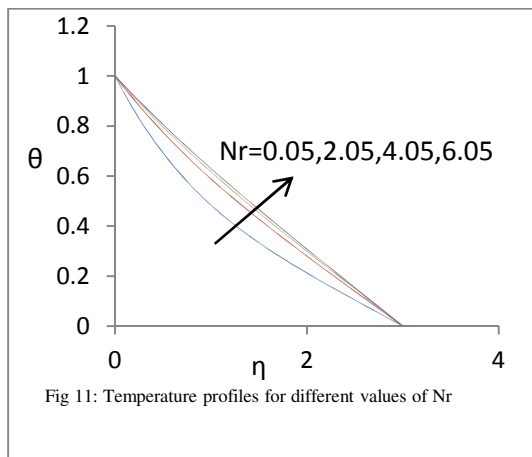
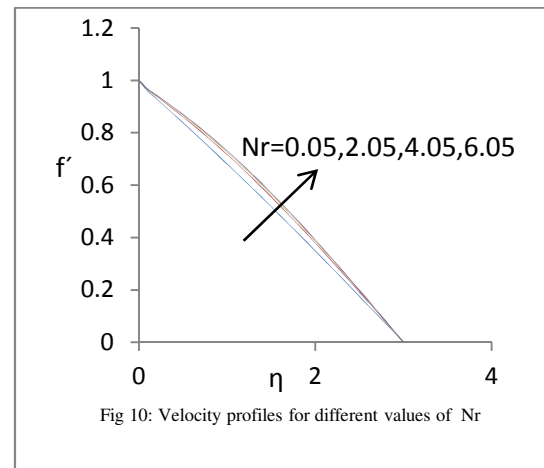
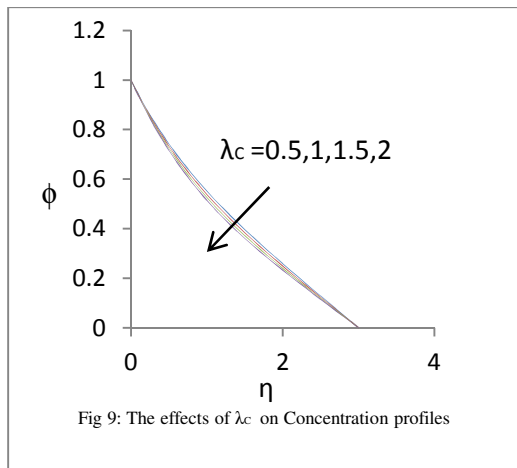
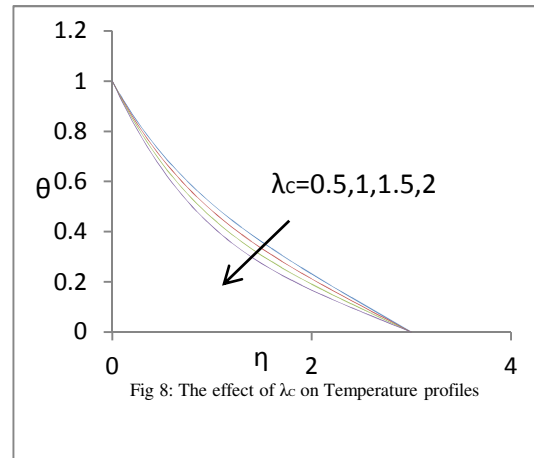
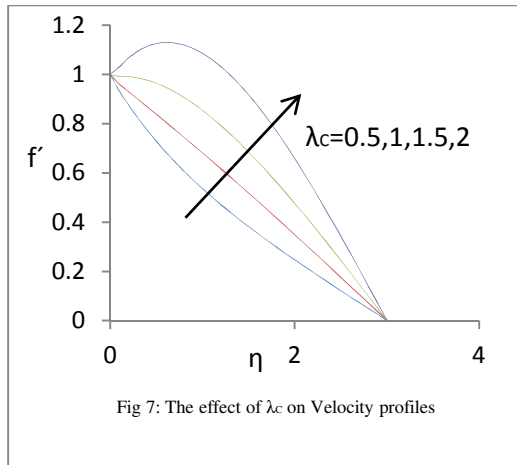
$$H_0[i] = E_0[i] + hE_1[i], H_1[i] = -2E_0[i] + h^2E_2[i] - hE_1[i]$$

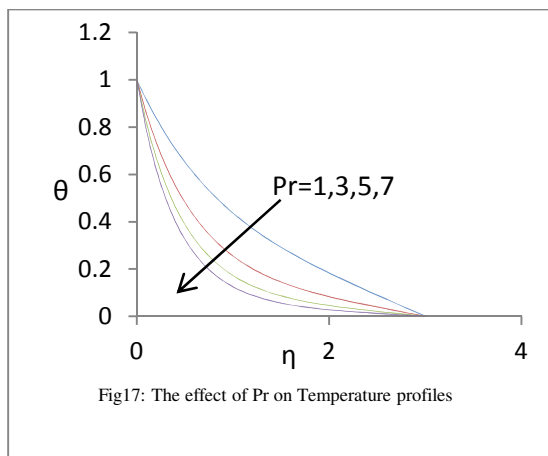
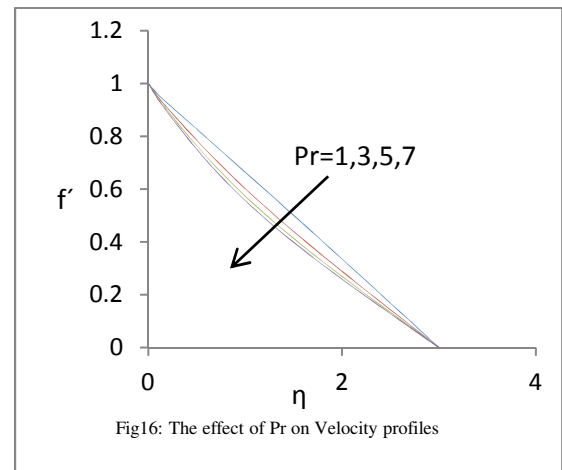
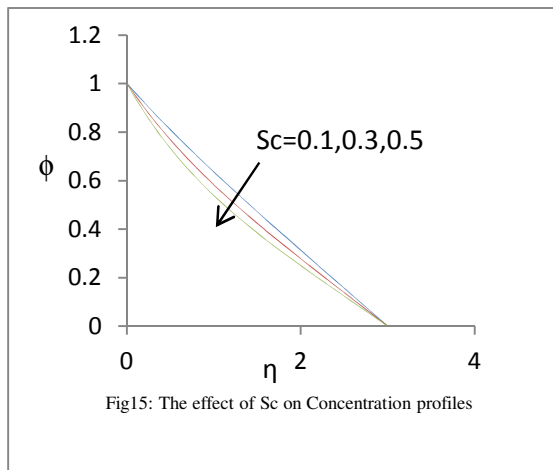
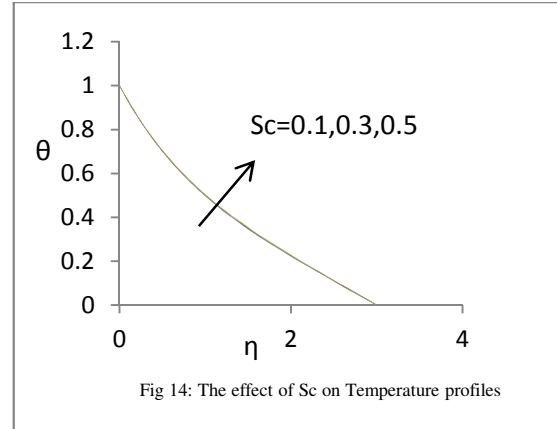
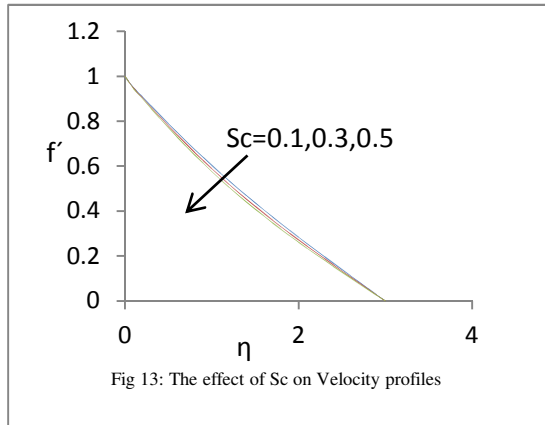
$$H_2[i] = E_0[i]$$

Here h represents the mesh size in η direction.

Equations (15), (16) and (17) are solved under the boundary conditions (10) by Thomas algorithm for various parameters entering into the problem and computations were carried out by using C programming.







RESULTS AND DISCUSSIONS:

A parametric study is performed to explore the effects of magnetic field parameter, Buoyancy parameter, and Thermal radiation parameter. In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations. The effect of magnetic field parameter on dimensionless velocity, temperature and concentration profiles with constant, buoyancy parameter, thermal radiation parameter, suction parameter, prandtl number and Schmidt number are presented in Fig. (1-3). It is observed that the velocity of the fluid decreases with the increase in magnetic field parameter whereas temperature and concentration increases.

Fig. (4-9) demonstrates the dimensionless velocity, temperature and concentration profiles for different values of buoyancy parameters (λ_T, λ_C) with constant thermal radiation parameter, suction parameter, prandtl number and Schmidt number and the uniform magnetic field. It is seen that the velocity increases with the increase of buoyancy parameter, whereas temperature and concentration decreases.

Fig. (10-12) illustrates the dimensionless velocity, temperature and concentration profiles for different values of Thermal radiation parameter N_r with constant Buoyancy parameter, suction parameter, prandtl number and Schmidt number and the uniform magnetic field. It is seen that the velocity and temperature of fluid increases with the increase of N_r , whereas concentration decreases, but with a very low concentration variation.

The effect of Schmidt number on the dimensionless velocity, temperature and concentration profiles for constant Buoyancy parameter, suction parameter, prandtl number and the uniform magnetic field as shown in Fig. (13-15). It is observed that velocity and concentration of fluid decreases and temperature increases with increase of Schmidt number

Fig. (16-17) illustrates the dimensionless velocity, and temperature profiles for different values of prandtl number with constant Buoyancy parameter, suction parameter, Schmidt number and the uniform magnetic field. It is seen that the velocity and temperature of fluid decreases with the increase of prandtl number.

CONCLUSIONS:

This paper presents numerical studies on the problem of influence of Buoyancy effects and thermal radiation of the nonlinear MHD flow with heat and mass transfer characteristics of an incompressible, viscous, electrically conducting and Boussinesq fluid along a permeable vertical stretching sheet. The solutions are found to be governed by different parameters such as buoyancy parameter, thermal radiation parameter, suction parameter, prandtl number and Schmidt number. Some of the important findings of the paper are:

- The effect of magnetic field is to reduce the velocity whereas to increase the temperature and concentration fields.
- Velocity of the fluid increases with the increase in the buoyancy parameters while the temperature and concentration decreases.
- Higher the thermal radiation parameter, the higher is the velocity and temperature fields and lower is the concentration field.
- The effect of Schmidt number is to decrease the velocity and concentration profiles whereas to increase the temperature profiles.
- Increase in the Prandtl number is to decrease the velocity and temperature profiles.

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