Heat and Mass Transfer of Laminar Boundary Layer Flow of Non-Newtonian Power Law Fluid past a Porous Flat Plate with Soret and Dufour effects

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ABSTRACT:

This article numerically studies the heat and mass transfer of a laminar boundary layer flow of a non-Newtonian power law fluid past a porous flat plate. The solution takes the suction/injection, power law index, Soret and Dufour effects into consideration. The governing boundary layer partial differential equations along with boundary conditions are first cast into a dimensionless form by a similarity transformation and the resulting ordinary differential equations are then solved numerically using implicit finite difference scheme. The influence of Soret and Dufour parameters, suction/injection, power law index, Prandtl number and Lewis number on non-dimensional velocity, temperature and concentration fields are discussed graphically. Tha variation of Power law index, Soret and Dufour numbers on heat and mass transfer rates is presented in tabular form.

Keywords: Non-Newtonian fluid, suction/injection, power law index, Soret and Dufour number, Finite Difference method.

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1. INTRODUCTION :

In the recent past, the study of flow and heat transfer of a non-Newtonian fluid has gained considerable interest because of its enormous applications in engineering. The majority of the studies dealt in the past are with the traditional Newtonian fluids. It is well known that most fluids which are encountered in chemical and allied processing applications do not satisfy the classical Newton's law and are accordingly known as non-Newtonian fluids. These types of fluids are frequently used in oil-engineering, biology, physiology, technology and industry. Because of such important applications, considerable efforts have been directed towards the analysis and understanding of such fluids. Hence to explain the rheological behavior of non-Newtonian fluids, different mathematical models have been proposed. Among these, power-law model is the most widely used for non-Newtonian fluids. The power-law fluids are the subclass of these non-Newtonian fluids which are very good in representing the Pseudo plastic and Dilatant nature of the fluids. In the existing literature a masterpiece of work has been done on heat and mass transfer of a power-law fluid. Schowalter [1] was the first one, who studied the application of a boundary layer to power-law pseudso-plastic fluids. The analysis of the flow field in a boundary-layer near a stretching sheet is an important part in fluid dynamics and heat transfer. This type of flow occurs in a number of engineering processes such as extrusion of plastic sheets, polymer processing, and metallurgy [2, 3]. Laminar boundary layer flow of non-Newtonian power-law fluid past a porous flat plate is examined by Jadhav [4]. Magneto-hydrodynamic flow of a power-law fluid over a stretching sheet investigated by Andersson and Bech [5]. Anderson and Dandapat [6] discussed the non-Newtonian power law fluid over a linearly stretching sheet. Parasad *et al*. [7] investigated the hydromagnetic flow and heat transfer of a non-Newtonian power-law fluid over a vertical stretching sheet. Ali [8, 9] investigated hydromagnetic flow and heat transfer over a non-isothermal permeable surface stretching with a power-law velocity with heat generation and suction/injection effects and in the presence of a non-uniform transverse magnetic field and boundary-layer flow of a viscous, incompressible, electrically conducting and heat-absorbing fluid along a semi-infinite vertical permeable moving plate in the presence of a uniform transverse magnetic field and thermal and concentration buoyancy effects respectively.

The study of hydro magnetic convective non-Newtonian fluid flows with heat and mass transfer in porous medium attracted many research due to its applications in many field like, soil physics, geophysics, aerodynamics and aeronautics. The interest in these new problems generates from their importance in liquid metals, electrolytes and ionized gases. Many authors like Choudary and Das [10], Rushikumar and Sivaraj [11], and Reddy et al. [12], investigated an unsteady free convective MHD non-Newtonian flow through a porous medium bounded by an infinite inclined porous plate. Saritha et.al [13] studied the effect of radiation on MHD non-Newtonian power-law fluid past over a non-linearly stretching surface with viscous dissipation. MHD effects on non-Newtonian power-law fluid past a continuously moving porous flat plate with heat flux and viscous dissipation was analyzed by Shashidar Reddy and Kishan [14].

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. Soret and Dufour effects on free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium with thermal radiation are studied by Tai and Char [15]. **Maleque [16] has investigated Magnetohydrodynamic Convective Heat and Mass Transfer Due to a rotating Disk with Thermal Diffusion Effect. Dufour and Soret effects on unsteady MHD convective heat and mass transfer flow due to a rotating disk was investigated by Maleque [17].** Mohammad Mehdi Rashidi et. al **[18]** examined Heat and mass transfer for MHD viscoelastic fluid flow over a vertical stretching sheet with Soret and Dufour effects.
Motivated by the above mentioned works, we investigate in this paper the effects of Soret and Dufour on heat and mass transfer of a steady, two dimensional flow of an electrically conducting, non-Newtonian power law fluid past a semi infinite porous flat plate. The solutions for the momentum, heat and mass

transfer equations are solved using implicit finite difference scheme. The effects of different involved parameters such as Suction/Injection, Magnetic number, Prandtl number, Lewis number, Soret and Dufour number on velocity, temperature and concentration fields are plotted and discussed.

2. MATHEMATICAL ANALYSIS :

Consider the flow of a steady, laminar, incompressible non-Newtonian power-law fluid past a semi infinite porous flat plate. The coordinate system is chosen such that X-axis is taken along the direction of the flow and y-axis normal to it. It is also assumed that a magnetic field with magnetic field intensity B_0 is applied along the Y-axis and that the magnetic Reynolds number is very small so that the induced magnetic field can be neglected. In addition, Hall Effect and electric field are assumed to be negligible. The fluid properties are assumed to be constant. We assumed that $T_w(x) = T_\infty + bx$ and $C_w(x) = C_\infty + cx$, where b and c are constants such that the uniform wall temperature T_w and concentration C_w are higher than those of their full stream values T_∞ , C_∞. **By invoking all the boundary layer approximations, the governing equations for the flow in this investigation can be written as**

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{K}{\rho}\frac{\partial}{\partial y}\left[-\frac{\partial u}{\partial y}\right]^{n}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2}+\frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

In the foregoing equations, u and v are the velocity components along the X and Y axes, n is the power-law index, K is the power-law fluid parameter, ρ is density, μ is the magnetic permeability, σ is the electrical conductivity of the fluid, $\alpha = k/\rho c_p$ is the thermal diffusivity, k is the thermal conductivity, c_p is the specific heat at a constant pressure, k_T is the thermal diffusion ratio, c_s is the concentration susceptibility, D_m is the coefficient of mass diffusivity, T is the temperature, C is the fluid concentration and T_m is the mean fluid temperature.

The boundary conditions associated with the present problem are as follows

$$u=0,\quad v=V_w,\qquad T=T_w(x)\quad and\quad C=C_w(x)\quad at\qquad y=0$$

$$u=U,\ v=0,\ T=T_\infty\quad C=C_\infty\quad as\quad y\to\infty \tag{5}$$

Where V_w is suction velocity. To facilitate the analysis, we follow the previous studies Jadhav [4] and Mohammad Mehdi Rashidi et al **[18]** and use the similarity variables

$$\left.\begin{aligned}\Psi(\eta) &= (\gamma x U^{2-n})^{\frac{1}{n+1}} f(\eta)\\ \eta &= y\left[\frac{U^{2-n}}{\gamma x}\right]^{\frac{1}{n+1}}\\ \theta(\eta) &= \frac{T-T_\infty}{T_w-T_\infty}\\ \varphi(\eta) &= \frac{C-C_\infty}{C_w-C_\infty}\end{aligned}\right\} \tag{6}$$

Where Ψ is the stream function defined in the usual way and *f* is the reduced stream function for the flow. Then the velocity components are defined using the similarity variables as

$$\left.\begin{aligned} u &= \frac{\partial \Psi}{\partial x} = Uf\,'(\eta) \\ v &= -\frac{\partial \Psi}{\partial y} = \frac{1}{n+1}\left(\gamma \frac{U^{2n-1}}{x^n}\right)^{\frac{1}{n+1}}(\eta f' - f) \end{aligned}\right\} \tag{7}$$

Introducing equation (6) and (7), the continuity equation is satisfied and the momentum, energy and concentration equations are transformed into ordinary differential equations as follows:

$$n(-f'')^{n-1}f''' + \frac{1}{n+1}ff'' - Mf' = 0 \tag{8}$$

$$\theta'' + Pr\left(\frac{1}{n+1}f\theta' - f'\theta\right) + Du\phi'' = 0 \tag{9}$$

$$\frac{1}{Le}\phi'' + Pr\left(\frac{1}{n+1}\phi' f - f'\phi\right) + Sr\theta'' = 0 \tag{10}$$

Where primes denote indicate differentiation with respect to η.

The transformed boundary conditions in dimensionless form are

$$\left.\begin{aligned} & f(0) = f_w(constant) \\ & f'(0) = 0, \quad f'(\infty) = 1 \\ & \theta(0) = 1, \; \theta(\infty) = 0 \\ & \phi(0) = 1, \quad \phi(\infty) = 0 \end{aligned}\right\} \tag{11}$$

Where $f_w = -\frac{(n+1)}{U}V_w Re^{1/n+1}$ is the suction/injection parameter ($f_w > 0$ for suction and $f_w < 0$ for injection),

$M = \frac{\sigma {B_0}^2 x}{\rho U}$ is the Magnetic parameter,

$Re_x = \frac{U^{2-n}x^n}{\nu}$ is the Reynolds number,

$Pr = \frac{Ux}{\alpha}Re_x^{-\frac{1}{n+1}}$ is the Prandtl number,

$Le = \frac{\alpha}{D_m}$ is the Lewis number,

$Du = \frac{D_m k_T}{c_s c_p} \cdot \frac{(C_w - C_\infty)}{(T_w - T_\infty)\alpha}$ is the Dufour number,

$Sr = \frac{D_m k_T}{T_m \alpha} \cdot \frac{(T_w - T_\infty)}{(C_w - C_\infty)}$ is the Soret number.

The physical quantities of engineering interest in this problem are the local Nusselt number and local Sherwood number, which are defined respectively by

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)} = -\theta'(0)Re_x^{1/n+1} \quad \text{and} \quad Sh_x = \frac{J_w x}{D_m(C_w - C_\infty)} = -\phi'(0)Re_x^{1/n+1}$$

Where the rate of heat transfer q_w and rate of mass transfer J_w are defined as

$$q_w = -k\left[\frac{\partial T}{\partial y}\right]_{y=0} \quad \text{and} \quad J_w = -D_m\left[\frac{\partial C}{\partial y}\right]_{y=0}$$

3. NUMERICAL SOLUTION:

The transformed governing equations (8), (9) and (10) are coupled and highly non-linear, so it is difficult to find the closed form solutions. Hence, the solutions of these equations with the boundary conditions (11) are solved numerically using implicit finite difference scheme.

To solve the system of transformed governing equations using finite difference scheme, first equation (8) is linearized using the Quasi linearization technique Bellman and Kalaba **[19]**.

Then equation (8) is transformed to

$$n\left[F'''(-f'')^{n-1} + f'''(-F'')^{n-1} - F'''(-F'')^{n-1}\right] + \frac{1}{n+1}\left[Ff'' + fF'' - FF''\right] - Mf' = 0 \qquad (12)$$

where F is assumed to be known and the above equation (12) can be expressed in the simplified form as

$$a_0 f''' + a_2 f'' + a_3 f' + a_4 f = a_5 - a_1[-f'']^{n-1} \qquad (13)$$

where $a_0[i] = n(-F'')^{n-1}$, $a_1[i] = nF'''$, $a_2[i] = \frac{1}{n+1}F$,

$a_3[i] = -M$, $a_4[i] = \frac{1}{n+1}F''$, $a_5[i] = nF'''(-F'')^{n-1} + \frac{1}{n+1}FF''$

Now equation (9) & (10) can be expressed in the simplified form as

$$c_0\theta'' + c_1\theta' + c_2\theta + c_3 = 0 \qquad (14)$$

$$e_0\phi'' + e_1\phi' + e_2\phi + e_3 = 0 \qquad (15)$$

where $c_0[i] = 1$, $c_1[i] = \frac{Pr}{n+1}F$, $c_2[i] = -Pr.F'$, $c_3[i] = Du.\emptyset''$

$e_0[i] = \frac{1}{Le}$, $e_1[i] = \frac{Pr}{n+1}F$, $e_2[i] = -Pr.F'$, $e_3[i] = Sr.\theta''$

Using implicit finite difference formulae, the equations (13), (14) & (15) are transformed to

$$b_0[i]f[i+2] + b_1[i]f[i+1] + b_2[i]f[i] + b_3[i]f[i-1] = b_4[i] \qquad (16)$$

$$d_0[i]\theta[i+1] + d_1[i]\theta[i] + d_2[i]\theta[i-1] + d_3[i] = 0 \qquad (17)$$

$$h_0[i]\phi[i+1] + h_1[i]\phi[i] + h_2[i]\phi[i-1] + h_3[i] = 0 \qquad (18)$$

where $b_0[i] = 2a_0[i]$, $b_1[i] = -6a_0[i] + 2ha_1[i] + h^2a_3[i]$, $b_2[i] = 6a_0[i] - 4ha_2[i] + 2h^3a_4[i]$

$b_3[i] = -2a_0[i] + 2ha_2[i] - h^2a_3[i]$, $b_5[i] = 2h^3\{a_5[i] - a_1[-F''[i]]^{n-1}\}$

$d_0[i] = 2c_0[i] + h\,c_1[i]$, $d_1[i] = -4c_0[i] + 2h^2 c_2[i]$, $d_2[i] = 2c_0[i] - hc_1[i]$, $d_3[i] = c_3[i]$

$h_0[i] = 2e_0[i] + h\,e_1[i]$, $h_1[i] = -4e_0[i] + 2h^2 e_2[i]$, $h_2[i] = 2e_0[i] - he_1[i]$, $h_3[i] = e_3[i]$

here 'h' represents the mesh size in η direction.

Since the equations governing the flow are nonlinear, iteration procedure is followed. To carry out the computational procedure, first the momentum equation (16) is solved which gives the values of *f* necessary for obtaining the solution of coupled energy equation (17) and concentration equations (18) under the boundary conditions (11) by Thomas algorithm. The numerical solutions of f are considered as $(n+1)^{th}$ order iterative solutions and F are the n^{th} order iterative solutions. To prove convergence of finite difference scheme, the computation is carried out for slightly changed value of h by running same program. No significant change was observed in the value. The convergence criterion used in this study is that the maximum change between the current and the previous iteration values in all the dependent variables satisfy 10^{-5}.

4. RESULTS AND DISCUSSION:

In this section, by applying the numerical values to different flow parameters, the effects on velocity, temperature and concentration fields are discussed. Graphical illustration of the results is very useful and practical to discuss the effect of different parameters. Numerical values are also tabulated in Table 1 & 2 to elucidate the effects of Power-law index, Soret and Dufour number on Nusslet number and Sherwood number. We considered the effects of Soret and Dufour so that their product remains constant at 0.004. It is observed from the tabular values that the heat transfer rate is decreasing for increasing values of Soret number (or simultaneously for decreasing values of Dufour number) but mass transfer rate increases for non-Newtonian fluids (i.e. pseudo plastic and dilatant fluids). Whereas in case of Newtonian fluids both heat transfer rate and mass transfer rate are increasing for increasing values of Soret number. It can also be noticed that with the increase in the power law index, heat and mass transfer coefficients are reduced.

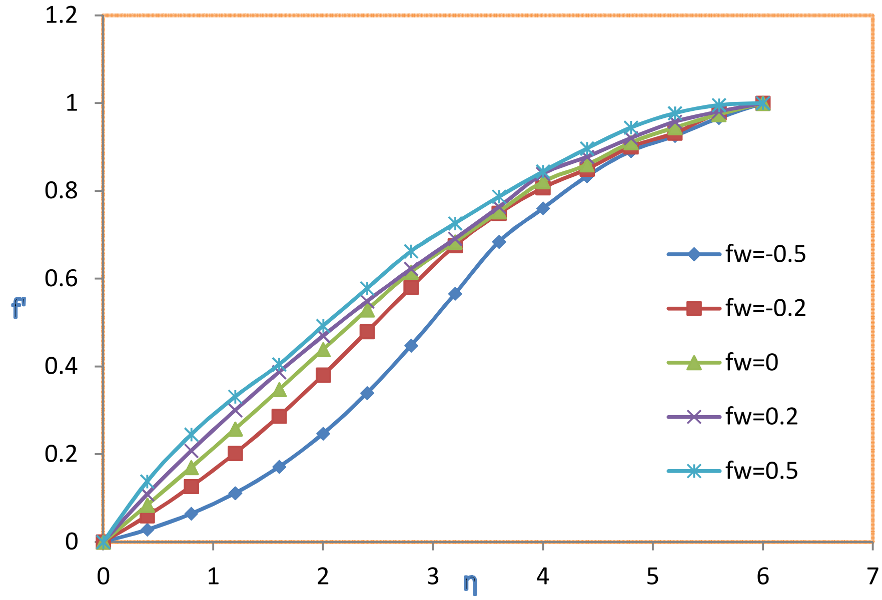
Figure (1) indicate the velocity profile *f'* for Newtonian and non-Newtonian fluids, it is evident from the figures that the effect of suction ($f_w > 0$) is to accelerate the velocity for Pseudo plastic fluid but to decelerate the velocity for Newtonian and Dilatant fluids and the effect of injection ($f_w < 0$) is to decelerate the velocity for Pseudo plastic fluid but to accelerate the velocity for Newtonian and Dilatant fluids. The influence of Magnetic field on the velocity profiles *f'* is shown in figure (2) for Newtonian and non-Newtonian fluids. It is evident from the figures that the pronounced effect of magnetic field parameter M on the velocity profile decreases with the increase in the magnetic parameter M i.e. the Laurent force which opposes the flow leads to an enhanced deceleration of the flow in both Newtonian and non-Newtonian fluids. Figure (3) is drawn for temperature profiles for different values of Prandtl number the cases of Newtonian and non-Newtonian fluids. The effect of Prandtl number is to reduce the temperature for both Newtonian and non-Newtonian fluids. Physically, fluids with smaller Prandtl number larger thermal diffusivity. Figure (4) depicts the variation of concentration profiles for Newtonian and non-Newtonian fluids with different values of Lewis number Le. Lewis number (diffusion ratio) is the ratio of Schmidt number and Prandtl number. The concentration profile is decreased due to the increase of Lewis number. Hence increase in the Lewis number reduces the concentration boundary layer thickness. The effects of Soret and Dufour numbers on temperature and concentration profiles for Newtonian and non-Newtonian fluids are plotted in Figures (5) and (6). The heat transfer caused by concentration gradient is called the diffusion thermo or Dufour effect. On the other hand mass transfer caused by temperature gradient is called Soret or thermal diffusion effect. It is evident from the figures that the increase in Soret number or decrease in Dufour number descends the temperature profiles and enhances the concentration distribution. We notice that, this behavior is a direct consequence of Soret effect, which produces a mass flux from lower to higher solute concentration driven by the temperature gradient. Hence increase in Soret number cools the fluid and reduces the temperature.

Table 1: Local Nusselt number $Nu_x/Re_x^{1/n+1}$ for f_w = 0.0, M = 0.1, Pr = 1.0 and Le = 1.0

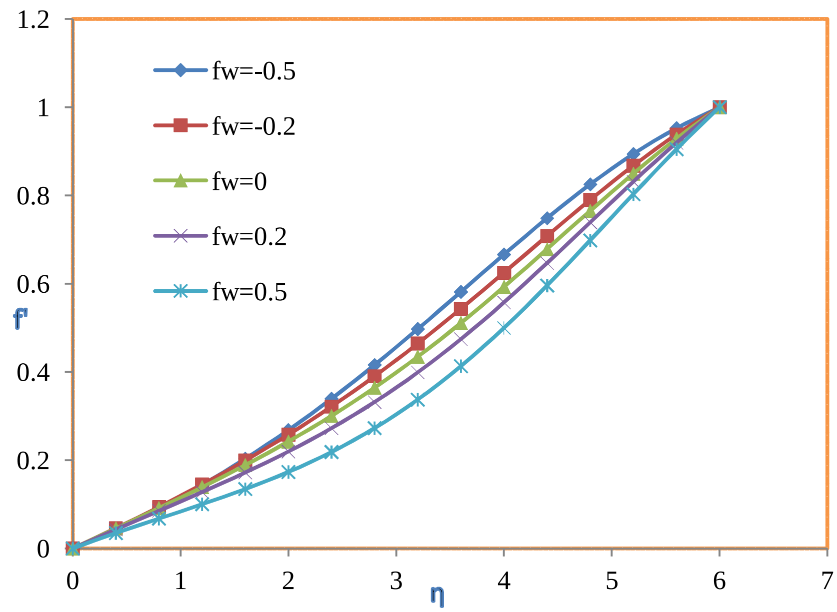
		$-\boldsymbol{\theta}'(\mathbf{0})$		
	(Sr, Du)	n=0.5	n=1.0	n=1.5
1	(0.05, 0.08)	9.728643	-2.79395	-2.3081
2	(0.1, 0.04)	7.161162	-0.10577	-1.40147
3	(0.2, 0.02)	4.116125	0.868104	-0.21564
4	(0.4, 0.01)	2.476433	0.902557	-0.1386

Table 2: Local Sherwood number $Sh_x/Re_x^{1/n+1}$ for f_w = 0.0, M = 0.1, Pr = 1.0 and Le = 1.0

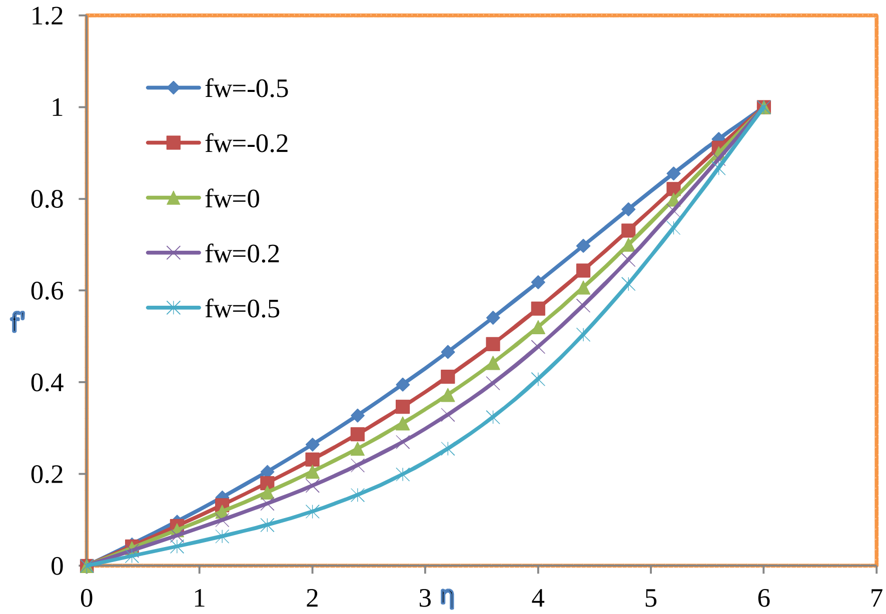
		$-\emptyset'(\mathbf{0})$		
	(Sr, Du)	n=0.5	n=1.0	n=1.5
1	(0.05, 0.08)	7.826346	-2.09009	-1.70994
2	(0.1, 0.04)	10.9477	-0.49668	-1.75576
3	(0.2, 0.02)	11.62069	1.51947	-1.86771
4	(0.4, 0.01)	12.2256	2.689975	-5.69396



(a) Pseudo Plastic fluids (n=0.5)

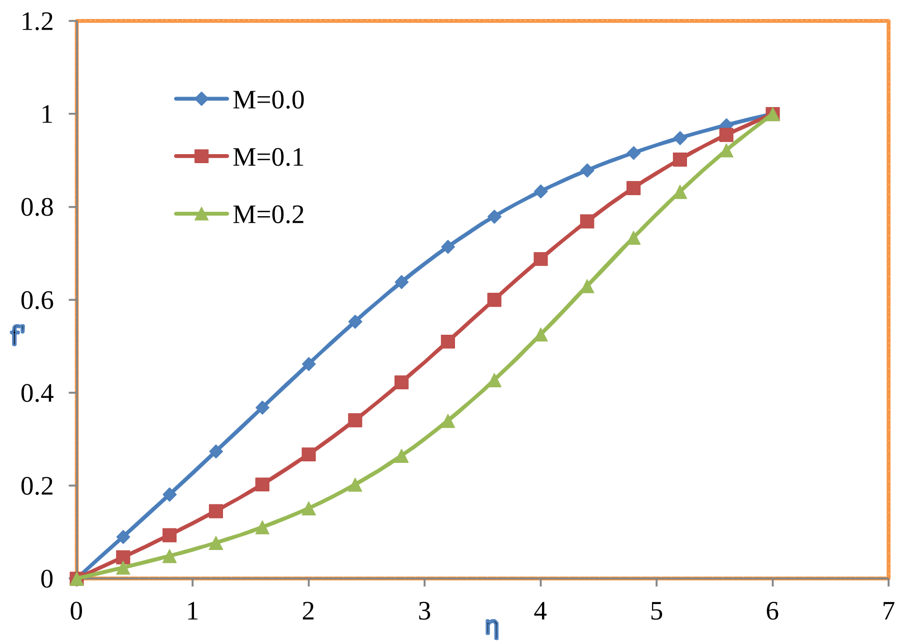


(b) Newtonian fluids (n=1.0)

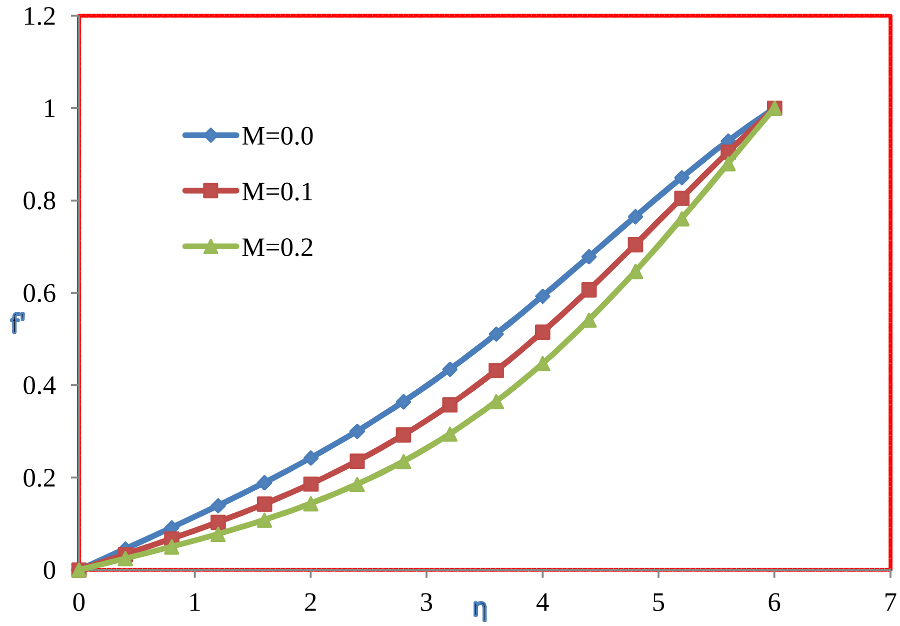


(c) Dilatant fluids (n=1.5)

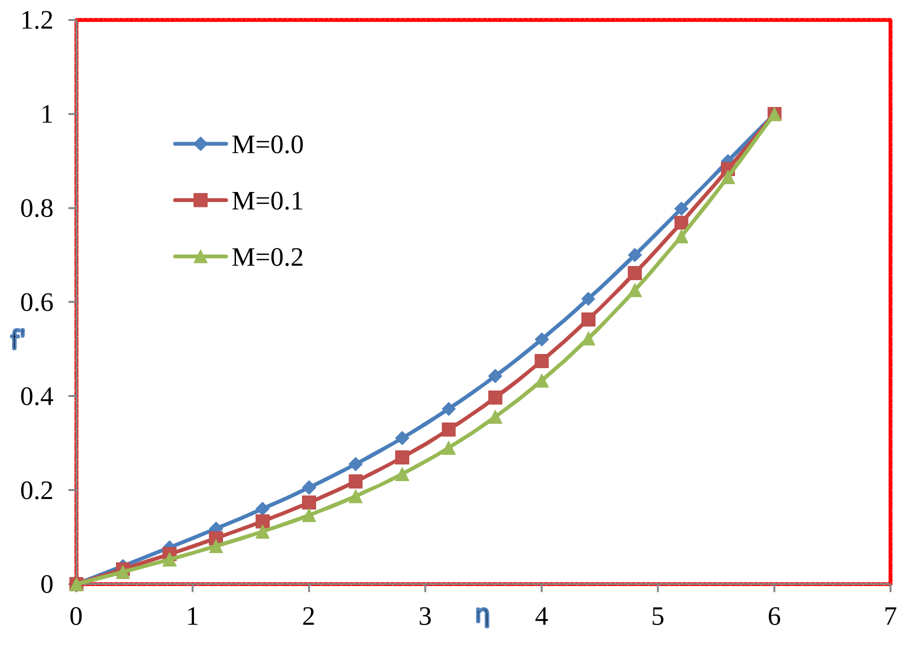
Figure 1: Velocity profiles for various values of Suction/injection parameter with Pr=1.0, Le=1.0, M=0.0, Sr=0.05, Du=0.08.



(a) Pseudo Plastic fluids (n=0.5)

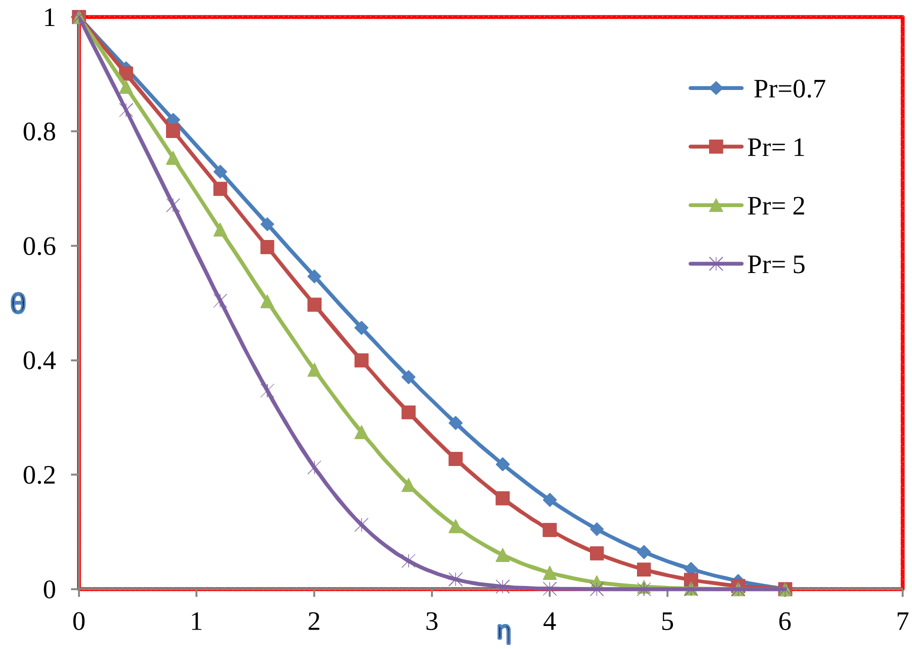


(b) Newtonian fluids (n=1.0)

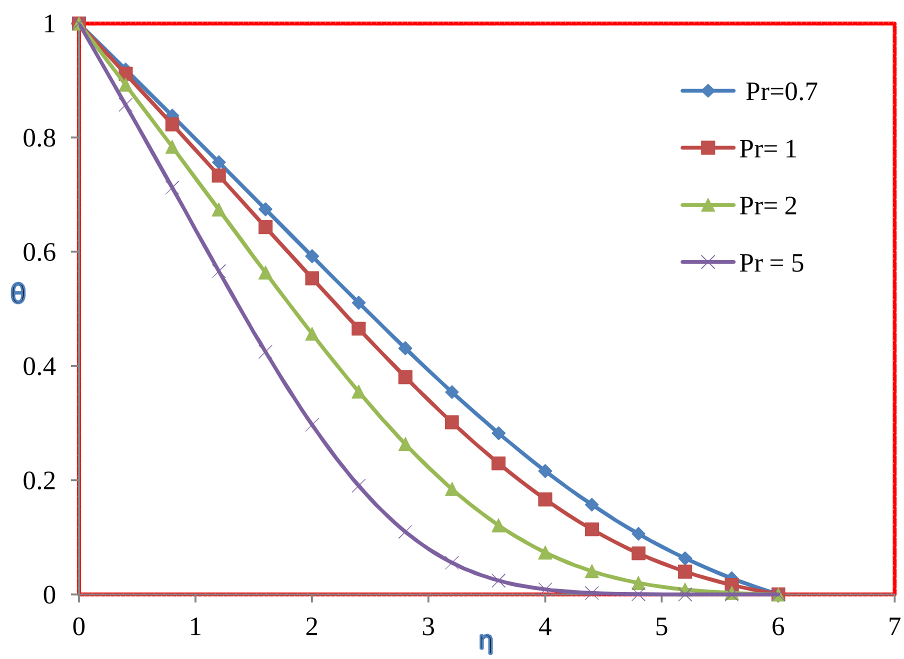


(c) Dilatant fluids (n=1.5)

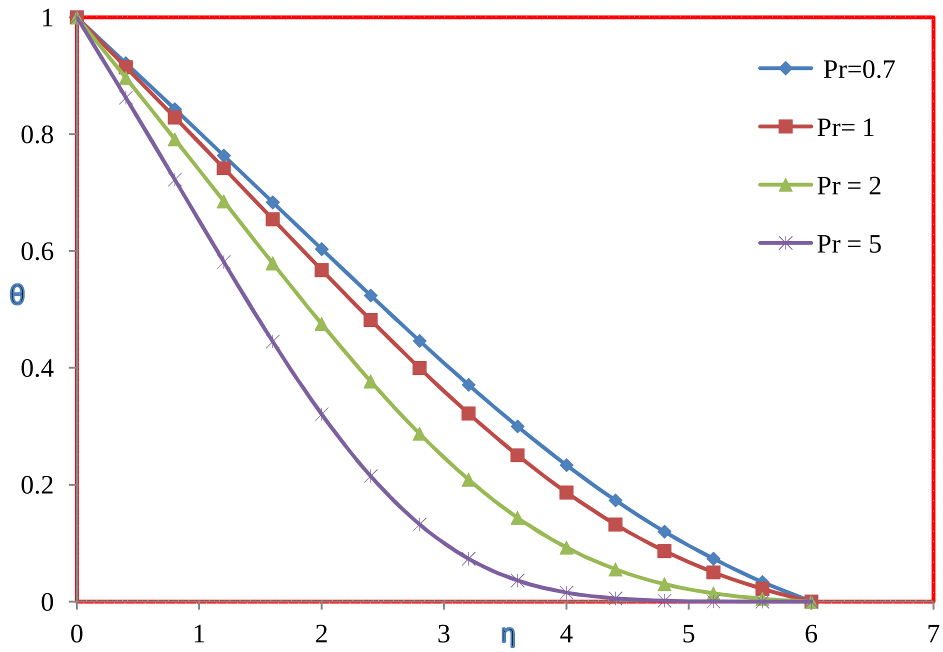
Figure 2: Velocity profiles for various values of Magnetic field parameter with Pr=1.0, Le=1.0, f_w=0.0, Sr=0.05, Du=0.08.



(a) Pseudo Plastic fluids (n=0.5)

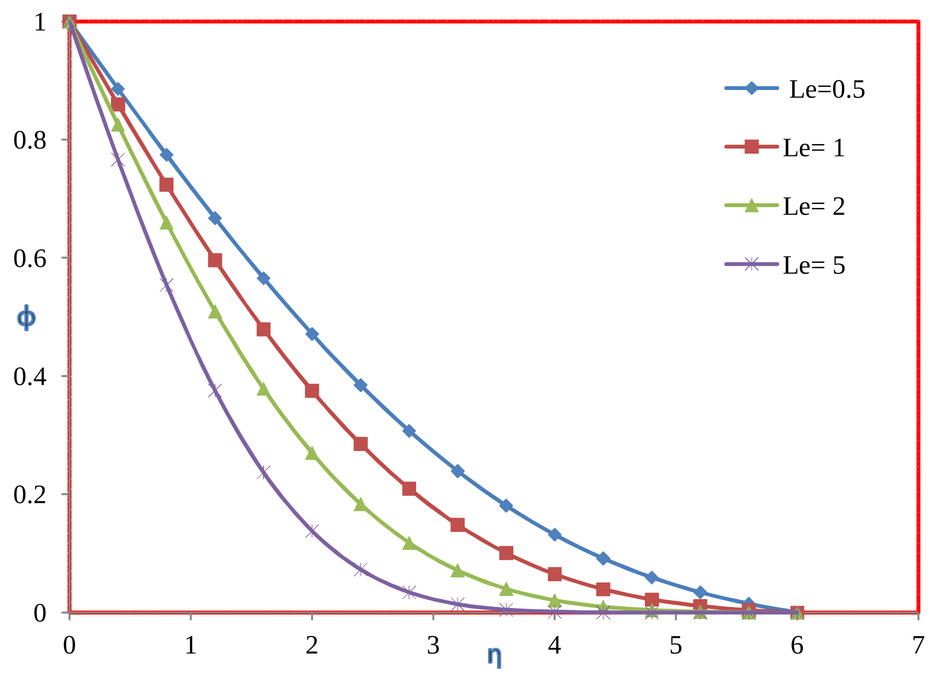


(b) Newtonian fluids (n=1.0)

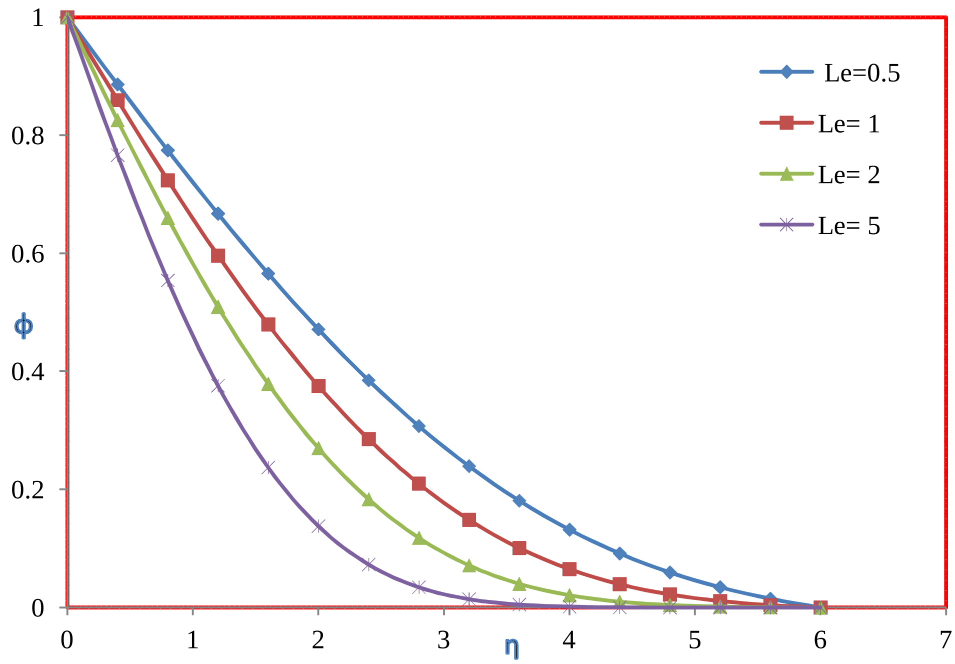


(c) Dilatant fluids (n=1.5)

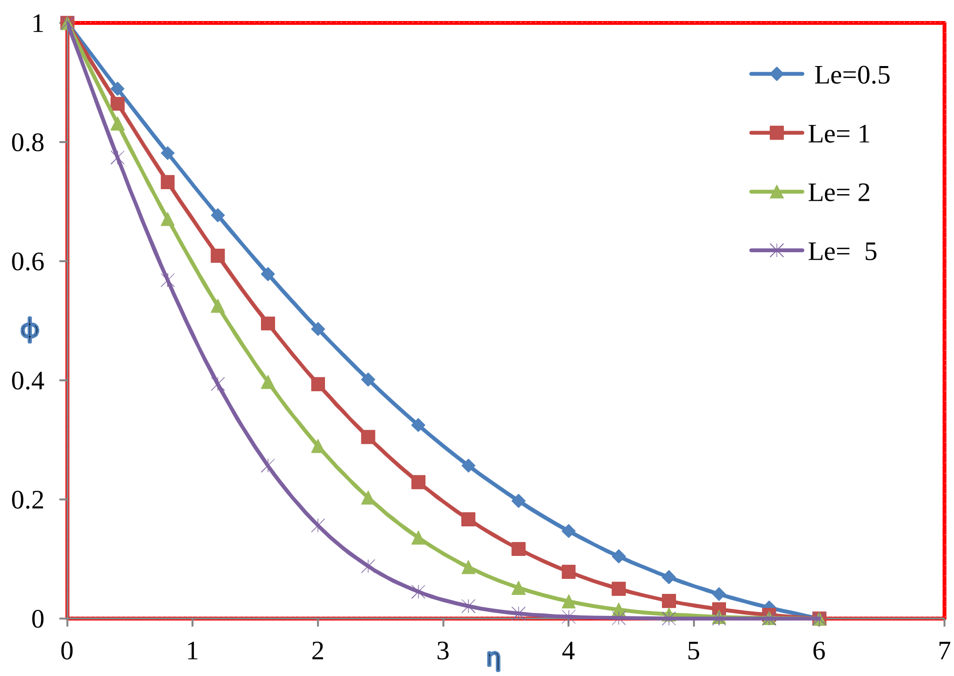
Figure 3: Temperature profiles for various values of Prandtl number with M=0.1, Le=1.0, f_w=0.0, Sr=0.05, Du=0.08.



(a) Pseudo Plastic fluids (n=0.5)

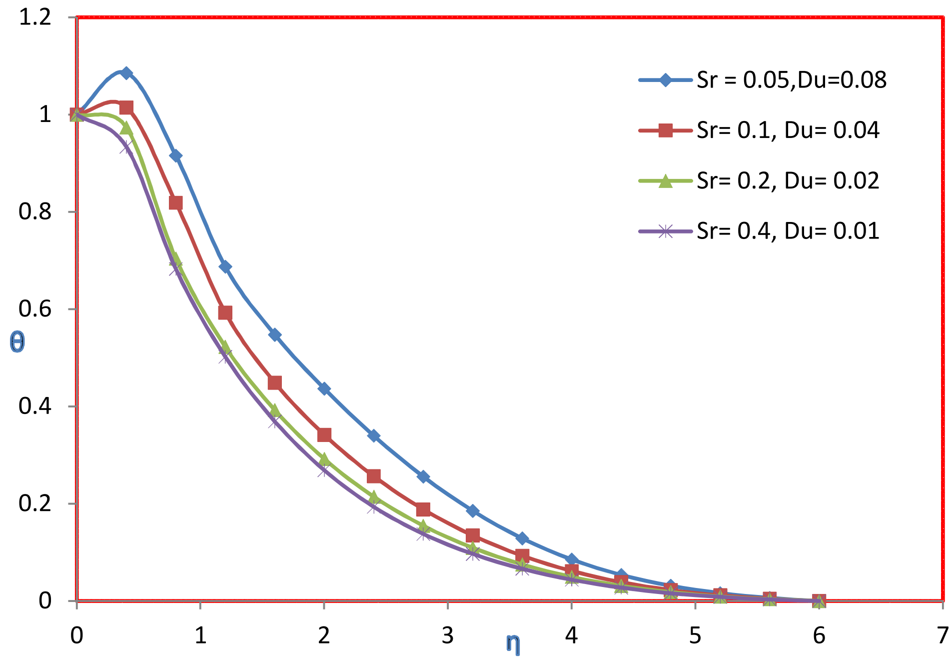


(b) Newtonian fluids (n=1.0)

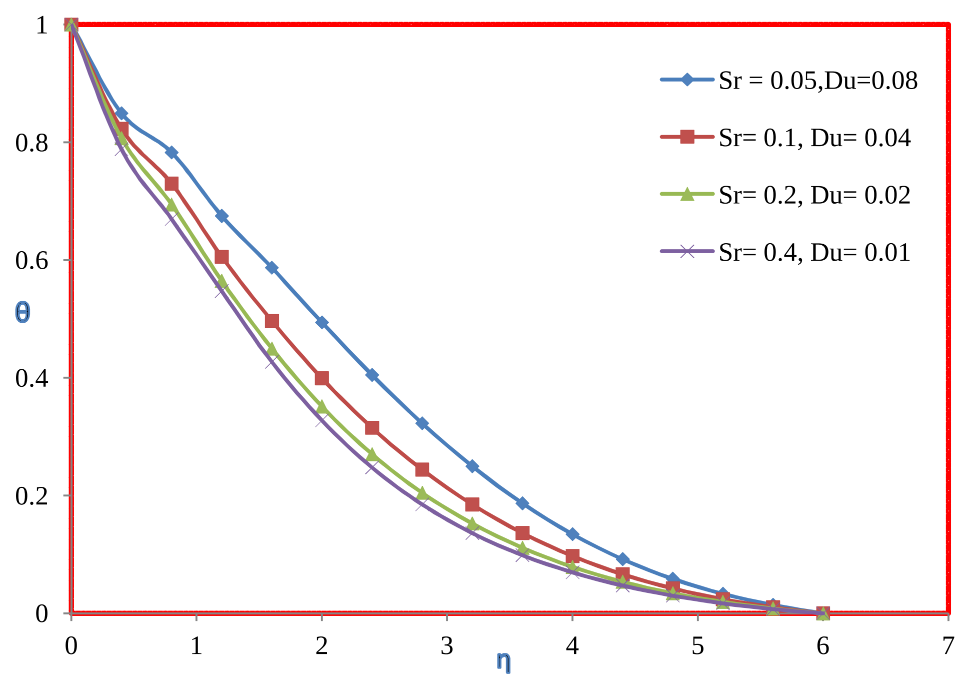


(c) Dilatant fluids (n=1.5)

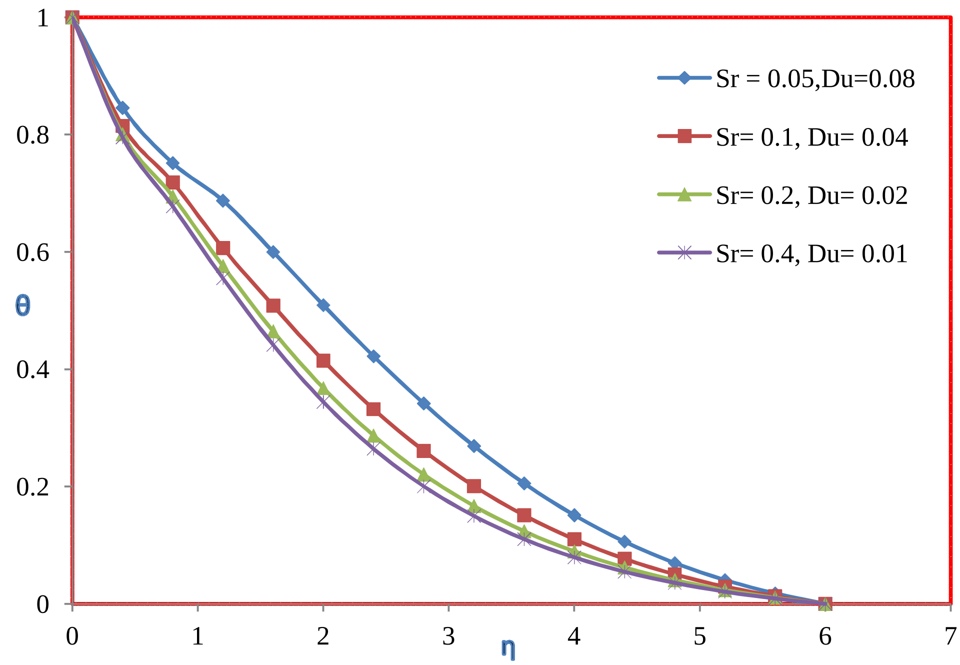
Figure 4: Concentration profiles for various values of Prandtl number with M=0.1, Le=1.0, f_w=0.0, Sr=0.05, Du=0.08.



(a) Pseudo Plastic fluids (n=0.5)

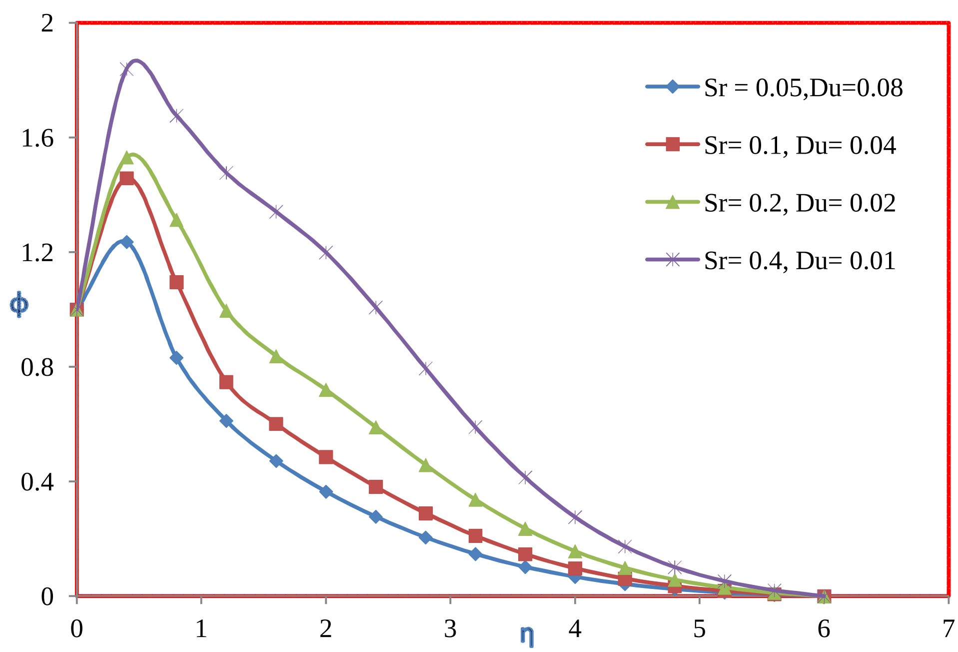


(b) Newtonian fluids (n=1.0)

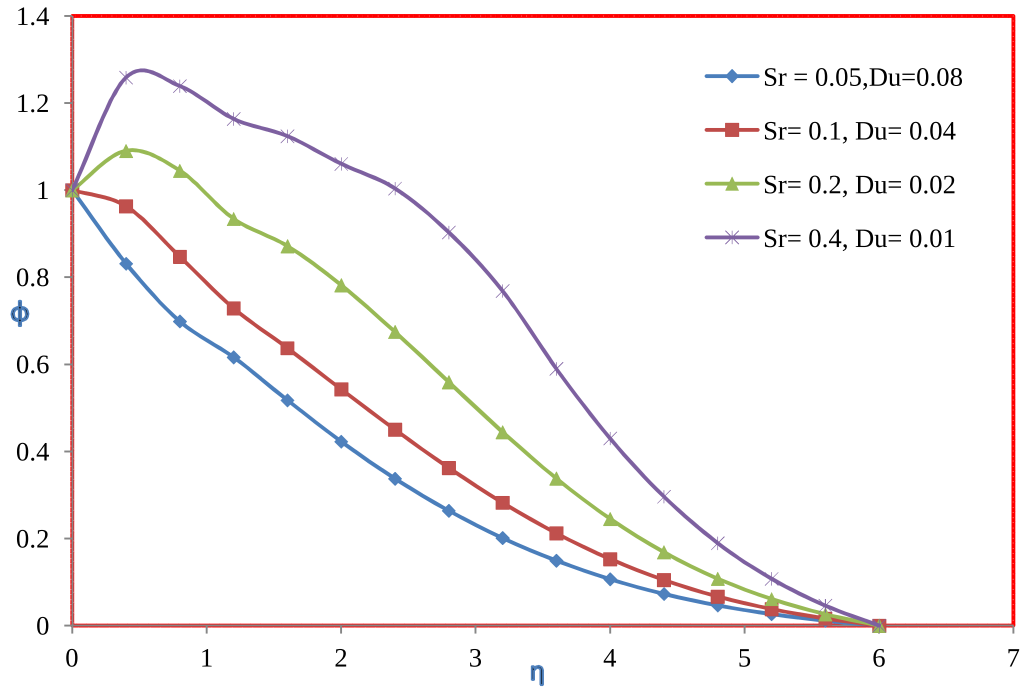


(c) Dilatant fluids (n=1.5)

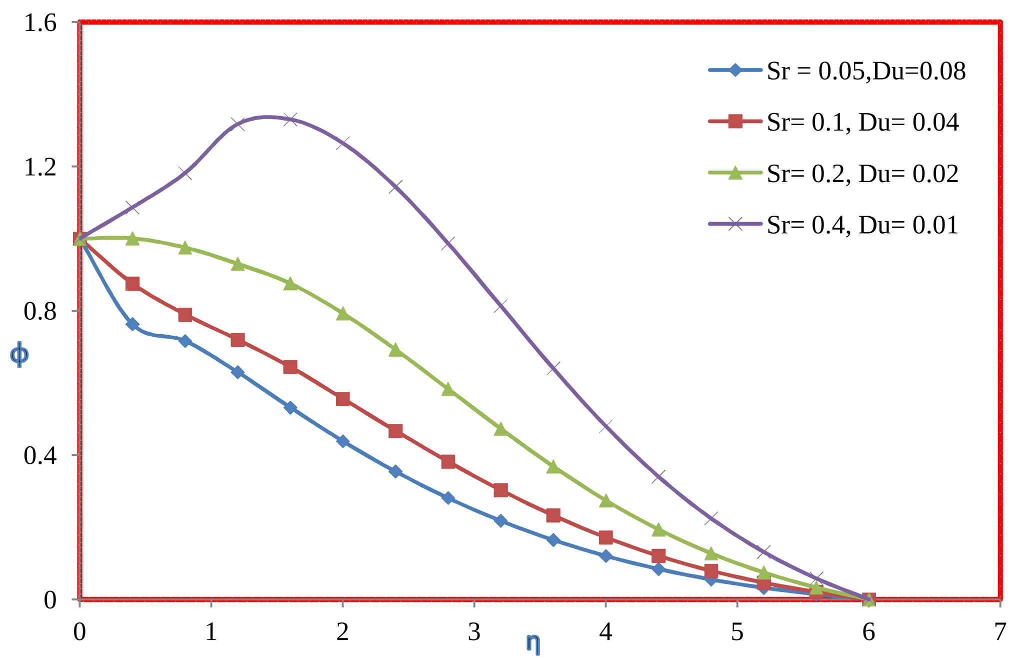
Figure 5: Temperature profiles for various values of Soret and Dufour numbers with M=0.1,Pr=1.0, Le=1.0, f_w=0.0.



(a) Pseudo Plastic fluids (n=0.5)



(b) Newtonian fluids (n=1.0)



(c) Dilatant fluids (n=1.5)

Figure 6: Concentration profiles for various values of Soret and Dufour numbers with M=0.1,Pr=1.0, Le=1.0, f_w=0.0.

5. CONCLUSION REMARKS:

This paper presents numerical studies on the problem of steady, laminar, incompressible, non-Newtonian power-law fluid past a semi infinite porous flat plate. The suction/injection, magnetic, Soret and Dufour effects are considered. The solutions are found to be governed by different parameters such as Suction/Injection, Magnetic number, Prandtl number, Lewis number, power-law index, Soret and Dufour number. Some of the important findings of the paper are:

(a) Soret number reduces the rate of heat transfer and enhances the rate of mass transfer for non-Newtonian fluids whereas reverse phenomenon is observed for Newtonian fluids.
(b) Power-law index decreases the rate of heat transfer and mass transfer.
(c) Effect of suction is to increase the velocity for Pseudo plastic fluid and to decrease the velocity for Newtonian and Dilatant fluids. Whereas reverse phenomenon is observed in case of injection.
(d) Effect of Prandtl number is to reduce the temperature boundary layer and the effect of Lewis number is decrease the concentration boundary layer.
(e) Increase in Soret number or decrease in the Dufour number is to reduces the temperature and enhance the concentration profiles.

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