Natural Convective Mass Transfer MHD Flow of Chemically Reactive Micropolar Fluid past a Vertical Porous Plate

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ABSTRACT

Magnetic field effects on a free convective mass transfer flow of chemically reactive micropolar fluid over a vertical porous plate are investigated in this work. A mathematical model related to the problem is developed from the basis of studying magnetohydrodynamics(MHD). A usual mathematical transformation is applied on the model to obtain a system of non-dimensional equations. Analytical solution of the dimensionless problem is obtained using perturbation technique. The influence of different parameters (Modified grashof number G_m ,Suction parameter f_w, Magnetic force number M, permeability of porous plate K, Micro-rotational number Δ, Vortex viscosity λ, Spin gradient viscosity number, Schmidt number S_c and Chemical reaction parameter C_r) on velocity, angular velocity and species concentration profiles are presented graphically. Based on these curves the results and conclusion are depicted.

Keywords: MHD, Micropolar fluid, Porous plate, Chemical reaction, Perturbation technique.

1. INTRODUCTION

Micropolar fluids are the combination of microstructure which are first observed Eringen (1960) by the micropolar fluid theory. The theory described about local effect increasing due to microstructure and the intrinsic motion of the fluid elements. Physically, micropolar fluids consist of small, rigid, cylindrical macromolecules with individual motion and are influenced by spin inertia. Kucaba-Pietal (2004), Khedr (2009) investigated colloidal, Muthu (2008) analyzed human and animal blood, Lockwood *et al.* (1987) driven liquid crystal as well as exotic lubricants. Kelson and Desseaux (2001) investigated the effect of surface circumstance on micropolar fluid flow. The unsteady micropolar fluid flow between two parallel porous plates was driven Srinivasacharya *et al.* (2001). Mixed convection micropolar fluid on a porous stretching sheet is investigated by Bhargava (2003). Mohanty *et al.* (2015) investigated heat and mass transfer effect on micropolar fluid on a porous stretching sheet.

Prathap kumer (2010) stuied on free convection flow of micropolar and viscous fluids through a vertical duct. Raptis (2011), Samiulhaq *et al.* (2012) and Seth *et al.* (2015) studied on free convective oscillatory flow and mass transfer with ramped temperature on a porous plate. Thereafter, Chamkha (2000), Chaudhary (2007), Samad and Mohebujjaman (2009), Eldabe (2011) and Seth (2015) have paid attention to the study of MHD free convection and mass transfer flows.

At present time, chemical and hydrometallurgical industries need the study of heat and mass transfer with chemical reaction. Ahmmed and Das (2013) analyzed an unsteady free convection with heat and mass transfer chemically reactive MHD flow. Raju *et al.* (2013) investigated an unsteady free convection and chemically reactive MHD flow through infinite vertical porous plate. Bakr (2011) driven the effect of chemical reaction an a micropolar fluid with oscillatory plate. Das *et al.* (1994) analyzed the effective of first order homogeneous chemical reaction of an unsteady micropolar fluid flow. Ibrahim *et al.* (2008), Anand Rao *et al*. (2012), Das (2012) and Raju *et al.* (2013) investigated the effect of chemical reaction on an unsteady MHD free convection fluid through semi-infinite vertical porous plate with heat absorption.

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Bakr (2011) analyzed the characteristic of a micropolar fluid velocity on oscillatory plate and constant heat source in a rotating frame. Kucaba-Pietal (2004), Khedr *et al*. (2009) investigated the micro inertia effects on the flow of a micropolar fluid past a semi-infinite plate.
Hence our main goal is to investigate a free convective mass transfer steady flow of a chemically reactive micropolar fluid past a semi-infinite porous plate.

2. ANALYSIS AND SOLUTION

2.1 MATHEMATICAL FLOW

A natural convective mass transfer steady flow of a chemically reactive micropolar fluid through a semi-infinite vertical porous plate is taken into account at the presence of magnetic field. The flow is considered vertically by x-direction and y-direction is represented horizontally. When the flow at rest, the species concentration level $C=C_{\infty}$ at all point, where C_{∞} be the concentration of uniform flow. It is also assumed that a magnetic field B of uniform strength is applied normal to the flow region. The physical configuration and co-ordinate system of the problem is presented in the following Fig.1.

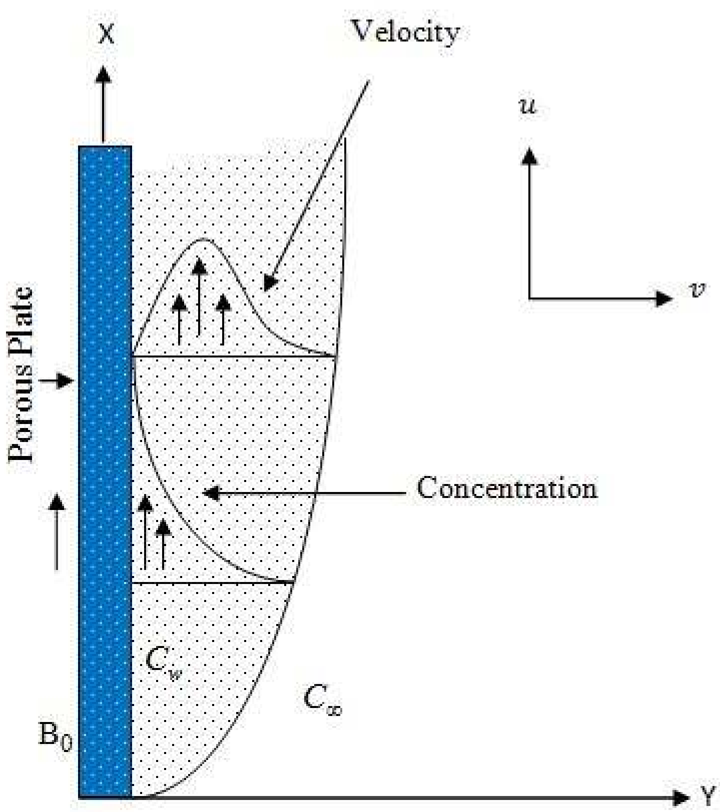

Fig. 1. Physical configuration of the flow

Within the framework of the above stated assumption, the governing equations under the boundary-layer approximations are given by,

Continuity Equation $\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0$ (1)

Momentum Equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta^{*}\left(C-C_{\infty}\right)+\left(\upsilon+\frac{\chi}{\rho}\right)\frac{\partial^{2}u}{\partial y^{2}}+\frac{\chi}{\rho}\frac{\partial\Gamma}{\partial y}-\frac{\sigma' u B_{0}^{\,2}}{\rho}-\frac{\upsilon}{K'}u \tag{2}$$

Angular Momentum Equation

$$u\frac{\partial\Gamma}{\partial x}+v\frac{\partial\Gamma}{\partial y}=\frac{\gamma}{\rho j}\left(\frac{\partial^{2}\Gamma}{\partial y^{2}}\right)-\frac{\chi}{\rho j}\left(2\Gamma+\frac{\partial u}{\partial y}\right) \tag{3}$$

Concentration Equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}-K_c\left(C-C_\infty\right) \tag{4}$$

with boundary condition,

$$u=0,\ \Gamma=-s\frac{\partial u}{\partial y},\ C=C_w \text{ at } y=0$$

$$u=0,\ \Gamma=0,\qquad C=C_\infty \text{ at } y\to\infty$$

where u is the velocity component, Γ is the velocity acting in z - direction (the rotation of Γ is in the $x-y$ plane), B_0 is the magnetic field component, g is local acceleration due to gravity, χ is the vortex viscosity, γ is the spin gradient viscosity, β^* is concentration expansion coefficient.

2.2 MATHEMATICAL FORMULATION

Since our goal is to attain analytical solutions of the problem so we introduce the following dimensionless variables,

$$\eta=y\sqrt{\frac{U_0}{2\upsilon x}},\psi=\sqrt{2\upsilon U_0 x}\,f(\eta),u=\frac{\partial\psi}{\partial y}\text{ and } v=-\frac{\partial\psi}{\partial x},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}\text{ and }\Gamma=\sqrt{\frac{U_0{}^3}{2\upsilon x}}g(\eta)$$

The dimensionless equations are,

$$(1+\Delta)f'''(\eta)+\Delta g'(\eta)+f(\eta)f''(\eta)+G_M\varphi(\eta)-(K+M)f'(\eta)=0 \tag{5}$$

$$\wedge g''(\eta)+f'(\eta)g(\eta)+f(\eta)g'(\eta)-2\lambda g(\eta)-\lambda f''(\eta)=0 \tag{6}$$

$$\varphi''(\eta)+S_c f(\eta)\varphi'(\eta)-S_cC_r\varphi(\eta)=0 \tag{7}$$

The associate boundary conditions

$$f(\eta)=f_w,\ f'(\eta)=0,\ g(\eta)=-sf''(\eta),\ \varphi(\eta)=1\ \ \eta=0$$

$$f'(\eta)=0,\ g(\eta)=0,\ \ \varphi(\eta)=0\ \ \eta\to\infty$$

Where, micro-rotational number $\Delta=\dfrac{\chi}{\rho\upsilon}$, modified Grashof number, $G_m=\dfrac{g\beta^*(C_w-C_\infty)2x}{U_0{}^2}$ permeability of porous plate, $K=\dfrac{2\upsilon x}{K'U_0}$, magnetic force number, $M=\dfrac{\sigma B_0{}^2 2x}{U_0\rho}$, Spin Gradient number, $\wedge=\dfrac{\gamma}{\upsilon\rho j}$, Vortex viscosity, $\lambda=\dfrac{2x\chi}{\rho jU_0}$,Schmidt number, $S_c=\dfrac{\upsilon}{D_m}$, chemical reaction parameter, $C_r=K_c\dfrac{2x}{U_0}$.

2.3 MATHEMATICAL ANALYSIS

Since the solution is sought for the large suction further transformation can be made **Arifuzzaman (2015)** as,

$$\xi=\eta f_w \tag{7}$$

$$f(\eta)=f_wF(\xi) \tag{8}$$

$$\phi(\eta)=f_w{}^2G(\xi) \tag{9}$$

$$g(\eta)=f_w{}^3H(\xi) \tag{10}$$

Now the model with small quantity,

$$(1+\Delta)F'''+\Delta H'+FF''+\varepsilon G_MG-(K+M)\varepsilon F'=0 \tag{11}$$

$$\wedge H''+F'H+FH'-2\lambda\varepsilon H-\lambda\varepsilon F''=0 \tag{12}$$

$$G''+S_cG'F-\varepsilon S_cC_rG=0 \tag{13}$$

The associate boundary conditions

$$F(\xi)=1,\quad F'(\xi)=0,\ G(\xi)=\varepsilon,\ H(\xi)=-sF''(\xi)\ \text{ at } \xi=0$$

$$F'(\xi)=0,\ G(\xi)=0,\quad H(\xi)=0\ \text{ at } \xi\to\infty$$

2.4 SOLUTION

Now for the large suction $\left(f_w > 1\right)$, ε will be very small. Therefore following **Bestman** (1990), F, G and H can be expended in terms of the small perturbation quantity ε,

$$F(\xi) = 1 + \varepsilon F_1(\xi) + \varepsilon^2 F_2(\xi) + \varepsilon^3 F_3(\xi) + ... \tag{14}$$

$$G(\xi) = \varepsilon G_1(\xi) + \varepsilon^2 G_2(\xi) + \varepsilon^3 G_3(\xi) + ... \tag{15}$$

$$H(\xi) = \varepsilon H_1(\xi) + \varepsilon^2 H_2(\xi) + \varepsilon^3 H_3(\xi) + ... \tag{16}$$

The dimensionless equations (5) - (7) transform into the following first order, second order and third order equations with their associated boundary conditions,

$$\left.\begin{aligned} &(1+\Delta) F_1''' + \Delta H_1' + F_1'' = 0 \\ &\wedge H_1'' + H_1' = 0 \\ &G_1'' + S_c G_1' = 0 \end{aligned}\right\} \tag{17}$$

$$F_1 = 0,\ G_1 = 1,\ H_1 = 0 \text{ at } \xi = 0$$

$$F_1' = 0,\ G_1 = 0,\ H_1 = 0 \text{ at } \xi \to \infty$$

$$\left.\begin{aligned} &(1+\Delta) F_2''' + \Delta H_2'(\xi) + F_1 F_1'' + F_2'' + G_M G_1(\xi) - (K+M) F_1' = 0 \\ &\wedge H_2'' + F_1' H_1 + F_1 H_1' + H_2' - 2\lambda H_1 - \lambda F_1'' = 0 \\ &G_2'' + S_c G_2' + S_c G_1' F_1 - S_c C_r G_1 = 0 \end{aligned}\right\} \tag{18}$$

$$F_2 = 0,\quad G_2 = 0,\ H_2 = 0 \text{ at } \xi = 0$$

$$F_2' = -\frac{1}{\varepsilon^2},\ G_2 = 0,\ H_2 = 0 \text{ at } \xi \to \infty$$

$$\left.\begin{aligned} &(1+\Delta) F_3''' + \Delta H_3' + F_2 F_1'' + F_1 F_2'' + F_3'' + G_M G_2 - (K+M) F_2' = 0 \\ &\wedge H_3'' + F_2' H_1 + F_1' H_2 + F_2 H_1' + F_1 H_2' + H_3' - 2\lambda H_2 - \lambda F_2'' = 0 \\ &G_3'' + S_c G_3' + S_c G_2' F_1 + S_c G_1' F_2 - S_c C_r G_2 = 0 \end{aligned}\right\} \tag{19}$$

$$F_3 = 0,\ G_3 = 0,\ H_3 = 0 \text{ at } \xi = 0$$

$$F_3' = \frac{1}{\varepsilon^3},\ G_3 = 0,\ H_3 = 0 \text{ at } \xi \to \infty$$

Now the solution of first order, second order and third order equations are given following,

$$F_1 = 0,\ G_1 = e^{-S_c \xi},\ H_1 = 0$$

$$F_2 = \frac{1}{A_2 \varepsilon^2} - \frac{\xi}{\varepsilon^2} - \frac{1}{A_2 \varepsilon^2} e^{-A_2 \xi} + A_3 e^{-S_c \xi},\ H_2 = 0 \text{ and } G_2 = -C_r e^{-S_c \xi}$$

$$F_3 = -\frac{1}{A_2 \varepsilon^3} + \frac{\xi}{\varepsilon^3} + \frac{1}{A_2 \varepsilon^3} e^{-A_2 \xi} + A_{15} e^{-A_2 \xi} + A_{16} e^{-S_c \xi} + A_{17} e^{-S_c \xi} + A_{18} e^{-A_2 \xi} - A_{19} e^{-S_c \xi}\ H_3 = A_8 e^{-A_2 \xi} + A_9 e^{-S_c \xi}$$

$$G_3 = A_4 e^{-S_c \xi} - A_5 e^{-S_c \xi} + A_6 \xi e^{-S_c \xi} - A_7 e^{-(S_c + A_2)\xi} + A_{20} e^{-2 S_c \xi}$$

Substituting the values of *F*, *H* and *G*, We get

The velocity equation,

$$f = f_w F(\xi)$$

$$\Rightarrow f = f_w + \left(\varepsilon^2 f_w A_3 + \varepsilon^3 f_w A_{16} + \varepsilon^3 f_w A_{17} e^{-S_c\xi} - \varepsilon^3 f_w A_{19}\right) e^{-S_c\xi} + \left(\varepsilon^3 f_w A_{15} + \varepsilon^3 f_w A_{18}\right) e^{-A_2\xi}$$

$$\Rightarrow f' = \left(-S_c\varepsilon^2 f_w A_3 - S_c\varepsilon^3 f_w A_{16} - S_c\varepsilon^3 f_w A_{17} e^{-S_c\xi} + S_c\varepsilon^3 f_w A_{19}\right) e^{-S_c\xi}$$

$$-\left(A_2\varepsilon^3 f_w A_{15} + A_2\varepsilon^3 f_w A_{18}\right) e^{-A_2\xi}$$

The angular velocity equation,

$$g(\eta) = f_w^3 H(\xi)$$

$$\Rightarrow g(\eta) = \varepsilon^3 f_w^3 A_8 e^{-A_2\xi} + \varepsilon^3 f_w^3 A_9 e^{-S_c\xi}$$

The concentration equation,

$$\phi(\eta) = f_w{}^2 G(\xi)$$

$$\Rightarrow \varphi(\eta) = f_w{}^2 \varepsilon e^{-S_c\xi} - \varepsilon^2 f_w{}^2 C_r e^{-S_c\xi} + \varepsilon^3 f_w{}^2 A_4 e^{-S_c\xi} - \varepsilon^3 f_w{}^2 A_5 e^{-S_c\xi} + \varepsilon^3 f_w{}^2 A_6 \xi e^{-S_c\xi}$$

$$-\varepsilon^3 f_w{}^2 A_7 e^{-(S_c+A_2)\xi}$$

3. RESULT AND DISCUSSION

For the purpose of the applicability of the present mathematical model, the analytical solution are driven using the perturbation method and the discussion is made for various values of parameters just as Modified grashof number G_m ,Suction parameter f_w, Magnetic force number *M*, permeability of porous plate *K*, Micro-rotational number Δ, Vortex viscosity λ, Spin gradient viscosity number $(\wedge)$, Schmidt number S_c and Chemical reaction parameter C_r. The fluid velocity, angular velocity and concentration versus the non-dimensional coordinate variable η are displayed in Figures.

The increase values of magnetic parameter create a drag force known as Lorent force. The velocity profiles are illustrated in Fig. 2. As it is observed, the velocity profiles curve climb up at the increase of magnetic force number. Afterwards, the suction parameter (f_w) stabilize the boundary layer growth. So the velocity profiles curve decline with go up suction parameters. Schmidt number decrease the molecular diffusivity. For this reason velocity curves downward due to Schmidt number (S_c). Modified Grashof number signifies the effect of buoyancy force to the viscous hydrodynamic force. So, velocity curves increase with modified Grashof number (G_m). The velocity profiles go down for permeability of prous plate (*K*). Then with increase of vortex viscosity, the velocity profiles plunge. Fig. 3 revels the angular velocity profiles. Firstly, angular velocity profiles decline with rise of suction parameter (f_w). Afterthat, it increases with the increase of modified Grashof number (G_m). But in Fig. 3(c) angular velocity profiles show flactution for Schmidt number (S_c). Then at the upsurge of micro-rotational number the angle velocity increase. At the end of the list of figure, angular velocity decline due to soar of spin gradient viscosity number and vortex viscosity.

Fig. 4 describes the concentration profiles. As it is noticed, concentration boundary layer is lowered down as chemical reaction parameter (C_r) climb up. Concentration curves is also declined as Schmidt number (S_c), suction parameter (f_w), micro-rotational parameter Δ increase.

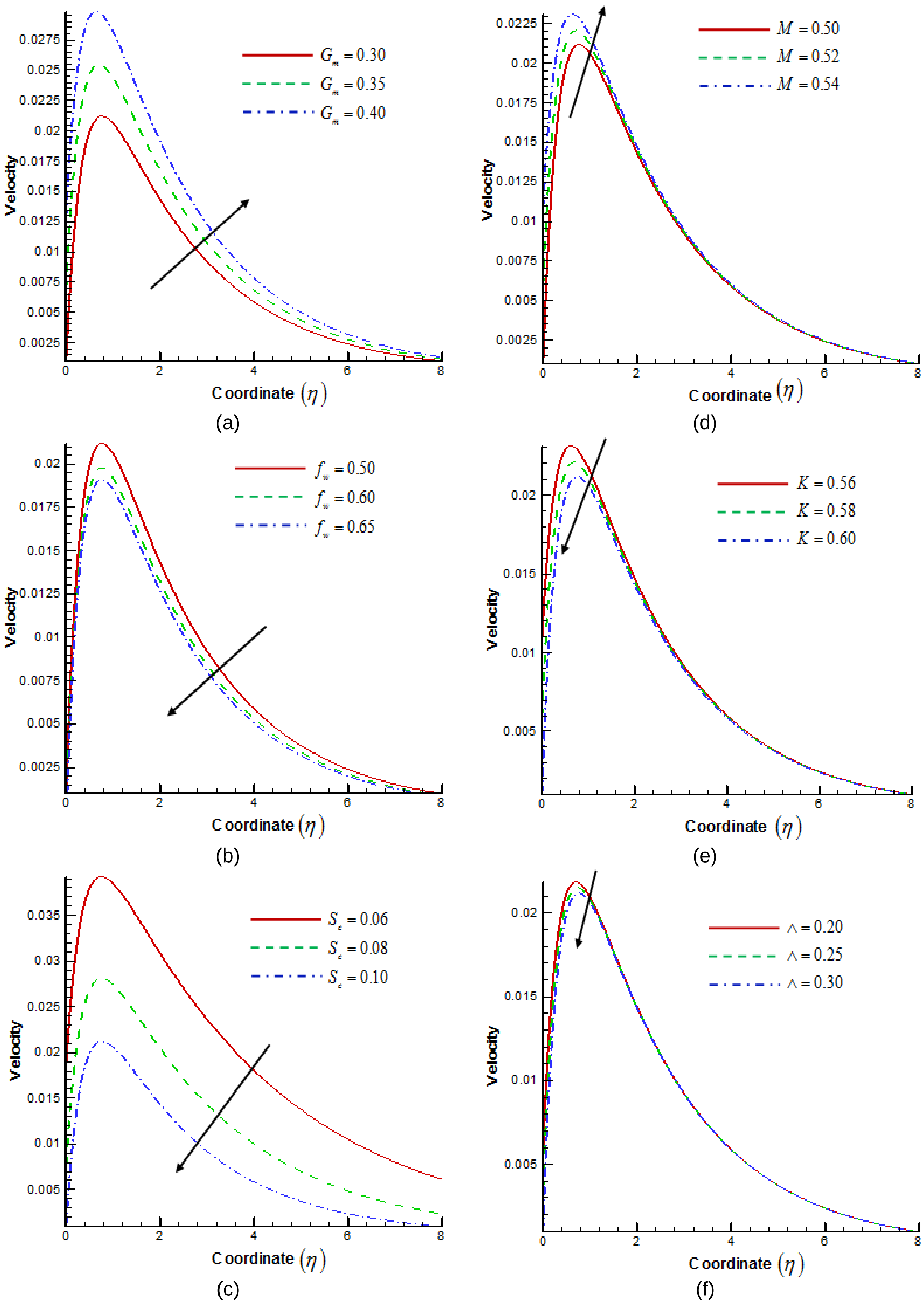

Fig. 2. Velocity profiles for different values of (a) modified Grashof number (b) suction parameter (c) Schmidt number (d) magnetic force number (e) permeability of porous plate (f) spin gradient viscosity number.

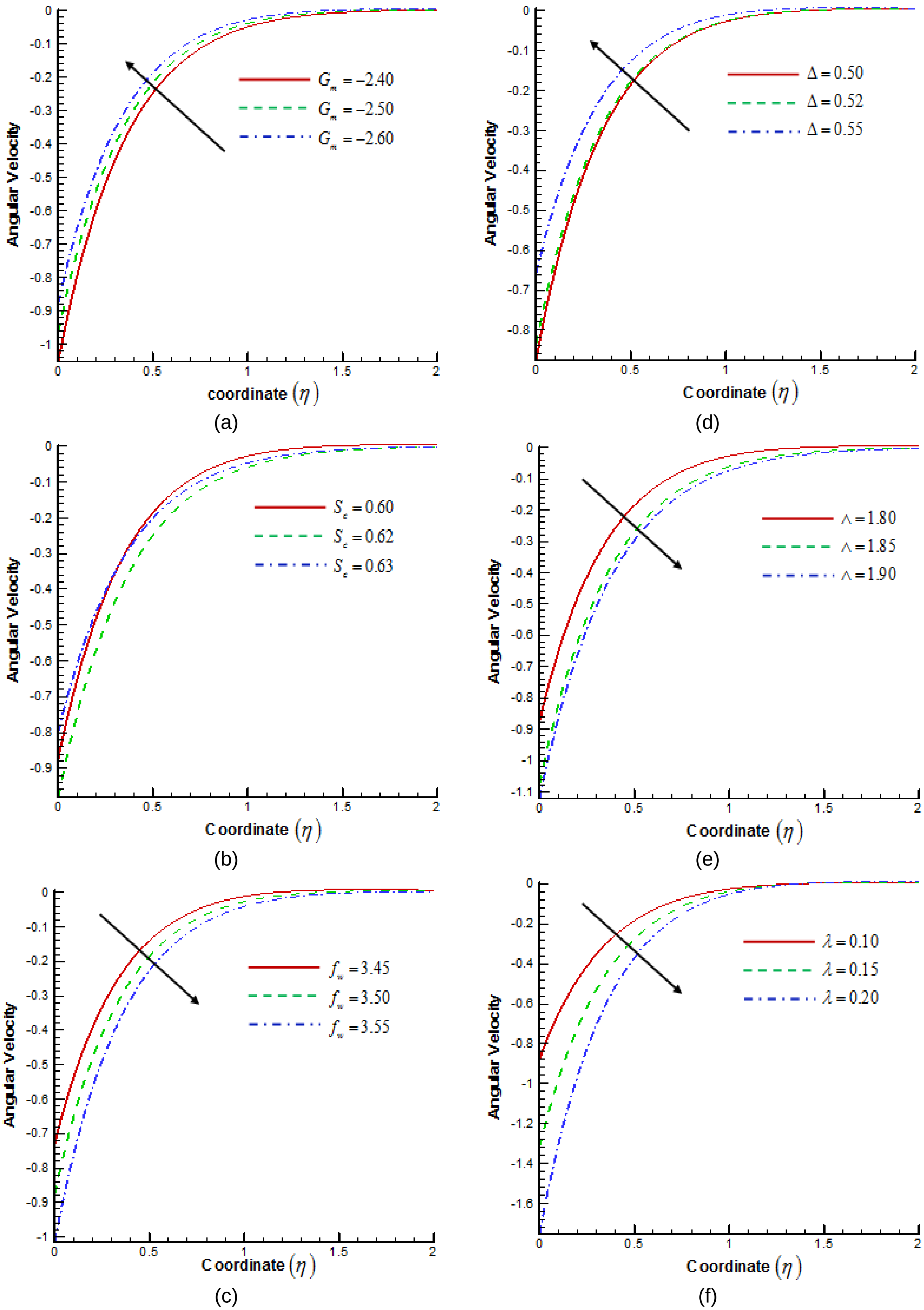

Fig. 3. Angular velocity profiles for different values of (a) modified Grashof number (b) Schmidt number (c) suction parameter (d) micro-rotational number (e) spin gradient viscosity number (f) vortex viscosity.

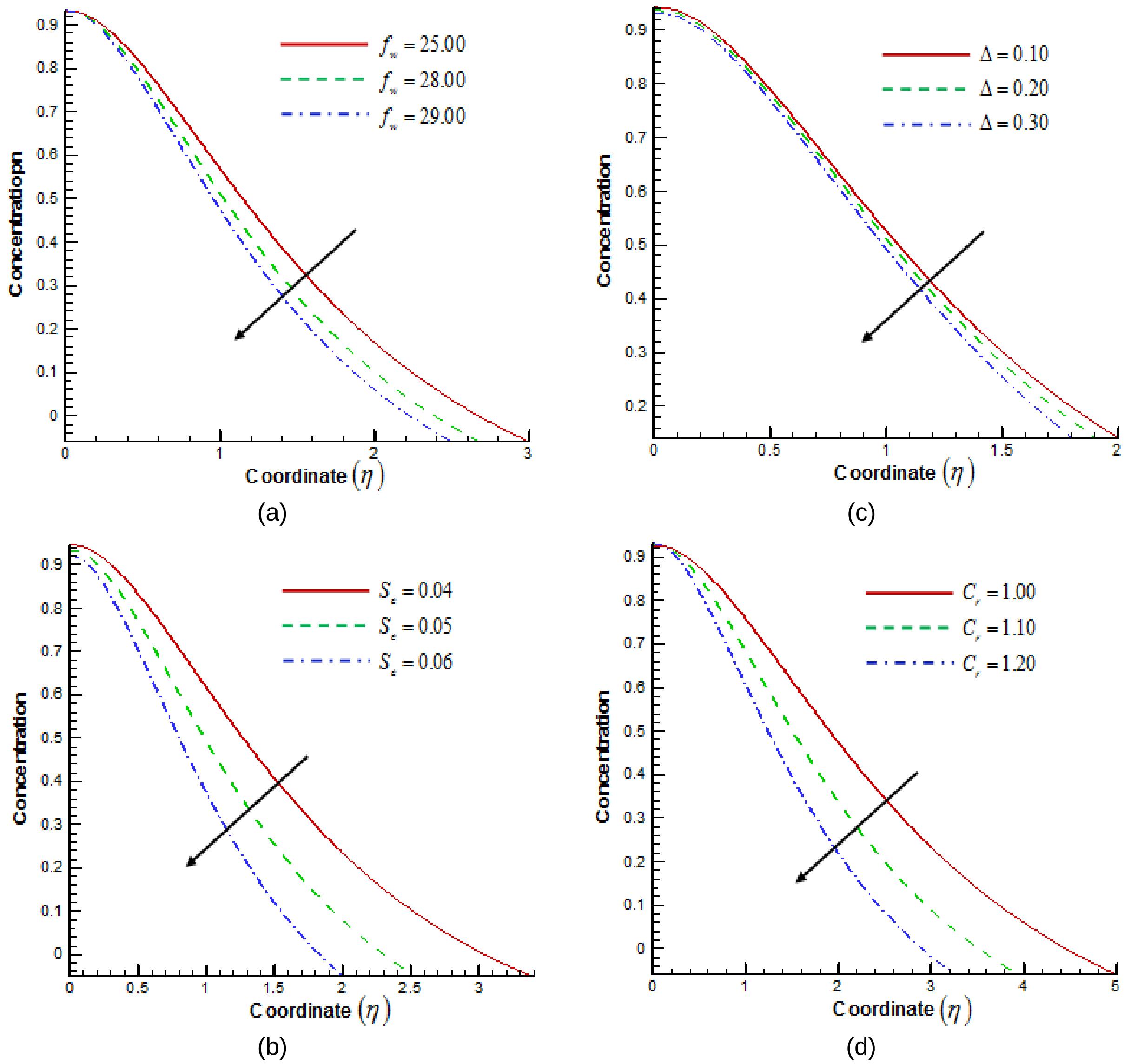

Fig. 4. Concentration profiles for different values of (a) suction parameter (d) Schmidt number (c) micro-rotational number (d) chemical reaction parameter.

4. CONCLUTION

Some of the important findings of the present work obtained from the graphical representation of the results are listed below:

1. The fluid velocity and angular velocity profiles decreases with the increase of Modified Grashof number.
2. The velocity and angular velocity profiles decreases with the increase of Suction parameter and also the concentration profile decreases with the increase of Suction parameter.
3. The velocity profile decreases and angular velocity profiles decreases with the increase of Schmidt number and also the concentration profile decreases with the increase of Schmidt number.
4. The velocity profile increase with Magnetic force number.
5. The velocity profiles decreases with the increase of Permeability of porous plate.
6. The concentration profile decreases with the increase of Chemical reaction parameter.
7. The angular velocity profile increases with the increase of Micro-rotational number and the concentration profile decreases with the increase of Micro-rotational number.
8. The angular velocity profile decreases with the increase of Spin gradient viscosity number.
9. The angular velocity profile decreases with the increase of Vortex viscosity.

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APPENDIX

$$A_1 = \frac{1}{\wedge}, \quad A_2 = \frac{1}{1+\Delta}, \quad A_3 = \frac{-A_2 G_M}{\left(-S_c{}^3 + A_2 S_c{}^2\right)}, \quad A_5 = \frac{S_c}{A_2 \varepsilon^2}, \quad A_6 = \frac{S_c}{\varepsilon^2}$$

$$A_7 = \frac{S_c^2}{A_2 \varepsilon^2 \left(A_2{}^2 + S_c A_2\right)}, \quad A_{20} = \frac{A_3}{2}, \quad A_8 = \frac{-\frac{A_2 \lambda}{\varepsilon^2}}{\left(A_2^2 - A_1 A_2\right)}, \quad A_9 = \frac{A_3 \lambda S_c^2}{S_c^2 - A_1 S_c}, \quad A_{10} = \Delta A_2{}^2 A_8$$

$$A_{11} = \Delta A_2 A_9 S_c, \quad A_{12} = G_m C_r A_2, \quad A_{13} = \frac{A_2 \left(K+M\right)}{\varepsilon^2}, \quad A_{14} = A_2 S_c A_3 \left(K+M\right), \quad A_{15} = \frac{A_{10}}{A_2{}^2}$$

$$A_{16} = \frac{A_{11}}{\left(-S_c^3 + S_c^2 A_2\right)}, \quad A_{17} = \frac{A_{12}}{\left(-S_c^3 + S_c^2 A_2\right)}, \quad A_{18} = \frac{A_{13}}{A_2{}^2}, \quad A_{19} = \frac{A_{14}}{\left(-S_c^3 + S_c^2 A_2\right)}$$