# MHD mixed convection flow of a nanofluid over a nonlinear stretching sheet with variable Brownian and thermophoretic diffusion coefficient

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**Abstract**

Investigation has been carried out to analyze the effects of variable wall temperature and concentration on Magnetohydrodynamic mixed convection flow of a nanofluid over a nonlinear stretching sheet with variable Brownian and thermophoretic diffusion coefficient. The governing differential equations were transformed into a set of non-linear coupled ordinary differential equations using similarity transformations. Results are shown graphically for the velocity profile, the temperature profile, and the concentration profile with different values of physical parameters like suction parameter, magnetic parameter, Grashof number, local modified Grashof number, thermal diffusivity, Prandtl number, Lewis number, the thermophoresis parameter and the Brownian motion parameter, the variable thermophoretic diffusion coefficient parameter and the variable Brownian motion diffusion coefficient parameter. A comparison with previously published work has been carried out and the results are found to be in good agreement. Finally, numerical values of pertinent physical quantities, such as the local Nusselt and local Sherwood numbers were presented graphically. It is found that Heat transfer rate decreases with the influence of Brownian motion and thermophoresis parameters and the local Sherwood number increases with the effect of both Brownian motion and thermophoresis parameters.

**Keywords**

Mixed convection; Magnetohydrodynamic; Brownian motion; Thermophoresis; Nonlinear Stretching parameter

## 1. Introduction

In fluid dynamics, the effects of external magnetic field on magnetohydrodynamic (MHD) flow over a stretching sheet are very important due to its applications in many engineering problems, such as glass manufacturing, geophysics, paper production, and purification of crude oil. A broad effort has been made to gain information regarding the stretching flow problems in various situations. The flow due to stretching of a flat surface was first investigated by Crane [1]. The effect of external magnetic field on the MHD flow over a stretching sheet was investigated by Pavlov [2]. The MHD flow and heat transfer over a stretching sheet with variable fluid viscosity has been discussed by Mukhopadhyay [3]. An excellent collection of articles on this topic can be found in [4-7]. Furthermore, many vital properties of MHD flow over stretching sheet were explored in various articles [8–10] in the literature. Several important investigations on the flow due to stretching/shrinking sheet are available in the literature [11–12]

All the above mentioned investigations deal with the flows over a linear stretching sheet. Cortell [13, 14] has worked on viscous flow and heat transfer over a nonlinearly stretching sheet. Awang and Hashim [15] obtained the series Solution for flow over a nonlinearly stretching sheet with chemical reaction and Magnetic field .The flow and heat transfer characteristics in a viscous fluid over a nonlinearly stretching sheet without heat dissipation effect was studied by Vajravelu [16]. The boundary layer flow of a nanofluid flow over a non-linearly stretching sheet was later studied by Rana and Bhargava [17]. The analytical solution of the boundary layer flow of an incompressible viscous fluid over a non-linear stretching sheet has been investigated by Hayat [18]. Approximate Solution of the Magneto-Hydrodynamic flow over a nonlinear stretching sheet has been studied by Eerdunbuhe and Temuerchaolu [19]. An excellent collection of articles on this topic can be found in [20-22].

Nanofluids are the suspension of nanometer-sized solid particles and fibers, which have been proposed as a means for enhancing the performance of heat transfer liquids currently available, such as water, toluene, oil and ethylene glycol mixture. Choi [23], was the first person who utilizes nanofluid. Choi et al. [24] affirmed that the addition of a one percent by volume of nanoparticles to usual fluids increases the thermal conductivity of the fluid up to approximately two times. Recently several modeling of the natural or mixed convection of nanofluids have been investigated numerically. The pioneer work on the boundary layer flow of a nanofluid over a stretching sheet has been carried out by Khan and Pop [25] using Buongiorno's model [26]. In his theory he explained that nanofluids have higher thermal conductivity compared to the base fluids. Some other recent articles describing the properties of nanofluid are cited in Refs.[27–31].

Mixed convection (or combined convection), one of the transport phenomena, is the composition of both natural and forced convection flow. These flow patterns are discovered simultaneously by both an external forcing mechanism and internal volumetric forces. Prasad et al. [32] analyzed the mixed convection heat transfer over a non-linear stretching surface with variable fluid properties. The mixed convection flow of a non-newtonian nanofluid over a non-linearly stretching sheet was discussed by Gorla and Kumari [33]. Sundeep and Sulochana[34] investigated the influence of non-uniform heat source/sink, mass transfer and chemical reaction on an unsteady mixed convection boundary layer flow of a magneto-micropolar fluid past a stretching/shrinking sheet in the presence of viscous dissipation and suction/injection. Double diffusive mixed convection flow of a non-Newtonian couple stress fluid over a vertical heated plate in a sparsely packed porous medium with variable fluid properties has been studied analytically and numerically by Dinesh[35]. Mustafa and Hayat [36] studied unsteady boundary layer flow of a Casson fluid due to an impulsively started moving flat plate. The Keller-Box method introduced by Keller [37] is one of the best numerical method. Basically it's a mixed finite volume method which consists in taking the average of a conservation law and of the associated constitutive law at the level of the same mesh cell. Sulochana[38] numerically analysed the effect of transpiration on magnetohydrodynamic stagnation-point flow of a Carreau nanofluid toward a stretching/shrinking sheet in the presence of thermophoresis and Brownian motion. Sulochana and Ashwinkumar[39] carried out the momentum, heat and mass transfer behaviour of magnetohydrodynamic flow towards a vertical rotating cone in porous medium with thermophoresis and Brownian motion effects. Sarif [40] obtained the numerical solution of the steady boundary layer flow and heat transfer over a stretching sheet with Newtonian heating by using Keller Box method.

Motivated by all the articles reviewed above and in particular, for more physical implications, this present investigation deals with the mixed convection flow of a nanofluid over a nonlinear stretching sheet with variable Brownian and thermophoretic diffusion coefficient by considering the effects of variable wall temperature and concentration. The basic governing equations are converted into ordinary differential equations by applying suitable similarity transformations and those equations were solved numerically by using finite difference method called as the Keller Box method.

## 2. Mathematical Formulation:

We consider the two-dimensional steady laminar MHD mixed convective flow of a nanofluid due to a stretching sheet situated at y = 0 with stretching velocity u $=C_1 x^n$, where $C_1$ is a constant and n is non linear stretching parameter. The fluid is electrically conducted due to an applied magnetic field B(x) normal to the stretching sheet. The magnetic Reynolds number is assumed small and so the induced magnetic field can be considered to be negligible. The wall temperature $T_w$ and the nanoparticle fraction $C_w$ are assumed constant at the stretching surface. When y tends to infinity, ambient temperature and concentration are $T_\infty$ and $C_\infty$, respectively. It is chosen that the coordinate system x-axis is along stretching sheet and y-axis is normal to the sheet.

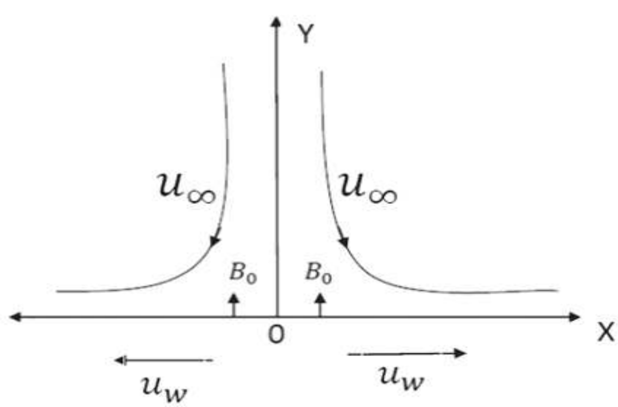



Fig 1 Physical model and coordinate system

The continuity, momentum, energy and concentration equations of incompressible nanofluid boundary layer flow are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \vartheta\frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) - \frac{\sigma B^2(x)}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left\{D_B(C)\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T(T)}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B(C)\frac{\partial^2 C}{\partial y^2} + \frac{D_T(T)}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Boundary conditions are

$$u(x,0) = U_W(x) = C_1 x^n \text{ }, v(x,0) = V_W(x) = C_2 x^m, T(x,0) = T_\infty + C_3 x^r, C(x,0) = C_\infty + C_4 x^r$$

$$\text{And } \ u(x,\infty) = 0, T(x,\infty) = T_\infty \text{ }, C(x,\infty) = C_\infty \tag{5}$$

where u , v are the velocity components along the x and y directions, respectively. T and C are the fluid temperature and concentration, respectively. ρ is the fluid density, g is the acceleration due to gravity, $\beta_T$ is the coefficient of thermal expansion, $\beta_C$ is the coefficient of expansion with concentration, $C_1, C_2, C_3, C_4$ are the constants, $U_W(x) = C_1 x^n$ is the stretching velocity of the plate, $V_w(x) = C_2 x^m$ is the transverse velocity at the surface, B(x)=$B_0 x^s$ is the applied magnetic field, where s = $\frac{n-1}{2}$, m = $\frac{n-1}{2}$, r = 2n-1, The stretching surface has a uniform temperature $T_w$ and the free stream temperature is $T_\infty$ with $T_w > T_\infty$. Also, it has a uniform concentration $C_w$ and the free stream concentration is $C_\infty$ with $C_w > C_\infty$.

In this study, $D_T(T)$ and $D_B(C)$ *are* the variable thermophoretic and Brownian motion diffusion coefficients, and assumed to vary linearly with temperature and volume fraction of the nanoparticles, respectively. We difine them as:

$$D_T(T) = D_{T_\infty}(1 + \frac{\varepsilon}{\Delta T}(T - T_\infty)),$$

$$D_B(T) = D_{B_\infty}(1 + \frac{\beta}{\Delta C}(C - C_\infty)). \tag{6}$$

where $\Delta T = (T_W - T_\infty), \Delta C = (C_W - C_\infty), T_W$ the surface temperature, $C_W$ the surface volume fraction of the nannoparticles, $\varepsilon$ the variable thermophoretic diffusion coefficient parameter, $\beta$ Brownian motion diffusion coefficient parameter, $D_{T_\infty}$ and $D_{B_\infty}$ are thermophoretic and Brawnian motion diffusion coefficients of the nanofluid far away from the sheet, respectively.

The stream function ψ(x, y) is defined by $u = \frac{\partial\psi}{\partial y}$ *and* $v = -\frac{\partial\psi}{\partial x}$, such that the continuity Eq.(1) is satisfied automatically. With the help of following similarity transformations, the non linear partial differential equations (2), (3) and (4) were transformed into coupled non linear ordinary differential equations satisfied

$$\eta = y\sqrt{\frac{(n+1)}{2}\frac{U(x)}{\vartheta x}} \text{ }, \psi = \sqrt{\frac{2}{(n+1)}\vartheta x U(x)} f(\eta), \ \theta(\eta) = \frac{T - T_\infty}{T_W - T_\infty} \text{ }, \phi(\eta) = \frac{C - C_\infty}{C_W - C_\infty} \tag{7}$$

The transformed ordinary differential equations are

$$f''' + ff'' - \frac{2}{(n+1)}\left[nf'^2 + Mf' - Gr\theta - Gc\phi\right] = 0 \quad (8)$$

$$\frac{1}{Pr}\theta'' + Nb(1 + \beta\phi)\phi'\theta' + Nt(1 + \varepsilon\theta)\theta'^2 + f\theta' - \frac{2(2n-1)}{(n+1)}f'\theta = 0 \quad (9)$$

$$\frac{1}{Le}\left((1 + \beta\phi)\phi'' + (1 + \varepsilon\theta)\frac{Nt}{Nb}\theta''\right) - \frac{2(2n-1)}{(n+1)}f'\phi + f\phi' = 0 \quad (10)$$

and the boundary conditions are transformed into

$f(0) = S, f'(0) = 1, g(0) = 1, h(0) = 1$

and $f'(\infty) = 0, g(\infty) = 0, h(\infty) = 0$ (11)

where the prime denotes differentiation with respect to $\eta$ and the parameters are given by:

$$S = -c_2\sqrt{\frac{2}{(n+1)\vartheta c_1}}, M = \frac{\sigma M_0^2}{\rho c_1}, Gr = \frac{g\beta_T(T_W - T_\infty)}{c_1^2 x^{2n-1}}, Gc = \frac{g\beta_C(C_W - C_\infty)}{c_1^2 x^{2n-1}},$$

$$\Pr = \frac{\vartheta}{\alpha}, \alpha = \frac{k}{\rho c_p}, Le = \frac{\vartheta}{D_{B\infty}}, Nt = \frac{(\rho c)_p D_{T\infty}(T_W - T_\infty)}{(\rho c)_f T_\infty \vartheta}, Nb = \frac{(\rho c)_p D_{B\infty}(C_W - C_\infty)}{(\rho c)_f \vartheta}. \quad (12)$$

Here, S, M, Gr, Gc, Pr, $\alpha$, Le, Nt and Nb, denote the suction parameter, magnetic parameter, Grashof number, local modified Grashof number, Prandtl number, thermal diffusivity, Lewis number, the thermophoresis parameter and the Brownian motion parameter, respectively.

And the physical quantities of the local Nusselt number $Nu_x$ and the local Sherwood number $Sh_x$ are defined as:

$$Nu_x = \frac{xq_w}{k(T_W - T_\infty)} \text{ and } Sh_x = \frac{xq_m}{D_B(C_W - C_\infty)} \quad (13)$$

where $q_w$ *and* $q_m$ are the wall heat and mass fluxes, respectively, and are given by

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \text{ and } q_m = -D_B\left(\frac{\partial c}{\partial y}\right)_{y=0} \quad (14)$$

Now equation (12) becomes

$$\frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0) \text{ and } \frac{Sh_x}{\sqrt{Re_x}} = -\emptyset'(0) \quad (15)$$

where $Re_x = \frac{u_w x}{\vartheta}$ is the Reynolds number.

## 3. Numerical procedure

The boundary value problem (8)–(10) is solved by a second order finite difference scheme known as the Keller Box method [37]. The numerical solutions are obtained in four steps as follows:

• Reduce the equations to a system of first order equations;

• write the difference equations using central differences;

• linearize the algebraic equations by Newton’s method, and write them in matrix–vector form; and

• solve the linear system by the block tri-diagonal elimination technique.

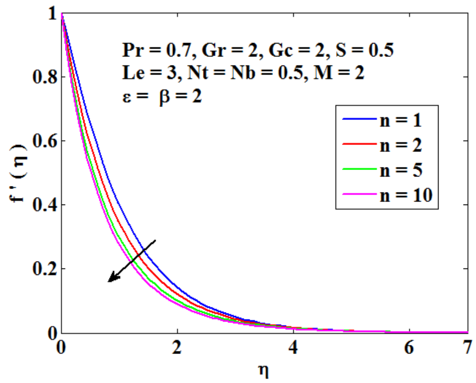
The step size $\Delta\eta$ and the position of the edge of the boundary layer $\eta_\infty$ are to be adjusted for different values of the parameters tomaintain accuracy. For numerical calculations, a uniformstep size of $\Delta\eta = 0.01$ is found to be satisfactory and the solutions are obtained with an error tolerance of $10^{-6}$ in all the cases. For brevity, the details of the solution procedure are not presented here.

## 4. Results and Discussion

The non-linear ordinary differential equations Eqs. (8) – (10) with the boundary conditions (11) were solved numerically by Keller Box method. The computation have been carried out for different values of governing parameters viz. suction parameter S, magnetic parameter M, Grashof number Gr, local modified Grashof number Gc, Prandtl number Pr, Lewis number Le, the thermophoresis parameter Nt and the Brownian motion parameter Nb, $\varepsilon$ the variable thermophoretic diffusion coefficient parameter and $\beta$ Brownian motion diffusion coefficient parameter. The velocity, temperature and concentration profiles for different governing parameters have also been examined for both values of non linear stretching parameters n=1, n =10.The results obtained in the study are compared with the existing literature and found in good agreement which is presented in the Table 1.

Table1: Comparison of Nusselt and Sherwood numbers when Pr = Le = 2 and M = Gr = Gc = S = $\varepsilon = \beta = 0$

| N | Nt | Nb | Rana and Bhargava [17] | | Present Result | |
|---|---|---|---|---|---|---|
| | | | - θ' ( 0 ) | -φ ' ( 0 ) | - θ' ( 0 ) | -φ ' ( 0 ) |
| 0.2 | 0.1 | 0.5 | 0.516 | 0.9012 | 0.5161 | 0.9014 |
| | 0.3 | | 0.4533 | 0.8395 | 0.4536 | 0.8386 |
| | 0.5 | | 0.3999 | 0.8048 | 0.3998 | 0.8039 |
| 3 | 0.1 | | 0.4864 | 0.8445 | 0.4766 | 0.8447 |
| | 0.3 | | 0.4282 | 0.7785 | 0.4279 | 0.7785 |
| | 0.5 | | 0.3786 | 0.7379 | 0.3782 | 0.7378 |
| 10 | 0.1 | | 0.4799 | 0.8323 | 0.4799 | 0.8322 |
| | 0.3 | | 0.4227 | 0.7654 | 0.4228 | 0.7654 |
| | 0.5 | | 0.3739 | 0.7238 | 0.3739 | 0.7232 |



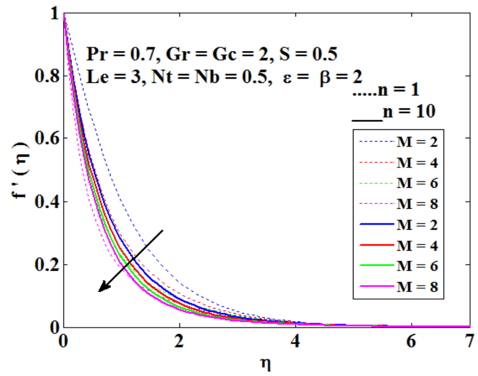
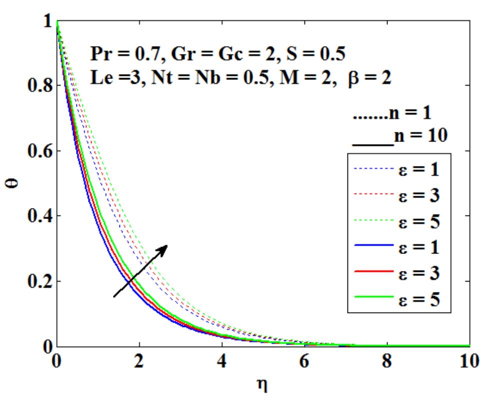




Fig 2 Velocity profile with variation in non linear stretching parameter n

Fig 3 Velocity profile with variation in M

The nature of velocity profile with variations in nonlinearly stretching parameter n and magnetic parameter has been displayed in figures 2 and 3. The velocity of the fluid is found to decrease with an increase in n. But the decrease of the velocity profile is negligible for large values of n since the coefficient $\frac{2n}{n+1}$ approaches to 2 when n→∞. Figure 3 shows the effect of magnetic parameter for nonlinear stretching parameters n=1, n =10. It can be observed that when the magnetic parameter M increases, the velocity decreases. This is because the transverse magnetic field creates the Lorentz force. It is a resistive force similar to the drag force which will result in the deceleration of the flow.



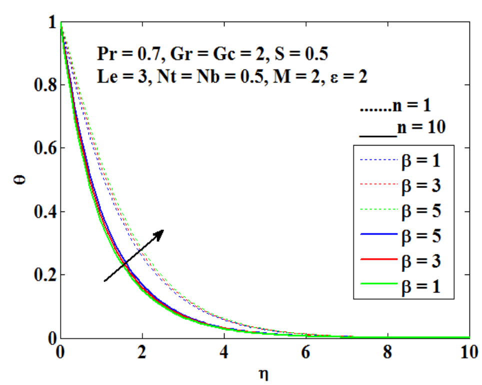
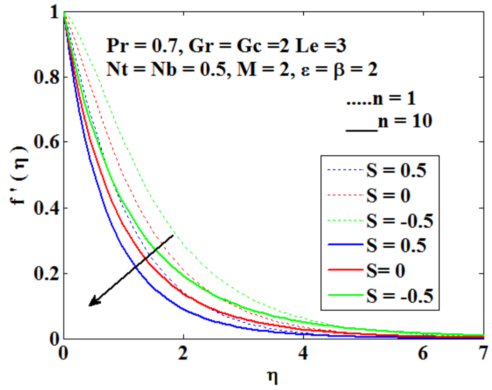




Fig 4 Temperature profile with variation in variable Thermoporetic diffusion coefficient $\varepsilon$

Fig 5 Temperature profile with variation in variable Brownian motion diffusion coefficient $\beta$

The effect of variable Thermoporetic diffusion coefficient parameter $\varepsilon$ and variable Brownian motion diffusion coefficient parameter $\beta$ on temperature of the nanofluid are displaced in figure 4 and figure 5. It is observed from the figures that increasing both the parameters can also increase the temperature of the fluid by keeping other parameters fixed. In this regard, temperature of the fluid is higher in injection situation than that of suction as it is revealed by the figures.



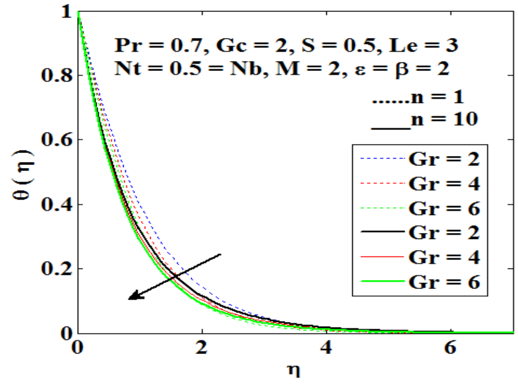
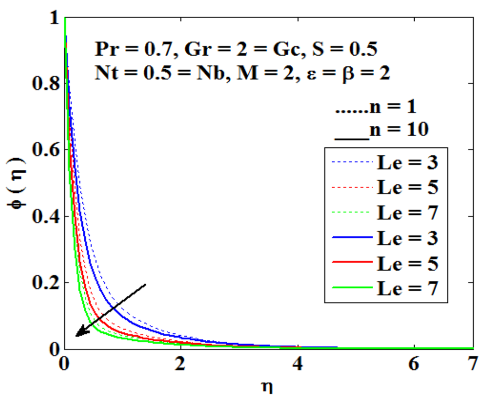




Fig 6 Velocity profile with variation in Suction parameter S

Fig 7 Temperature profile with variation in Gr

Figure 6 shows the effect of velocity profile with respect to the variation in suction parameter S. It can be noticed that when the values of 'S' increase, the velocity profile graph decreases. Due to increase of suction parameter the amount of fluid particles were drawn into the wall hence the boundary layer decreases. Fig 7 reveals the effect of Grashoff number Gr on temperature profile, it is observed that temperature slightly decreases with increasing values of local Grashoff number Gr.



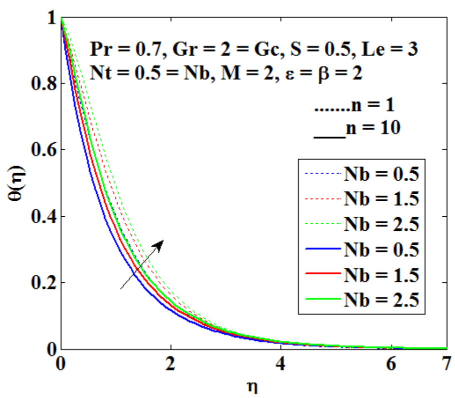




Fig 8 Concentration profile with variation in Lewis number Le

Fig 9 Temperature profile with variation in in Brownian motion parameter

Fig 8 presents the effect of Lewis number on dimensionless nanoparticle concentration. An increase in Lewis values will reduce the profile of nanoparticle concentration and larger Le values will also suppress concentration profile. This is probably due to the fact that mass transfer rate increases as Lewis number increases. It also reveals that the concentration gradient at surface of the sheet increases. Moreover, the concentration at the surface of a sheet decreases as the values of Le increase. Fig 9 shows the influence of Brownian motion parameter on temperature profile. It clearly indicates that the thermal boundary layer thickness increases with an increase in Brownian motion parameter Nb.

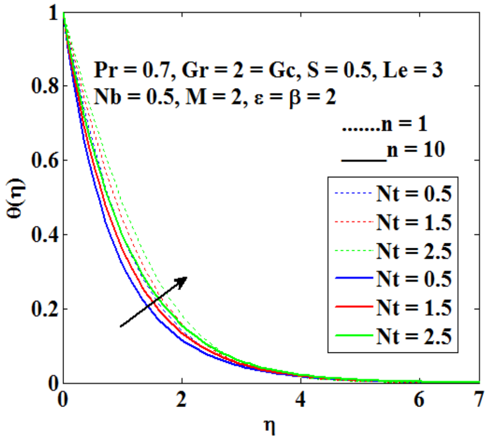



Fig 10 Temperature profile with variation Nt

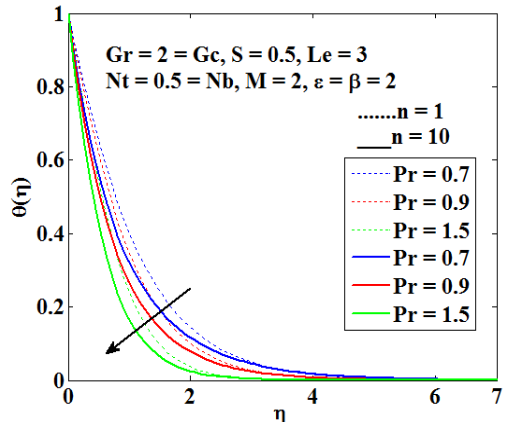



Fig 11 Temperature profile with variation in Pr

Figure 10 shows the influence of thermoporesis parameter Nt on nanoparticle concentration. From the figure it is clear that nanoparticle concentration increases with increasing values of thermoporetic parameter Nt. The enhancement of thermophoretic effects causes the migration of nanoparticles from the hot surface to the cold ambient fluid as a consequence of this the temperature increases in the boundary layer. The effect of Prandtl number Pr on the heat transfer process is shown by the Fig. 11. This graph reveals that an increase in Prandtl number Pr results in a decrease in the temperature distribution, because, thermal boundary layer thickness decreases with an increase in Prandtl number Pr. In short, an increase in the Prandtl number means slow rate of thermal diffusion. The graph also shows that as the values of Prandtl number Pr increase, the wall temperature decreases.

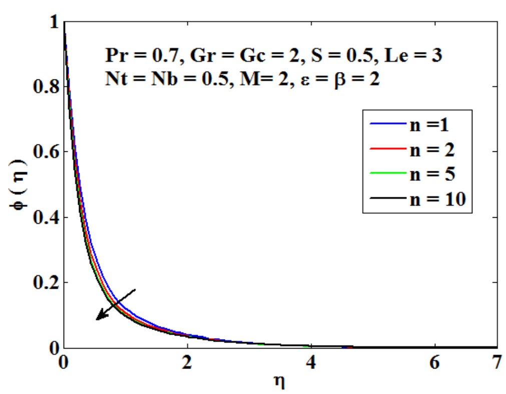



Fig12 Concentration profile with variation in non linear stretching parameter n

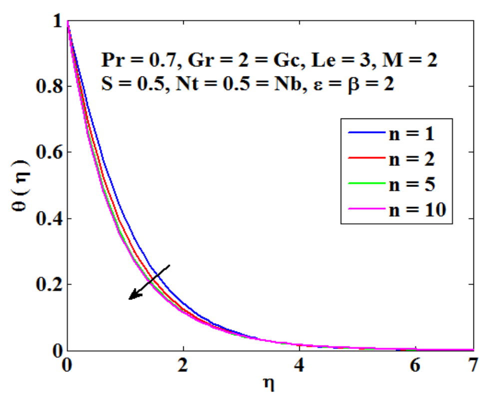



Fig 13 Temperature profile with variation in nonlinear stretching parameter n

Fig 12 dipicts the nature of nanoparticle volume fraction with variation in nonlinearly stretching parameter n. It shows that nanoparticle concentration decreases with an increase in n. The nature of temperature profile with variation in non linearly stretching parameter n has been depicted in Fig 13. It can be observed that temperature decreases with an increase in n.

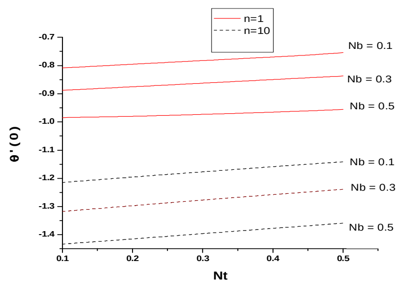



Fig 14 Variation of local Nusselt number – $\theta'(0)$ with Nt for different values of Nb.

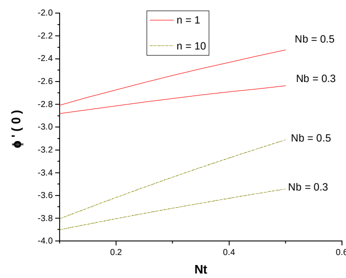



Fig 15 Variation of local Sherwood number $-\emptyset'(0)$ with Nt different values of Nb

Fig 14 shows the influence of both the Brownian motion parameter Nb and thermophoresis parameter Nt on local Nusselt number – $\theta'(0)$. As both parameters increase, the heat transfer rate on the surface of a sheet decreases. This indicates that an increment in thermophoresis parameter induces resistance to the diffusion of mass. This results in the reduction of heat transfer rate on the surface.

Fig 15 depicts the variation of local Sherwood number $-\emptyset'(0)$ in response to a change in Brownian motion parameter Nb. The graph shows that the local Sherwood number increases as Nb increases and also increases with an increase in Nt.

## 5. Conclusion

Investigation has been carried out numerically to study the effects of Brownian motion and thermophoresis on MHD mixed convection flow of a nanofluid over a nonlinear stretching sheet with variable temperature and concentration. The transformed nonlinear ordinary differential equations are solved by using Keller Box Method. The obtained numerical results are compared with previously published work and they are found to be in excellent agreement. The effects of governing parameters on the flow and heat transfer characteristics are thickness decreases with the effect of magnetic parameter and suction parameter.presented graphically and quantitatively. The main observations of the present study are as follows:

1. Influence of non linear stretching parameter decreases both the velocity of the fluid as well as temperature.
2. The boundary layer thickness is increases with an increase in both variable Thermophorotic diffusion coefficient parameter and variable Brownian motion diffusion coefficient parameter.
3. The velocity of the fluid is decreases with an increase in both Magnetic parameter and Suction parameter.
4. Thermal boundary layer thickness decreases with an increase in both Grashof number and Prandtl number.

5. The thickness of thermal boundary layer increases with an increase in both Brownian motion and thermophoresis parameters.
6. An increase in nanoparticle concentration decreases both the Lewis number and nonlinear stretching parameter.
7. Heat transfer rate decreases with the influence of Brownian motion and thermophoresis parameters.
8. The local Sherwood number increases with the effect of both Brownian motion and thermophoresis parameters.

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