

# Magnetic properties of a quasi-two-dimensional Heisenberg antiferromagnet $\alpha$ -RbCrF<sub>4</sub>

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## ABSTRACT

We synthesized a quasi-two-dimensional Heisenberg antiferromagnet on a square lattice formed in  $\alpha$ -RbCrF<sub>4</sub> by improving the pretreatment method before primary sintering. From X-ray diffraction measurements, the crystal structure was found to consist of a TlAlF<sub>4</sub>-type structure, which shows a good two-dimensionality. Temperature dependence of magnetic susceptibility shows a broad peak, which indicates typical low-dimensional antiferromagnets, at  $T_{\text{max}} \approx 47$  K. Furthermore, a sharp peak which indicates an antiferromagnetic phase transition also appeared at  $T_N = 29.3(2)$  K. Following several previous theoretical investigations, we estimated intra-layer ( $J_{\text{intra}}$ ) and inter-layer ( $J_{\text{inter}}$ ) exchange interactions to be  $J_{\text{intra}}/k_B = -6.6(1)$  K and  $J_{\text{inter}}/J_{\text{intra}} \approx 0.05$ , respectively. As a result, we found that  $\alpha$ -RbCrF<sub>4</sub> is a quasi-two-dimensional Heisenberg antiferromagnet.

**Keywords:** Quasi-two-dimensional magnet, X-ray diffraction, magnetic susceptibility,  $\alpha$ -RbCrF<sub>4</sub>

## 1. INTRODUCTION

Recently, magnetoelectric multiferroic materials have received much attention because of the possibility of either "magnetic control of ferroelectric domains" or "electric control of magnetic domains"[1,2,3]. In ferromagnetics and ferroelectrics, the switching from one domain orientation to another occurs because of the application of external perturbation, which changes the preferred, lowest energy orientation of the order parameter from one state to another. In addition to magnetic and electric fields, mechanical stress can have a switching effect in ferroelastic materials. In addition to magnetic and electric fields, mechanical stress can have a switching effect in ferroelastic materials. Using point groups of prototypic and ferroic phases, Aizu classified the cases where ferromagnetism, ferroelectricity, and ferroelasticity coexist and completely couple with each other[4].

Many investigations have been reported for successive structural phase transitions in  $A^I M^{\text{III}} F_4$  compounds, where the  $M^{\text{III}} F_6$  octahedra are centered in a square-based

parallelepiped of  $A^+$  cations in the so-called  $TlAlF_4$ -type structure[5]. Figure 1 shows an aristotype  $A^I M^{III} F_4$  structure. The corner-sharing  $M^{III} F_6$  octahedra result in a square lattice with each layer separated by  $A^I$  cations, leading to a good two-dimensionality in  $A^I M^{III} F_4$  compounds. For example, in non-magnetic compounds  $TlAlF_4$  and  $RbAlF_4$ [6,7], internal strains were investigated and the switching of ferroelastic domains by uniaxial stress was demonstrated, although a ferroelastic-ferroelectric effect can not be expected because of the non-polar space group of these materials. Furthermore, in magnetic compounds with  $S = 5/2$ ,  $RbFeF_4$  and  $CsFeF_4$ [8,9,10,11], an orthorhombic (mmm)-tetragonal (4/mmm) structural phase transition causes spontaneous strain, and then, an antiferromagnetic phase transition occurs far below the structural phase transition temperature. Temperature ( $T$ ) dependence of magnetic susceptibilities ( $\chi$ ) of  $RbFeF_4$  and  $CsFeF_4$  shows a typical two-dimensional antiferromagnetic behavior. On the other hand, in magnetic compound with  $S = 1$ ,  $CsVF_4$ [12,13], a sharp  $\chi(T)$  peak corresponding to an antiferromagnetic phase transition appeared at a magnetic field ( $H$ ) of 200 Oe, and the  $\chi(T)$  curves for field cooling (FC) and zero-field cooling (ZFC) overlapped. At  $H \geq 5$  kOe, a ferromagnetic moment was induced by the magnetic field and the splitting of  $\chi(T)$  curves for the FC and ZFC appeared. Therefore, we hope that mechanical stress can be used to switch the magnetic and/or ferroelectric domains in ferroelastic  $A^I M^{III} F_4$  compounds.

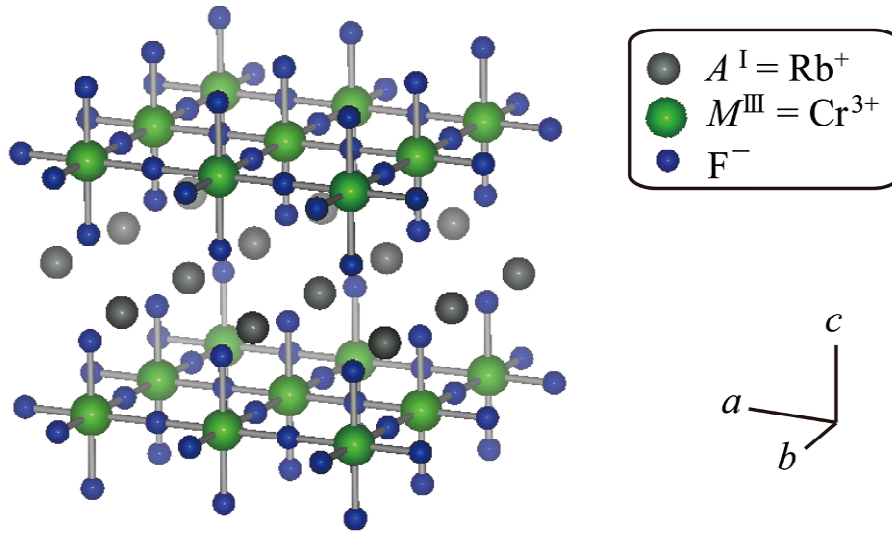
Previously, we have intensively studied a series of chromium fluorides,  $A^I CrF_4$  ( $A^I = K$  and  $Cs$ ), because of their highly frustrated magnetic structures such as triangular spin tubes[14,15]. Table 1 presents the structural phase diagram of  $A^I CrF_4$  ( $A^I = K, Rb, Cs$ ), as previously reported by Kozak[16]. In equilateral triangular spin tube  $CsCrF_4$ , no structural isomer exists below the melting point. However, a structural isomer was observed in  $KCrF_4$  and  $RbCrF_4$  when the sintering temperature was varied. In  $KCrF_4$ , non-equilateral triangular spin tube  $\alpha$ - $KCrF_4$  was crystallized below 768 °C [15], whereas  $\beta$ - $KCrF_4$  consisting of the  $CsCrF_4$ -type structure was crystallized above 768 °C. In  $RbCrF_4$ ,  $\alpha$ - $RbCrF_4$  consisting of a  $TlAlF_4$ -type structure was crystallized below 750 °C [17], whereas  $\beta$ - $RbCrF_4$ , consisting of a  $CsCrF_4$ -type structure, is crystallized above 750 °C. The magnetic properties of  $RbCrF_4$  were studied without any distinction between the  $\alpha$ - and  $\beta$ -phases[18,19].

We have previously observed that the magnetic susceptibility of  $CsCrF_4$  is strongly affected by a small amount of paramagnetic impurities and/or imperfect crystallization[14,15]. A crystallization method that enables the synthesis of high-quality  $\alpha$ - $RbCrF_4$  must be developed to confirm the magnetic ground state. Figure 1 shows the aristotype structure of  $\alpha$ - $RbCrF_4$ , which we believe a quasi-two-dimensional Heisenberg antiferromagnet with  $S = 3/2$ . In this study, we obtained highly crystalline  $\alpha$ - $RbCrF_4$  and performed X-ray diffraction (XRD) and magnetic susceptibility measurements to confirm that high-quality samples were prepared and to confirm the heretofore unreported magnetic properties of  $\alpha$ - $RbCrF_4$ .

77 **Table 1: Structural phase diagram of  $A^I\text{CrF}_4$  ( $A^I = \text{K, Rb, and Cs}$ ) in Ref. 16.**

	$\alpha$ - phase (low- $T$ phase)	Critical Temperature ( $^{\circ}\text{C}$ )	$\beta$ - phase (high- $T$ phase)
$\text{KCrF}_4$	non-equilateral triangular spin tube	768	$\text{CsCrF}_4$ -type
$\text{RbCrF}_4$	square lattice ( $\text{TlAlF}_4$ -type)	750	$\text{CsCrF}_4$ -type
$\text{CsCrF}_4$	equilateral triangular spin tube	No structural isomer below melting point.	

78



80 **Figure 1: Schematic of aristotype structure of  $A^I M^{\text{III}}\text{F}_4$  compound. Each layer is**  
81 **stacked without translation in the  $ab$ -plane. In  $\alpha$ - $\text{RbCrF}_4$ ,  $A^I$  and  $M^{\text{III}}$  correspond to**  
82  **$\text{Rb}^+$  and  $\text{Cr}^{3+}$ , respectively.**

83

## 84 2. SAMPLE PREPARATION

85

86 We prepared polycrystalline samples of  $\alpha$ - $\text{RbCrF}_4$  using a method similar to that employed  
87 in the synthesis of high-quality  $\text{CsCrF}_4$ , i.e., using a conventional solid-state reaction  
88 method[15]. We mixed the  $\text{RbF}$  and  $\text{CrF}_3 \cdot 4\text{H}_2\text{O}$  starting materials in accordance with the  
89 stoichiometry and then heated them at  $200\text{ }^{\circ}\text{C}$  for more than 48 h under vacuum with  
90  $P < 1 \times 10^{-3}\text{ Pa}$  to dehydrate the crystals. Sintering was then performed at various  
91 temperatures below  $750\text{ }^{\circ}\text{C}$ . The final sample color was dark green. To further purify the  
92 samples, we improved the pretreatment method before primary sintering, as discussed in the  
93 next section.

95 To examine the sample crystal structure and phase purity, we performed powder XRD  
 96 measurements at room temperature. The XRD data were collected for  $5^\circ < 2\theta < 70^\circ$  by a  
 97 Philips X'pert Pro MPD using the Bragg-Brentano geometry with Cu  $K\alpha$  radiation. Because  
 98 the antiferromagnetic ground state in  $\text{CsVF}_4$  was found to be broken by  $H \geq 5$  kOe [12,13], a  
 99 weaker magnetic field should be applied to  $\alpha$ - $\text{RbCrF}_4$ . The temperature dependence of  $\chi$   
 100 was measured using a superconducting quantum interference device magnetometer  
 101 (Quantum Design, MPMS-XL) from 2 K to 350 K. In this study, we defined the magnetic  
 102 susceptibility as  $\chi \equiv M/H$ . The FC and ZFC data were collected after applying  $H = 10$  Oe  
 103 and 1 kOe at  $T = 350$  K and 2 K, respectively. Because the  $\chi(T)$  curves for the FC and ZFC  
 104 data overlapped, the ZFC data were omitted in the following discussion.

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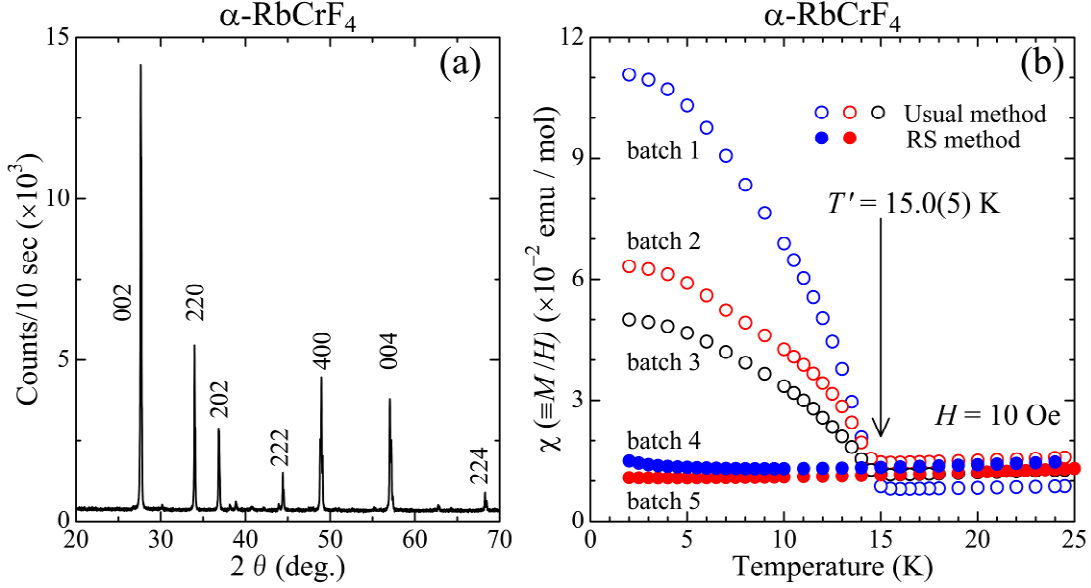
### 106 3. EXPERIMENTAL RESULTS AND DISCUSSION

107

108 Figure 2(a) presents the sharp XRD peak profiles obtained after sintering at  $640^\circ\text{C}$ .  
 109 According to the previous studies, the space group of  $\alpha$ - $\text{RbCrF}_4$  is  $Pmmn$  ( $2a \times 2b \times c$ ), and  
 110 the lattice constants are  $2a = 7.348 \text{ \AA}$  and  $c = 6.442 \text{ \AA}$  [16,17,20]. On the basis of the  $Pmmn$   
 111 ( $2a \times 2b \times c$ ) space group, we can denote the all fundamental peaks using the indices shown  
 112 in Fig. 2(a). However, additional XRD peaks appeared and their origin may be impurities  
 113 and/or distorted square lattice caused by the tilting of  $\text{CrF}_6$  octahedra. In  $A^I M^{\text{III}} \text{F}_4$ , the  
 114  $Pmmn$  ( $2a \times 2b \times c$ ) space group is expected a ferroelastic state from Aizu's  $4/mmmFmmm$   
 115 notation.

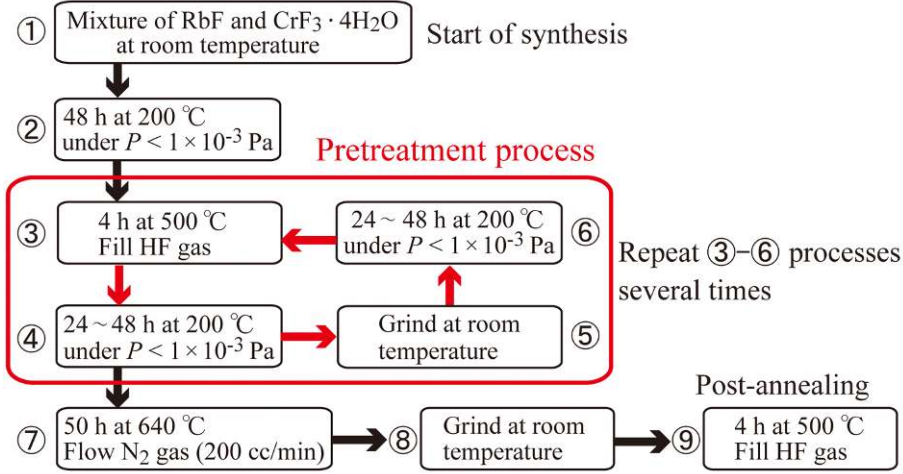
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117 We determined the best sintering temperature to be  $640^\circ\text{C}$  and then post-annealed the  
 118 samples at  $500^\circ\text{C}$  under HF gas; we denote this as the usual method[15]. As shown in Fig.  
 119 2(b), we measured the  $T$  dependence of  $\chi$  for  $\alpha$ - $\text{RbCrF}_4$  at  $H = 10$  Oe to prevent the  
 120 saturation of impurity-induced weak ferromagnetic moments at high magnetic fields. As  
 121 observed in  $\chi(T)$  data for the samples obtained by the usual method, an anomaly indicating  
 122 a weak ferromagnetic moment appeared at  $T' = 15.0(5) \text{ K}$ . As shown in Fig. 2(b), when we  
 123 crystallized several samples under the same conditions, the transition temperatures  $T'$   
 124 remained unchanged; however, the values of  $\chi$  at 2.0 K varied widely among different  
 125 batches. We believe that this magnetic transition at  $T'$  is due to extrinsic properties, i.e., the  
 126 presence of some magnetic impurities and/or poor crystallizations.



**Figure 2: (a) X-ray diffraction pattern for  $\alpha$ -RbCrF<sub>4</sub> after sintering at 640 °C. Fundamental peaks are denoted by the indices on the basis of the Pmmn ( $2a \times 2b \times c$ ) space group. (b) Low-temperature magnetic susceptibility [ $\chi(T)$ ] for samples obtained using the usual method in the three different batches (open circles) and the RS method in the two batches (closed circles). The all datasets agree with one another above  $T' = 15.0(5)$  K. However, an anomaly indicating the weak ferromagnetic moment appeared in the data for the samples obtained by the usual method, whereas no anomaly appeared in the data for the samples obtained by the RS method.**

To further purify polycrystalline  $\alpha$ -RbCrF<sub>4</sub>, we improved the pretreatment of the sample before primary sintering at 640 °C. Figure 3 presents the scheme of the improved pretreatment, i.e., processes 3-6 were repeated several times before primary sintering. Finally, the samples were heated at 640 °C for 50 h under N<sub>2</sub> gas flow (200 cc/min) and then post-annealed at 500 °C under HF gas. We will denominate the improved pretreatment "the return to synthetic precursor method" (abbreviated as the RS method). As observed in Fig. 2(b), when we compare the data for the samples obtained using the RS method with those obtained using the usual method, the  $\chi(T)$  curves for both methods agree with each other above 15 K; however, no anomaly appeared below 15 K in the data for the samples obtained by the RS method and its tendency showed high reproducibility. On the other hand, the weak ferromagnetic moment at 2.0 K for the samples using the usual method showed a large sample dependence. Therefore, we concluded that the impurity-induced weak ferromagnetic moments appeared in the samples using the usual method. Regrettably, the samples obtained by the two methods are indistinguishable based on the XRD data. In future, we will refine the superstructure for  $\alpha$ -RbCrF<sub>4</sub> by another experimental methods such as EXAFS or XANFS experiment to obtain high-quality  $\alpha$ -RbCrF<sub>4</sub> where the magnetic phase transition at  $T'$  will be absent.



**Figure 3: Schematic of the improved pretreatment for  $\alpha$ -RbCrF<sub>4</sub>. Processes 3-6 were repeated several times before primary sintering at 640 °C. We will denominate the improved pretreatment "the return to synthetic precursor method" (abbreviated as the RS method).**

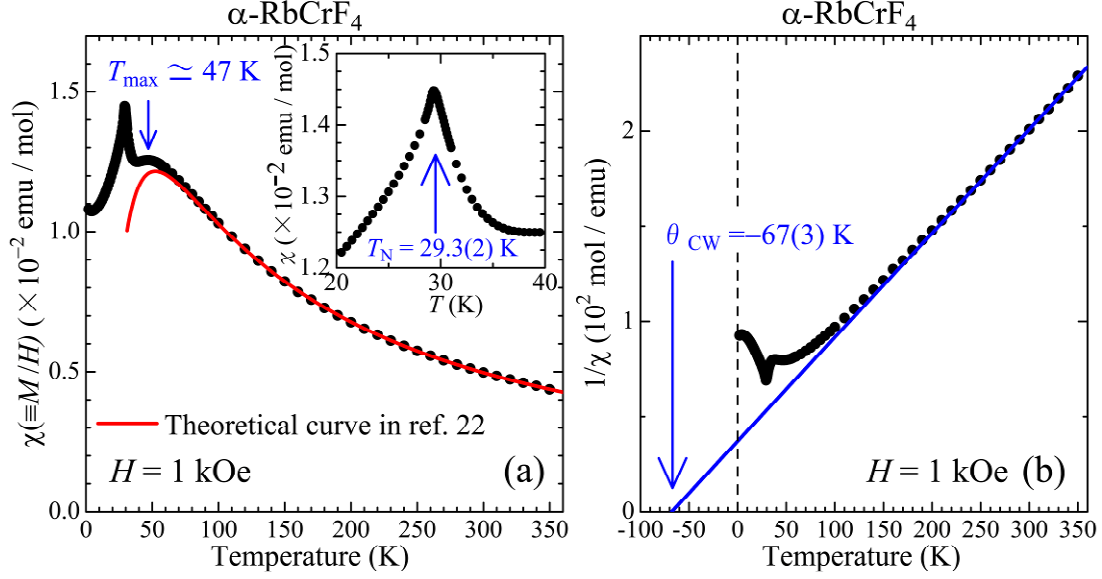
Figure 4(a) shows the  $\chi(T)$  curves at  $H = 1$  kOe over the entire temperature range for the high-quality  $\alpha$ -RbCrF<sub>4</sub>. A broad  $\chi(T)$  peak indicating a typical low-dimensional antiferromagnetic behavior appeared at  $T_{\max} \approx 47$  K [21]. As observed in the inset of Fig. 4(a), a sharp  $\chi(T)$  peak appeared with the curvature similar to that of CsVF<sub>4</sub> at  $H = 200$  Oe [12]. Therefore, we conclude that an antiferromagnetic phase transition occurs at  $T_N = 29.3(2)$  K in  $\alpha$ -RbCrF<sub>4</sub>. Figure 4(b) shows the  $1/\chi(T)$  curve. We fitted the  $1/\chi(T)$  data above 250 K to the Curie-Weiss law [ $\chi(T) = C/(T - \Theta_{\text{CW}})$  and  $C = N_A g^2 \mu_B J(J+1)/(3k_B)$ ]; the resulting Weiss temperature ( $\Theta_{\text{CW}}$ ) and effective magnetic moment were  $-67(3)$  K and  $3.98(3) \mu_B$ , respectively. The effective magnetic moment agrees with the spin-only value  $3.87(3) \mu_B$  within the experimental error. Applying the molecular field theory to solve the Hamiltonian given by

$$H_{\text{cal}} = -2 \sum_{\langle i, j \rangle} J_{i, j} S_i \cdot S_j, \quad (3.1)$$

the Weiss temperature is written as

$$\Theta_{\text{CW}} = \frac{2zJS(S+1)}{3k_B}, \quad (3.2)$$

where  $z$  is the number of nearest neighbors and  $J$  is the isotropic exchange interaction, provided that this equation is valid for  $T > |\Theta_{\text{CW}}|$ . Applying  $\Theta_{\text{CW}} = -67(3)$  K and  $z = 4$ , we estimated the intra-layer exchange interaction to be  $J_{\text{intra}}/k_B = -6.7(3)$  K.



177

178 **Figure 4: (a) Temperature dependence of the magnetic susceptibility ( $\chi$ ) for the high-**  
 179 **quality  $\alpha$ -RbCrF<sub>4</sub>. A broad  $\chi(T)$  peak appeared at  $T_{\max} \approx 47$  K. The solid line is the**  
 180 **best fit of the theoretical formula reported by Lines in Ref. 22, which yields**  
 181  $J_{\text{intra}}/k_B = -6.6$  K and  $\chi_{\text{const}} = -9.4 \times 10^{-5}$  emu/mol. The inset shows a sharp peak for

182  $\chi(T)$ , indicating an antiferromagnetic phase transition at  $T_N = 29.3(2)$  K. (b)

183 **Temperature dependence of  $1/\chi(T)$ . The solid line is the Curie-Weiss law; the**  
 184 **resulting Weiss temperature is  $\Theta_{\text{CW}} = -67(3)$  K.**

185

186 For a quadratic-layer Heisenberg antiferromagnet, the magnetic susceptibility reported by  
 187 Lines can be expressed in the high-temperature region [ $\chi_{2D}(T)$ ] by a series expansion[22].  
 188 To subtract the  $T$ -independent term ( $\chi_{\text{const}}$ ), i.e., the diamagnetic contribution and Van Vleck  
 189 term, we attempted to use [ $\chi_{2D}(T) + \chi_{\text{const}}$ ] with the experimental data in the 200-350 K  
 190 temperature range far above  $|\Theta_{\text{CW}}|$ . For the best-fit curve shown in Fig. 4(a), the values of  
 191  $J_{\text{intra}}$  and  $\chi_{\text{const}}$  are determined to be  $-6.6(1)$  K and  $-9.4(1) \times 10^{-5}$  emu/mol, respectively.  
 192 The values of  $J_{\text{intra}}$  obtained by the different analyses are in good agreement. Using the  
 193 relation between  $J_{\text{intra}}$  and  $T_{\max}$ , we further obtained  
 194  $J_{\text{intra}}/k_B = -T_{\max}/[1.12 \times S(S+1) + 0.10] = -5.5(2)$  K [22]. This value is smaller than the others.  
 195 The broad  $\chi(T)$  peak in  $\alpha$ -RbCrF<sub>4</sub> is necessary to re-consider not only the development of  
 196 an antiferromagnetic short-range order accompanied with the inter-layer exchange  
 197 interaction ( $J_{\text{inter}}$ ) but also the existence of the inequivalent magnetic sites, i.e., the tilting of  
 198 CrF<sub>6</sub> octahedra in the opposite manner[20]. Based on the theoretical investigations of the  
 199 effect of  $J_{\text{inter}}$  by Ginsberg,  $\chi_{2D}(T)$  must be modified at  $T < |\Theta_{\text{CW}}|$ , but the value of  $T_{\max}$  is  
 200 almost invariant[23]. According to Yasudas' theoretical investigations of the relation between  
 201  $J_{\text{inter}}$  and  $T_N$  in both quantum and classical two-dimensional antiferromagnets, we obtained



202  $J_{\text{inter}}/J_{\text{intra}} \approx 0.05$  from Fig. 3 in Ref. 24, where we roughly estimated  
 203  $(k_{\text{B}}T_{\text{N}})/[2|J_{\text{intra}}|S(S+1)] \approx 0.6$ . Based on the magnetic susceptibility and neutron diffraction  
 204 experiments on CsVF<sub>4</sub>, the magnetic ground state may change from the antiferromagnetic  
 205 state to some other state with ferromagnetic moments at a critical magnetic field ( $H_{\text{c}}$ )[12,13].  
 206 In  $\alpha$ -RbCrF<sub>4</sub>, we re-expressed  $H_{\text{c}}$  at  $T = 0$  K as  $|g_1 - g_2|\mu_{\text{B}}H_{\text{C}} = 6|J_{\text{inter}}|$ , where  $g_1$  and  $g_2$   
 207 correspond to the g-value of each inequivalent magnetic site in antiferrodistortive CrF<sub>6</sub>  
 208 octahedra[12]. Because  $|g_1 - g_2| \approx 0.01 - 0.05$  in Cr<sup>3+</sup> compounds[25], we roughly estimated  
 209 the critical field as  $H_{\text{C}} \approx 2 \times 10^3 - 1 \times 10^4 |J_{\text{inter}}|/(g\mu_{\text{B}})$ . Consequently, we found that the  $\chi(T)$   
 210 data at  $H = 1$  kOe shown in Fig. 4 indicate the presence of an intrinsic antiferromagnetic  
 211 ground state. Thus, the absence of the differences between the FC and ZFC  $\chi(T)$  curves is  
 212 due to the absence of a ferromagnetic moment.

## 213 4. CONCLUSIONS

214 We successfully synthesized high-quality  $\alpha$ -RbCrF<sub>4</sub>, a quasi-two-dimensional Heisenberg  
 215 antiferromagnet with  $S = 3/2$  using the RS method, which is an improved pretreatment  
 216 method. The RS method can be applied to many powdered polycrystalline or  
 217 monocrystalline fluorides before primary sintering. XRD experiments revealed that  $\alpha$ -  
 218 RbCrF<sub>4</sub> consists of a TAlF<sub>4</sub>-type structure, which shows a good two-dimensionality.  
 219 Magnetic susceptibility experiments did not find any extrinsic anomaly due to impurities at  
 220  $T' = 15.0(5)$  K. The intrinsic  $\chi(T)$  curve exhibited a broad  $\chi(T)$  peak at  $T_{\text{max}} \approx 47$  K and  
 221 showed the occurrence of an antiferromagnetic phase transition at  $T_{\text{N}} = 29.3(2)$  K. Using the  
 222  $\chi(T)$  data in the high- $T$  region, we obtained  $J_{\text{intra}}/k_{\text{B}} = -6.6(1)$  K and  $J_{\text{inter}}/J_{\text{intra}} \approx 0.05$ . As a  
 223 result, the  $\chi(T)$  data at  $H = 1$  kOe indicate the presence of an intrinsic antiferromagnetic  
 224 ground state. If the degree of the good two-dimensionality is increased, the  
 225 antiferromagnetic ground state may change under low magnetic fields because of the  
 226 competition between the Zeeman energy and  $J_{\text{inter}}$ . We expect that the singular curvature of  
 227 the  $\chi(T)$  peak at  $T_{\text{N}} = 29.3$  K is because of the existence of inequivalent magnetic sites, i.e.,  
 228 the different tilting schemes of the F<sup>-</sup> octahedra surrounding the Cr<sup>3+</sup> ions at different sites. In  
 229 future, the superstructure will be determined to clarify whether ferromagnetism,  
 230 ferroelectricity, and ferroelasticity coexist in this material. Furthermore, the temperature  
 231 dependence of the heat capacity under high magnetic fields and high field magnetization  
 232 process will be measured to confirm the magnetic-field-induced phase transition.

## 233 ACKNOWLEDGEMENTS

234 We are grateful to T. Goto, and T. Masuda for the stimulating discussions. We received  
 235 valuable support from the Frontier Science Research Center in Kagoshima University for  
 236 performing the powder XRD experiments. This research was partly supported by the Kurata  
 237 Memorial Hitachi Science and Technology Foundation, and JSPS KAKENHI Grant Numbers  
 238 JP22014009 and JP23684029.



244 **COMPETING INTERESTS**

245

246 The authors declare that no competing interests exist.

247

248 **AUTHORS' CONTRIBUTIONS**

249

250 All aspects of this work were carried out in close collaboration between both authors. Both  
251 authors read and approved the final manuscript.

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