Natural Convective Mass Transfer MHD Flow of Chemically Reactive Micropolar Fluid past a Vertical Porous Plate

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ABSTRACT

Magnetic field effects on a free convective mass transfer flow of chemically reactive micropolar fluid over a vertical porous plate are investigated in this work. A mathematical model related to the problem is developed from the basis of studying magnetohydrodynamics(MHD). A usual mathematical transformation is applied on the model to obtain a system of non-dimensional equations. Analytical solution of the dimensionless problem is obtained using perturbation technique. The influence of different parameters (Modified Grashof number G_m ,Suction parameter f_w, Magnetic force number M, permeability of porous plate K, Micro-rotational number Δ, Vortex viscosity λ, Spin gradient viscosity number, Schmidt number S_c and Chemical reaction parameter C_r) on velocity, angular velocity and species concentration profiles are presented graphically. Based on these curves the results and conclusion are depicted.

Keywords: MHD, Micropolar fluid, Porous plate, Chemical reaction, Perturbation technique*.*

1. INTRODUCTION

Micropolar fluids are the combination of microstructure which are first observed Eringen (1960) by the micropolar fluid theory. The theory described about local effect increasing due to microstructure and the intrinsic motion of the fluid elements. Physically, micropolar fluids consist of small, rigid, cylindrical macromolecules with individual motion and are influenced by spin inertia. Kucaba-Pietal (2004), Khedr (2009) investigated colloidal, Muthu (2008) analyzed human and animal blood, Lockwood *et al.* (1987) driven liquid crystal as well as exotic lubricants. Kelson and Desseaux (2001) investigated the effect of surface circumstance on micropolar fluid flow. The unsteady micropolar fluid flow between two parallel porous plates was driven Srinivasacharya *et al.* (2001). Mixed convection micropolar fluid on a porous stretching sheet is investigated by Bhargava (2003). Mohanty *et al.* (2015) investigated heat and mass transfer effect on micropolar fluid on a porous stretching sheet.

Prathap kumer (2010) studied on free convection flow of micropolar and viscous fluids through a vertical duct. Raptis (2011), Samiulhaq *et al.* (2012) and Seth *et al.* (2015) studied on free convective oscillatory flow and mass transfer with ramped temperature on a porous plate. Thereafter, Chamkha (2000), Chaudhary (2007), Samad and Mohebujjaman (2009), Eldabe (2011) and Seth (2015) have paid attention to the study of MHD free convection and mass transfer flows.

At present time, chemical and hydrometallurgical industries need the study of heat and mass transfer with chemical reaction. Ahmmed and Das (2013) analyzed an unsteady free convection with heat and mass transfer chemically reactive MHD flow. Raju *et al.* (2013) investigated an unsteady free convection and chemically reactive MHD flow through infinite vertical porous plate. Bakr (2011) driven the effect of chemical reaction on micropolar fluid with oscillatory plate. Das *et al.* (1994) analyzed the effective of first order homogeneous chemical reaction of an unsteady micropolar fluid flow. Ibrahim *et al.* (2008), Anand Rao *et al*. (2012), Das (2012) and Raju *et al.* (2013) investigated the effect of chemical reaction on an unsteady MHD free convection fluid through semi-infinite vertical porous plate with heat absorption.

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Bakr (2011) analyzed the characteristic of a micropolar fluid velocity on oscillatory plate and constant heat source in a rotating frame. Kucaba-Pietal (2004), Khedr *et al*. (2009) investigated the micro inertia effects on the flow of a micropolar fluid past a semi-infinite plate.

Hence our main goal is to investigate a free convective mass transfer steady flow of a chemically reactive micropolar fluid past a semi-infinite porous plate.

2. FORMULATION OF THE PROBLEM

A natural convective mass transfer steady flow of a chemically reactive micropolar fluid through a semi-infinite vertical porous plate is taken into account at the presence of magnetic field. The flow is considered vertically by x-direction and y-direction is represented horizontally. When the flow at rest, the species concentration level $C=C_\infty$ at all point, where C_∞ be the concentration of uniform flow. It is also assumed that a magnetic field B of uniform strength is applied normal to the flow region. The physical configuration and co-ordinate system of the problem is presented in the following Fig.1.

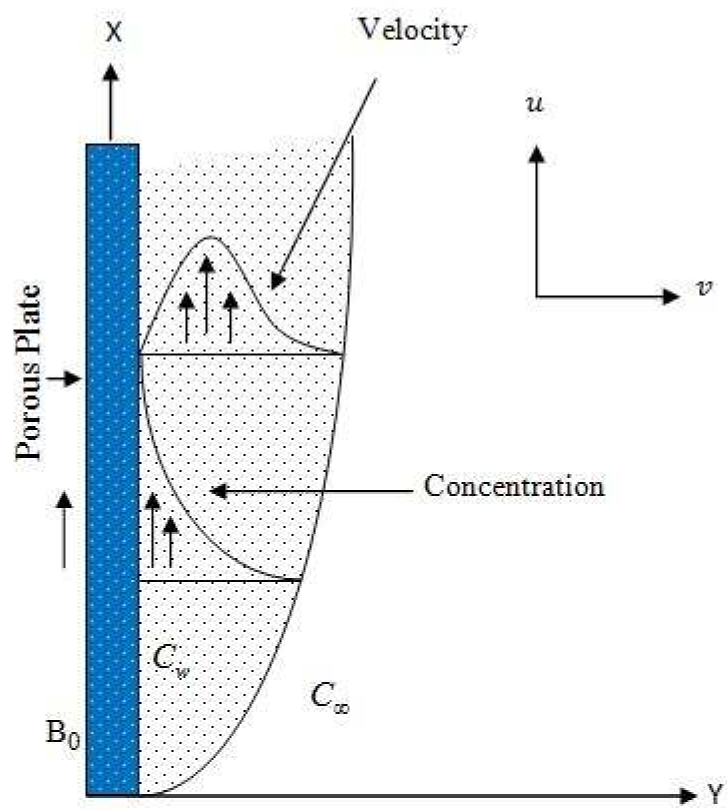

Fig. 1. Physical configuration of the flow

Within the framework of the above stated assumption, the governing equations under the boundary-layer approximations are given by,

Continuity Equation $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ (1)

Momentum Equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta^{*}\left(C-C_\infty\right)+\left(\upsilon+\frac{\chi}{\rho}\right)\frac{\partial^2 u}{\partial y^2}+\frac{\chi}{\rho}\frac{\partial \Gamma}{\partial y}-\frac{\sigma' u B_0{}^2}{\rho}-\frac{\upsilon}{K_1}u \quad (2)$$

Angular Momentum Equation

$$u\frac{\partial \Gamma}{\partial x}+v\frac{\partial \Gamma}{\partial y}=\frac{\gamma}{\rho j}\left(\frac{\partial^2 \Gamma}{\partial y^2}\right)-\frac{\chi}{\rho j}\left(2\Gamma+\frac{\partial u}{\partial y}\right) \quad (3)$$

Concentration Equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-K_c\left(C-C_\infty\right) \tag{4}$$

with boundary condition,

$$u=0,\ \Gamma=-s\frac{\partial u}{\partial y},\ C=C_w \text{ at } y=0$$

$$u=0,\ \Gamma=0,\qquad C=C_\infty \text{ at } y\to\infty$$

where u is the velocity component, Γ is the velocity acting in z - direction (the rotation of Γ is in the $x-y$ plane), B_0 is the magnetic field component, g is local acceleration due to gravity, χ is the vortex viscosity, γ is the spin gradient viscosity, β^* is concentration expansion coefficient.

2.1 Non-dimensional Form

Since our goal is to attain analytical solutions of the problem so we introduce the following dimensionless variables,

$$\eta=y\sqrt{\frac{U_0}{2\upsilon x}},\psi=\sqrt{2\upsilon U_0 x}\,f(\eta),u=\frac{\partial\psi}{\partial y} \text{ and } v=-\frac{\partial\psi}{\partial x},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \text{ and } \Gamma=\sqrt{\frac{U_0^{\ 3}}{2\upsilon x}}g(\eta)$$

The dimensionless equations are,

$$(1+\Delta)f'''(\eta)+\Delta g'(\eta)+f(\eta)f''(\eta)+G_M\varphi(\eta)-(K+M)f'(\eta)=0 \tag{5}$$

$$\wedge g''(\eta)+f'(\eta)g(\eta)+f(\eta)g'(\eta)-2\lambda g(\eta)-\lambda f''(\eta)=0 \tag{6}$$

$$\varphi''(\eta)+S_c f(\eta)\varphi'(\eta)-S_cC_r\varphi(\eta)=0 \tag{7}$$

The associate boundary conditions

$$f(\eta)=f_w,\ f'(\eta)=0,\ g(\eta)=-sf''(\eta),\ \varphi(\eta)=1\ \ \eta=0$$

$$f'(\eta)=0,\ g(\eta)=0,\ \ \varphi(\eta)=0\ \ \eta\to\infty$$

Where, micro-rotational number $\Delta=\frac{\chi}{\rho\upsilon}$, modified Grashof number, $G_m=\frac{g\beta^*(C_w-C_\infty)2x}{U_0^{\ 2}}$ permeability of porous plate, $K=\frac{2\upsilon x}{K_1U_0}$, magnetic force number, $M=\frac{\sigma B_0^{\ 2}2x}{U_0\rho}$, Spin Gradient number, $\wedge=\frac{\gamma}{\upsilon\rho j}$, Vortex viscosity, $\lambda=\frac{2x\chi}{\rho jU_0}$, Schmidt number, $S_c=\frac{\upsilon}{D_m}$, chemical reaction parameter, $C_r=K_c\frac{2x}{U_0}$.

3 ANALITICAL SOLUTION

Since the solution is sought for the large suction further transformation can be made **Arifuzzaman (2015)** as,

$$\xi=\eta f_w \tag{7}$$

$$f(\eta)=f_wF(\xi) \tag{8}$$

$$\phi(\eta)=f_w^{\ 2}G(\xi) \tag{9}$$

$$g(\eta)=f_w^{\ 3}H(\xi) \tag{10}$$

Now the model with small quantity,

$$(1+\Delta)F'''+\Delta H'+FF''+\varepsilon G_MG-(K+M)\varepsilon F'=0 \tag{11}$$

$$\wedge H''+F'H+FH'-2\lambda\varepsilon H-\lambda\varepsilon F''=0 \tag{12}$$

$$G''+S_cG'F-\varepsilon S_cC_rG=0 \tag{13}$$

The associate boundary conditions

$$F(\xi)=1,\quad F'(\xi)=0,\ G(\xi)=\varepsilon,\ H(\xi)=-sF''(\xi) \text{ at } \xi=0$$

$$F'(\xi)=0,\ G(\xi)=0,\quad H(\xi)=0 \text{ at } \xi\to\infty$$

Now for the large suction $\left(f_w > 1\right)$, ε will be very small. Therefore following **Bestman** (1990), F, G and H can be expended in terms of the small perturbation quantity ε,

$$F(\xi) = 1 + \varepsilon F_1(\xi) + \varepsilon^2 F_2(\xi) + \varepsilon^3 F_3(\xi) + ... \tag{14}$$

$$G(\xi) = \varepsilon G_1(\xi) + \varepsilon^2 G_2(\xi) + \varepsilon^3 G_3(\xi) + ... \tag{15}$$

$$H(\xi) = \varepsilon H_1(\xi) + \varepsilon^2 H_2(\xi) + \varepsilon^3 H_3(\xi) + ... \tag{16}$$

The dimensionless equations (5) - (7) transform into the following first order, second order and third order equations with their associated boundary conditions,

$$\left.\begin{aligned} &(1+\Delta) F_1''' + \Delta H_1' + F_1'' = 0 \\ &\wedge H_1'' + H_1' = 0 \\ &G_1'' + S_c G_1' = 0 \end{aligned}\right\} \tag{17}$$

$F_1 = 0,\ G_1 = 1,\ H_1 = 0$ at $\xi = 0$

$F_1' = 0,\ G_1 = 0,\ H_1 = 0$ at $\xi \to \infty$

$$\left.\begin{aligned} &(1+\Delta) F_2''' + \Delta H_2'(\xi) + F_1 F_1'' + F_2'' + G_M G_1(\xi) - (K+M) F_1' = 0 \\ &\wedge H_2'' + F_1' H_1 + F_1 H_1' + H_2' - 2\lambda H_1 - \lambda F_1'' = 0 \\ &G_2'' + S_c G_2' + S_c G_1' F_1 - S_c C_r G_1 = 0 \end{aligned}\right\} \tag{18}$$

$F_2 = 0,\quad G_2 = 0,\ H_2 = 0$ at $\xi = 0$

$F_2' = -\dfrac{1}{\varepsilon^2},\ G_2 = 0,\ H_2 = 0$ at $\xi \to \infty$

$$\left.\begin{aligned} &(1+\Delta) F_3''' + \Delta H_3' + F_2 F_1'' + F_1 F_2'' + F_3'' + G_M G_2 - (K+M) F_2' = 0 \\ &\wedge H_3'' + F_2' H_1 + F_1' H_2 + F_2 H_1' + F_1 H_2' + H_3' - 2\lambda H_2 - \lambda F_2'' = 0 \\ &G_3'' + S_c G_3' + S_c G_2' F_1 + S_c G_1' F_2 - S_c C_r G_2 = 0 \end{aligned}\right\} \tag{19}$$

$F_3 = 0,\ G_3 = 0,\ H_3 = 0$ at $\xi = 0$

$F_3' = \dfrac{1}{\varepsilon^3},\ G_3 = 0,\ H_3 = 0$ at $\xi \to \infty$

Now the solution of first order, second order and third order equations are given following,

$F_1 = 0,\ G_1 = e^{-S_c \xi},\ H_1 = 0$

$F_2 = \dfrac{1}{A_2 \varepsilon^2} - \dfrac{\xi}{\varepsilon^2} - \dfrac{1}{A_2 \varepsilon^2} e^{-A_2 \xi} + A_3 e^{-S_c \xi}$, $H_2 = 0$ and $G_2 = -C_r e^{-S_c \xi}$

$F_3 = -\dfrac{1}{A_2 \varepsilon^3} + \dfrac{\xi}{\varepsilon^3} + \dfrac{1}{A_2 \varepsilon^3} e^{-A_2 \xi} + A_{15} e^{-A_2 \xi} + A_{16} e^{-S_c \xi} + A_{17} e^{-S_c \xi} + A_{18} e^{-A_2 \xi} - A_{19} e^{-S_c \xi}$ $H_3 = A_8 e^{-A_2 \xi} + A_9 e^{-S_c \xi}$

$G_3 = A_4 e^{-S_c \xi} - A_5 e^{-S_c \xi} + A_6 \xi e^{-S_c \xi} - A_7 e^{-(S_c + A_2)\xi} + A_{20} e^{-2 S_c \xi}$

Substituting the values of *F*, *H* and *G*, We get

The velocity equation,

$$f = f_w F(\xi)$$

$$\Rightarrow f = f_w + \left(\varepsilon^2 f_w A_3 + \varepsilon^3 f_w A_{16} + \varepsilon^3 f_w A_{17} e^{-S_c\xi} - \varepsilon^3 f_w A_{19}\right) e^{-S_c\xi} + \left(\varepsilon^3 f_w A_{15} + \varepsilon^3 f_w A_{18}\right) e^{-A_2\xi}$$

$$\Rightarrow f' = \left(-S_c\varepsilon^2 f_w A_3 - S_c\varepsilon^3 f_w A_{16} - S_c\varepsilon^3 f_w A_{17} e^{-S_c\xi} + S_c\varepsilon^3 f_w A_{19}\right) e^{-S_c\xi}$$

$$-\left(A_2\varepsilon^3 f_w A_{15} + A_2\varepsilon^3 f_w A_{18}\right) e^{-A_2\xi}$$

The angular velocity equation,

$$g(\eta) = f_w^3 H(\xi)$$

$$\Rightarrow g(\eta) = \varepsilon^3 f_w^3 A_8 e^{-A_2\xi} + \varepsilon^3 f_w^3 A_9 e^{-S_c\xi}$$

The concentration equation,

$$\phi(\eta) = f_w^{\,2} G(\xi)$$

$$\Rightarrow \varphi(\eta) = f_w^{\,2}\varepsilon e^{-S_c\xi} - \varepsilon^2 f_w^{\,2} C_r e^{-S_c\xi} + \varepsilon^3 f_w^{\,2} A_4 e^{-S_c\xi} - \varepsilon^3 f_w^{\,2} A_5 e^{-S_c\xi} + \varepsilon^3 f_w^{\,2} A_6 \xi e^{-S_c\xi}$$

$$-\varepsilon^3 f_w^{\,2} A_7 e^{-(S_c + A_2)\xi}$$

4. RESULT AND DISCUSSION

For the purpose of the applicability of the present mathematical model, the analytical solution are driven using the perturbation method and the discussion is made for various values of parameters just as Modified grashof number G_m ,Suction parameter f_w, Magnetic force number *M*, permeability of porous plate *K*, Micro-rotational number Δ, Vortex viscosity λ, Spin gradient viscosity number $(\wedge)$, Schmidt number S_c and Chemical reaction parameter C_r. The fluid velocity, angular velocity and concentration versus the non-dimensional coordinate variable η are displayed in Figures.

The increase values of magnetic parameter create a drag force known as Lorentz force. The velocity profiles are illustrated in Fig. 2. As it is observed, the velocity profiles curve climb up at the increase of magnetic force number. Afterwards, the suction parameter (f_w) stabilizes the boundary layer growth. So the velocity profiles curve decline with go up suction parameters. Schmidt number decreases the molecular diffusivity. Modified Grashof number signifies the effect of buoyancy force to the viscous hydrodynamic force. So, velocity curves increase with modified Grashof number (G_m). The velocity profiles go down for permeability of prous plate (*K*). Then with increase of vortex viscosity, the velocity profiles plunge. Fig. 3 revels the angular velocity profiles. Firstly, angular velocity profiles decline with rise of suction parameter (f_w). After that, it increases with the increase of modified Grashof number (G_m). But in Fig. 3(c) angular velocity profiles show fluctuation for Schmidt number (S_c). Then at the upsurge of micro-rotational number the angle velocity increase. At the end of the list of figure, angular velocity decline due to soar of spin gradient viscosity number and vortex viscosity.

Fig. 4 describes the concentration profiles. As it is noticed, concentration boundary layer is lowered down as chemical reaction parameter (C_r) climb up. Concentration curves are also declined as Schmidt number (S_c), suction parameter (f_w), micro-rotational parameter Δ increase.

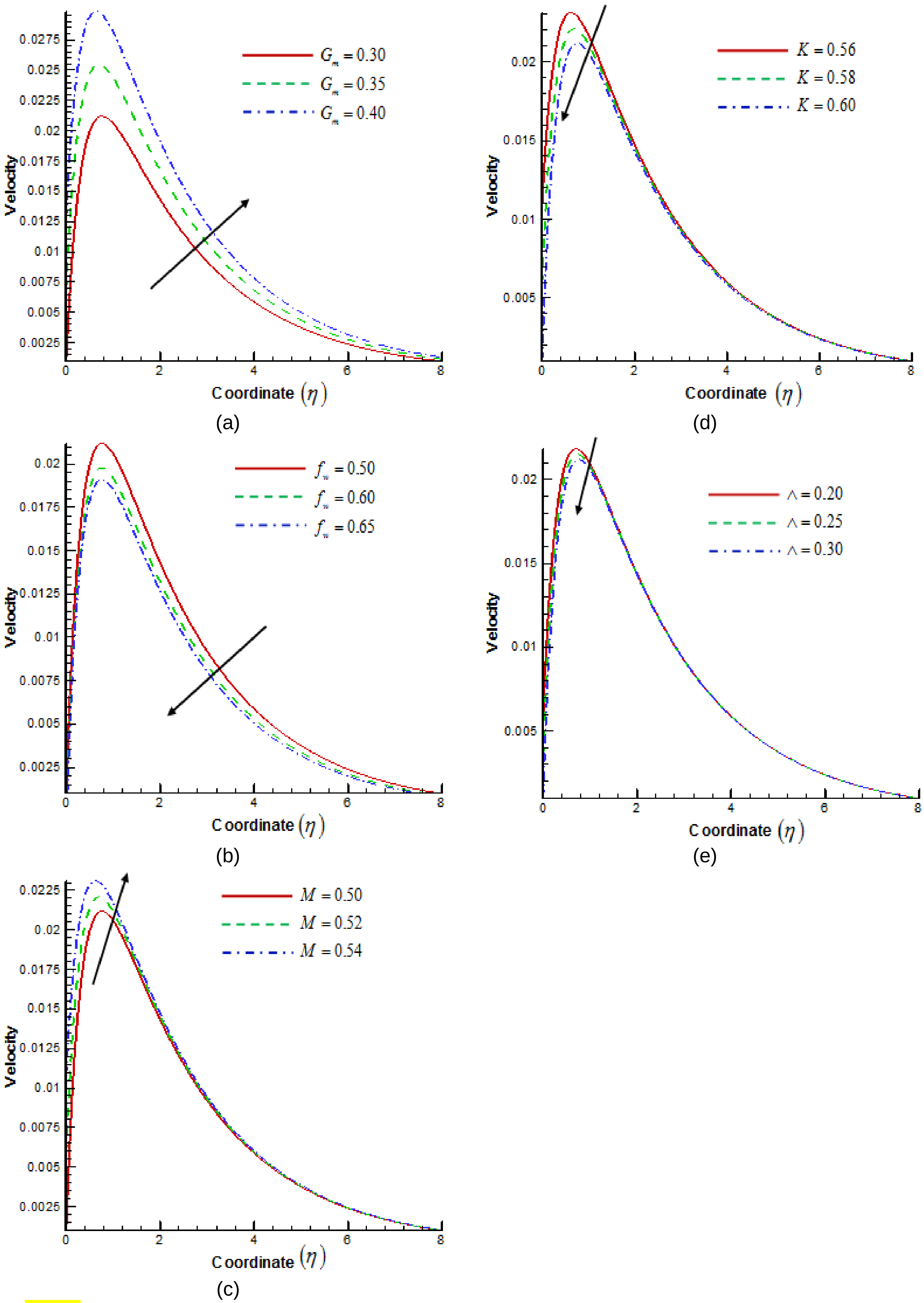

Fig. 2. Velocity profiles for different values of (a) modified Grashof number (b) suction parameter (c) magnetic force number (d) permeability of porous plate (e) spin gradient viscosity number.

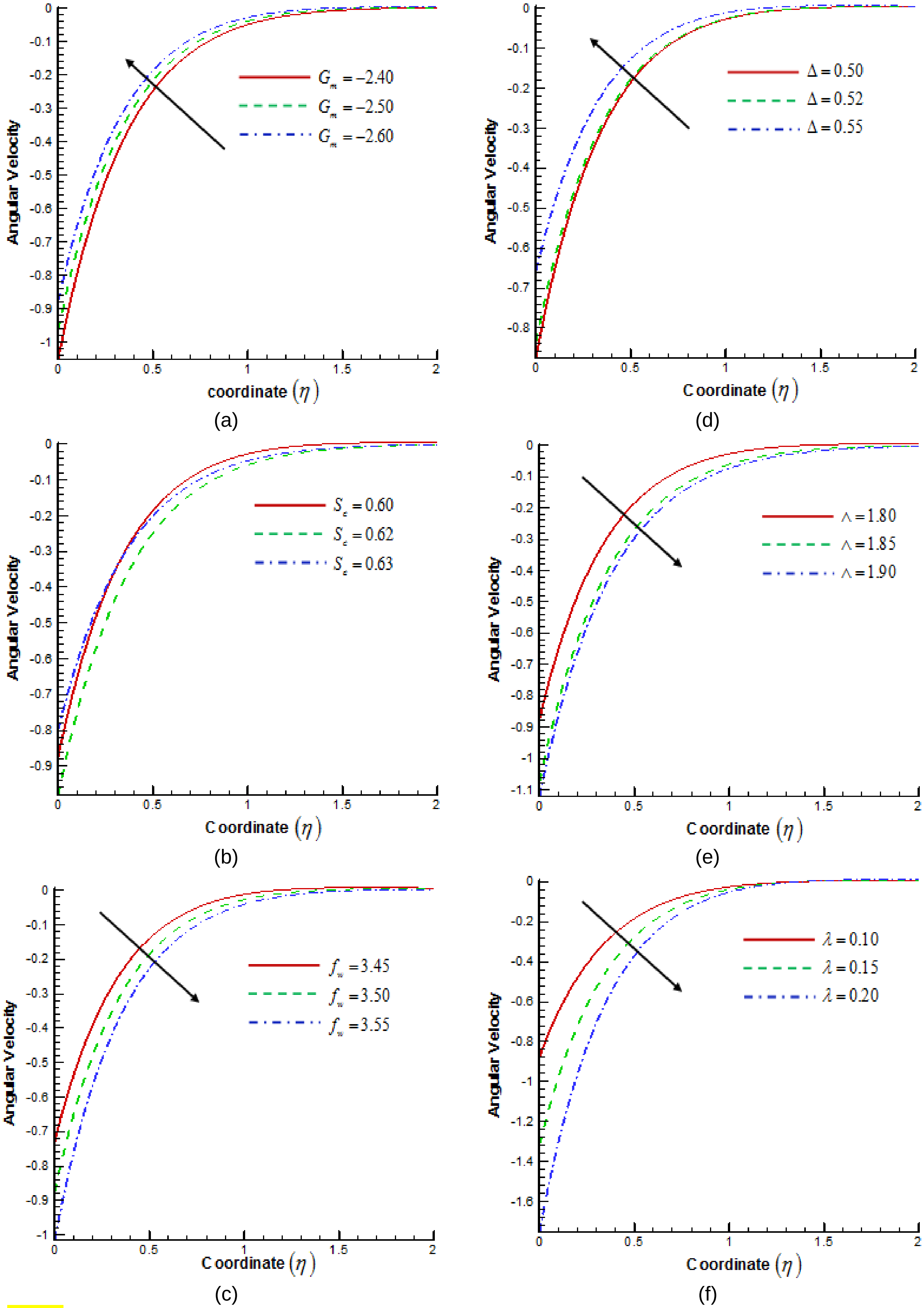

Fig. 3. Angular velocity profiles for different values of (a) modified Grashof number (b) Schmidt number (c) suction parameter (d) micro-rotational number (e) spin gradient viscosity number (f) vortex viscosity.

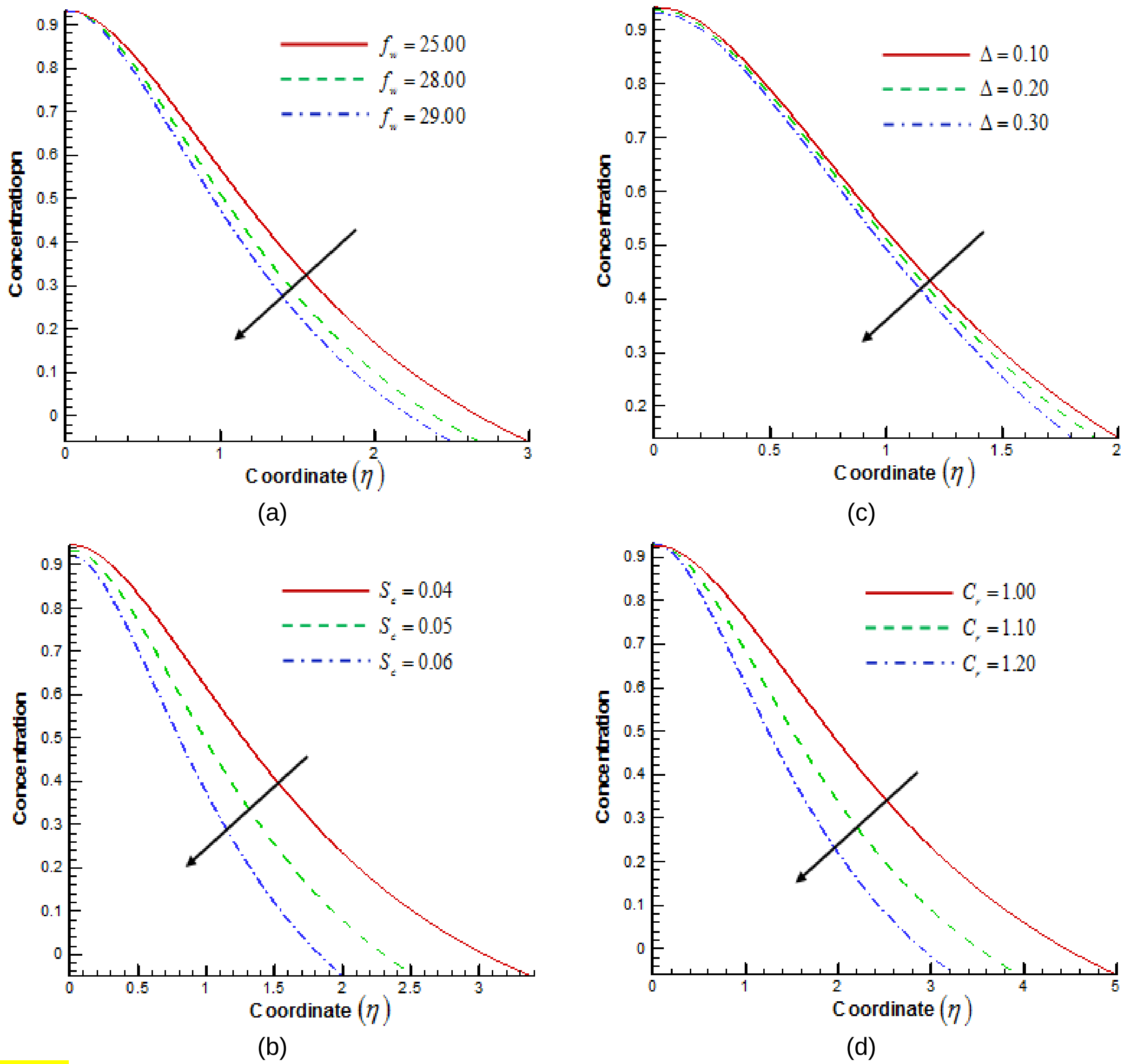

Fig. 4. Concentration profiles for different values of (a) suction parameter (d) Schmidt number (c) micro-rotational number (d) chemical reaction parameter.

4. CONCLUTION

Some of the important findings of the present work obtained from the graphical representation of the results are listed below:

1. The fluid velocity and angular velocity profile decreases with the increase of Modified Grashof number.
2. The velocity and angular velocity profile decreases with the increase of Suction parameter and also the concentration profile decreases with the increase of Suction parameter.
3. The velocity profile decreases and angular velocity profiles decreases with the increase of Schmidt number and also the concentration profile decreases with the increase of Schmidt number.
4. The velocity profile increase with Magnetic force number.
5. The velocity profiles decreases with the increase of Permeability of porous plate.
6. The concentration profile decreases with the increase of Chemical reaction parameter.
7. The angular velocity profile increases with the increase of Micro-rotational number and the concentration profile decreases with the increase of Micro-rotational number.
8. The angular velocity profile decreases with the increase of Spin gradient viscosity number.
9. The angular velocity profile decreases with the increase of Vortex viscosity.

REFERENCES

1. Eringen AC. Theory of micropolar fluids, Journal of Applied Mathematics and Mechanics. 1960; 6: 1–18.
2. Kucaba-Pietal A. Microchannels flow modelling with the micropolar fluid theory. Bulletin of the Polish Academy of Science. 2004; 52(3): 209-214.
3. Khedr MEM, Chamkha AJ, Bayomi M. MHD Flow of a Micropolar Fluid past a Stretched Permeable Surface with Heat Generation or Absorption. Nonlinear Analysis: Modelling and Control. 2009; 14(1): 27–40.
4. Muthu P, Rathish Kumar V and Peeyush Chandra. A study of micropolar fluid in an annular tube with application to blood flow. Journal of Mechanics in Medicine and Biology. 2008; 8(4): 561–576.
5. Lockwood F. Benchaitra M and Friberg S. ASLE Tribology Transactions. Astrophys Space 1987;30:539.
6. Prathap Kumar J, Umavathi JC, Chamkha J, Pop I. Fully developed free convective flow of micropolar and viscous fluids in a vertical channel. Applied Mathematical Modelling. 2010; 34: 1175–1186.
7. Raptis A. Free convective oscillatory flow and mass transfer past a porous plate in the presence of radiation for an optically thin fluid. Thermal Science. 2011; 15: 849-857.
8. Samiulhaq I, Khan F, Ali and Shafie S. MHD free convection flow in a porous medium with thermal diffusion and ramped wall temperature. Journal of the Physical Society of Japan. 2012; 81: 044401.
9. Seth GS, Sharma R and Sarkar S. Natural convection heat and mass transfer flow with Hall current, rotation, radiation and heat absorption past an accelerated moving vertical plate with ramped temperature. Journal of Applied Fluid Mechanics. 2015; 8: 7-20.
10. Chamkha AJ and Khaled ARA. Hydromagnetic combined heat and mass transfer by natural convection from a permeable surface embedded in a fluid saturated porous medium. International Journal Numerical Methods Heat Fluid Flow. 2000; 10: 455-477.
11. Kelson NA, Desseaux A. Effect of surface condition on flow of micropolar fluid driven by a porous stretching sheet. International Journal of Engineering Science. 2001; 39: 1881–1897.
12. Chaudhary RC and Jain A. Combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium. Romanian Journal of Physics. 2007; 52: 505-524.
13. Samad MA, Mohebujjaman M. MHD heat and mass transfer free convection flow along a vertical stretching sheet in presence of magnetic field with heat generation. Research Journal of Applied Sciences, Engineering and Technology. 2009; 1(3): 98-106.
14. Eldabe NTM, Elbashbeshy EMA, Hasanin WSA and Elsaid EA. Unsteady motion of MHD viscous incompressible fluid with heat and mass transfer through porous medium near a moving vertical plate. International Journal of Energy Technology and policy. 2011; 3: 1–11.
15. Das UN, Dekha R and Soungalgekar VM. Effects on mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Forschungim Ingenieurwesen. 1994; 60: 284-287.
16. Ibrahim FS, Elaiw AM and Bakr AA. Effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Communication Nonlinear Science and Numerical Simulation. 2008; 13: 1056–1066.
17. Annad Rao J, Sivaiah, S. and Srinivasa Raju, R. "Chemical Reaction Effects on an Unsteady MHD Free Convection Fluid Flow past a Semi-infinite Vertical plate Embedded in a Porous Medium with Heat Absorption." Journal of Applied Fluid Mechanics. 2012; 3: 63-70.
18. Das K. Magnetohydrodynamics free convection flow of a radiating and chemically reacting fluid past an impulsively moving plate with ramped wall temperature. Journal of Applied Mechanic. 2012; 79: 17-25.
19. Raju MC, Varma SVK, Rao RRK. Unsteady MHD free convection and chemically reactive flow past an infinite vertical porous plate. Journal of Future Engineering and Technology. 2013; 8(3): 35-40.
20. Bakr AA. Effects of chemical reaction on MHD free convection and mass transfer flow of a micropolar fluid with oscillatory plate velocity and constant heat source in a rotating frame of reference. Communication Nonlinear Science and Numerical Simulation. 2011; 16: 698–710.
21. Srinivasacharya D, Ramana murthy JV, Venugopalam D. Unsteady stokes flow of micropolar fluid between two parallel porous plates. International Journal of Engineering Science. 2001; 39:1557–1563.
22. Bhargava R, Kumar L, Takhar HS. Finite element solution of mixed convection microploar fluid driven by a porous stretching sheet. International Journal of Engineering Science. 2003; 41: 2161–2178.
23. Mohanty B, Mishra SR, Pattanayak HB. Numerical investigation on heat and mass transfer effect of micropolar fluid over a stretching sheet through porous media. Alexandria Engineering Journal. 2015; 54: 223–232.
24. Ahmed N. and Das KK. Unsteady MHD mass transfer flow past a suddenly moving vertical plate in a porous medium in rotating system. International Journal of Engineering Science and Technology. 2013; 5: 1906-1923.
25. Arifuzzaman SM. Manjirul Islam, Mohidul Haque. Combined Heat and Mass Transfer Steady Flow of Viscous Fluid over a Vertical Plate with Large Suction. International Journal of Engineering Science, Technology and society. 2015; 3(5): 236-242.

APPENDIX

$$A_1 = \frac{1}{\wedge}, \quad A_2 = \frac{1}{1+\Delta}, \quad A_3 = \frac{-A_2 G_M}{\left(-S_c{}^3 + A_2 S_c{}^2\right)}, \quad A_5 = \frac{S_c}{A_2 \varepsilon^2}, \quad A_6 = \frac{S_c}{\varepsilon^2} \quad A_7 = \frac{S_c^2}{A_2 \varepsilon^2 \left(A_2{}^2 + S_c A_2\right)},$$

$$A_{20} = \frac{A_3}{2}, \quad A_8 = \frac{-\dfrac{A_2 \lambda}{\varepsilon^2}}{\left(A_2^2 - A_1 A_2\right)}, \quad A_9 = \frac{A_3 \lambda S_c^2}{S_c^2 - A_1 S_c}, \quad A_{10} = \Delta A_2{}^2 A_8 \quad A_{11} = \Delta A_2 A_9 S_c, \quad A_{12} = G_m C_r A_2,$$

$$A_{13} = \frac{A_2\left(K+M\right)}{\varepsilon^2}, \quad A_{14} = A_2 S_c A_3 \left(K+M\right), \quad A_{15} = \frac{A_{10}}{A_2{}^2}, \quad A_{16} = \frac{A_{11}}{\left(-S_c^3 + S_c^2 A_2\right)}, \quad A_{17} = \frac{A_{12}}{\left(-S_c^3 + S_c^2 A_2\right)},$$

$$A_{18} = \frac{A_{13}}{A_2{}^2}, \quad A_{19} = \frac{A_{14}}{\left(-S_c^3 + S_c^2 A_2\right)}$$