Fermi problem and superluminal signals in quantum electrodynamics

Abstract

Using as an example the Fermi problem dealing with nonstationary transformation of optical excitation from one atom to another the reason of superluminal signals appearance in quantum electrodynamics is clearing. It is shown that the calculation using the conventional methods in Heisenberg and Schrödinger representations in nonstationary problems lead to different results. The Schrödinger representation predicts the existents of specified quantum superluminal signals. In Heisenberg representation the superluminal signals are absent. The reason of non-identity of representations is close connected with using of the adiabatic hypothesis.

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1. Introduction

In 1932 year E. Fermi [1] by developing the Dirac theory [2] of quantum transpositions had considered the problem dealing with nonstationary transformation radiation between excited atom and another atom being in its ground state. He calculated the probability of such process as a corresponded matrix element squared. It was shown that the radiation transformation has the retarded character and is described by character construction $t = R / c$. Here t is the time of excitation transformation, R is the distance between atoms, c is the light velocity in vacuum. The result was repeated in many following theoretic papers [3,4] .

The detailed analysis of Fermi calculations performed in the paper [5] had shown that the retarded character of signal defined by formula $t = R / c$ is only the approximation connected with using the pole approximation. More punctual calculations show the appearance of a small superluminal forerunner placed before the classical electromagnetic wave front at a distance of the order of one wave length. The author supposes that such a fact does not have the physical sense. In his paper [5] he tried to proof this fact in general form using the Heisenberg representation.

In paper [6] the Fermi result was analyzed again. The appearance of the superluminal forerunner forces the authors to clean the reason of its appearance and revise the Dirac theory [2] of quantum transpositions. In the paper [6] one postulates the incorrectness of representation of quantum transpositions probability as a square of consequence matric elements. One proposes to evaluate the observed values as quantum average values of consequence quantum operators. Such average values have to be calculated in Heisenberg representation. This way leads to the exact realization of the expression $t = R/c$ and likes the method proposed in [5]. The Schrödinger representation in [6] was not investigated.

In a paper [6] as in a paper [5] the authors using the Heisenberg representation came to the conclusion of impossibility of the appearance in quantum electrodynamics the superluminal signal.

Last years the interest for the optical superluminal signals has risen supplementary. Such signals were discovered in many experimental investigations [7-14]. The necessity of their theoretical description has appeared. All attempts of theory constructions in present days (the fluctuations excluded) deal extremely with the classical representation of the internal structure of electromagnetic field [15-19]. The exception represents the paper [20]. In this work using the interaction representation and non-equality $\left\langle \hat{E}^2 \right\rangle \geq \left\langle \hat{E} \right\rangle^2$, $\hat{E}$ being the strength operator of electromagnetic field, one shows the appearance in electrodynamics of excited media the superluminal signals. Such signals do not have the classical analogs. For the appearance of such signals the inversion population of atom states in media is not necessary. Such superluminal signals were experimentally observed and evidently are in a good coincidence with experimental data [13]. The reason and their appearance conditions in connection with experiments mentioned above are very interesting. In present work such questions are solved using the Fermi problem as example. In such a way one shows that the quantum radiation transfer in quantum electrodynamics at the finite times in Heisenberg and Schrödinger representations are described in different ways. Other words these representations are non-identical. Such result possesses not only the methodic significance. The fact is that the superluminal signals appear only in Schrödinger representation. In Heisenberg representation they are absent. This fact permits to understand the result differences in the calculations using the different methods. Namely this fact opens the possibility for prediction the analogous results in other situations.

We doubt not in the results of calculations in papers [5] and [6] but we doubt in the finite conclusions in these works. In these works the conclusions about the absent of superluminal signals in quantum electrodynamics follows from Heisenberg representation. But in these works the analysis using Schrödinger representation is absent. In the following we shall revise the solutions of Fermi-problem by using both representations. We shall show that nonidentity of Schrödinger and Heisenberg representations in nonstationary quantum's problems is naturally and connected closely with using in quantum electrodynamics the adiabatic hypothesis.

2. The state of the problem

Let us suppose that the test atom (1) being in its ground state is placed at the point $\mathbf{R}_1$ and is attacked by the radiation of excited atom (2) placed in the point $\mathbf{R}_2$. The excited atom begins interact with electromagnetic field at a moment of time t_0. Each atom possesses only one electron. We neglect the spin variables. One supposes the atoms are placed in wave zone at a large distance between them that permits to neglect in the exchange effect and in the longitudinal electromagnetic field. Suppose that each atom possesses only two energetic levels. But these levels may have energetic sublevels. The excited and ground states of primary excited atom (2) are describes consequently by indexes j_{ex} and j_g. The energetic states of no-excited atom (1) are described by indexes i_{ex} and i_g. The Hamiltonian of the problem in Schrödinger representation and quasi-resonant approximation is written in the following form

$$\hat{H} = \hat{H}^0 + \hat{H}', \quad \hat{H}^0 = \int \hat{\psi}_1^+(\mathbf{r}_1)\hat{H}_1\hat{\psi}_1(\mathbf{r}_1)d\mathbf{r}_1 + \int \hat{\psi}_2^+(\mathbf{r}_2)\hat{H}_2\hat{\psi}_2(\mathbf{r}_2)d\mathbf{r}_2 + \hat{H}_{ph}$$

$$\hat{H}' = -\frac{e}{mc}\int \hat{\psi}_1^+(\mathbf{r}_1)\hat{p}_{\mathbf{r}_1}^{\nu_1}\hat{A}^{\nu_1}(\mathbf{r}_1)\hat{\psi}_1(\mathbf{r}_1)d\mathbf{r}_1 - \frac{e}{mc}\int \hat{\psi}_2^+(\mathbf{r}_2)\hat{p}_{\mathbf{r}_2}^{\nu_2}\hat{A}^{\nu_2}(\mathbf{r}_2)\hat{\psi}_2(\mathbf{r}_2)d\mathbf{r}_2\theta(t-t_0), \quad (1)$$

$\theta(t-t_0)$ being the Heaviside step function that fixed the moment of time appearance of the interaction of radiated atom with electromagnetic field. Over the repeated indexes one supposes the summation,

$$\hat{\psi}_1(\mathbf{r}_1) = \sum_i \psi_i(\mathbf{r}_1 - \mathbf{R}_1)\hat{b}_i, \quad \hat{\psi}_2(\mathbf{r}_2) = \sum_j \psi_j(\mathbf{r}_2 - \mathbf{R}_2)\hat{b}_j, \quad \hat{H}_{ph} = \sum_{\mathbf{k}\lambda} \hbar ck\left(\hat{\alpha}_{\mathbf{k}\lambda}^+\hat{\alpha}_{\mathbf{k}\lambda} + \frac{1}{2}\right),$$

$$\hat{A}^{\nu}(\mathbf{r}) = \sum_{\mathbf{k}\lambda}\sqrt{\frac{\hbar c}{2kV}}e_{\mathbf{k}\lambda}^{\nu}\left(\hat{\alpha}_{\mathbf{k}\lambda}e^{i\mathbf{kr}} + \hat{\alpha}_{\mathbf{k}\lambda}^+e^{-i\mathbf{kr}}\right).$$

The waves functions ψ_i and ψ_j denote the behavior of electrons in atoms (1) and (2), $\hat{b}_i^+$ and $\hat{b}_j^+$ denote the electron creations operators at the same states. By $\hat{\alpha}_{\mathbf{k}\lambda}$ and $\hat{\alpha}_{\mathbf{k}\lambda}^+$ the annihilation and creation photon operators in states $(\mathbf{k},\lambda)$ are denoted. Here $\mathbf{k}$ is the photon wave vector, λ is the index of its polarization. The photons have only the transversal polarization λ=(1,2). The rationalized Gauss unite system is used. For the electrons fulfil numbers equal to unity the form of operator commutation relations does not change the finite results. That is why for the sake of simplicity one supposes all the operators being the Bose-Einstein field operators.

Instead of Schrödinger representation it will be convenient to use the equivalent interaction representation. If $\Psi(t)$ is the system wave function in Schrödinger representation than in interaction representation the wave function $\tilde{\Psi}(t)$ has the following view

$$\tilde{\Psi}(t) = \exp\left(i\frac{\hat{H}^0}{\hbar}t \right)\Psi(t) .$$

For the initial state in which the atom (1) is in its ground state and atom (2) is in excited state and photons are absent the view of wave function is the following

$$\tilde{\Psi}^0 = \hat{b}_{i_g}^+ \hat{b}_{j_{ex}}^+ \left|0\right\rangle ,$$

where $\left|0\right\rangle$ being the wave function of vacuum state. If the photon field differs from the vacuum state and any conglomerate of free photons with fulfil numbers $\mathbf{N}(\mathbf{k}) = ..., N_{\mathbf{k}\lambda}, ...$ is present than the wave function of such state will be denoted as $\left|\mathbf{N}(\mathbf{k})\right\rangle$. After the appearance in space of excited atom (2) the wave function $\tilde{\Psi}(t)$ of total system at any moment of time $t > t_0$ may be expressed as a set over the self-functions of $\hat{H}^0$ operator

$$\tilde{\Psi}(t) = \sum_{ij} c_{ij}^{(1)}(t)\hat{b}_i^+\hat{b}_j^+\left|0\right\rangle + \sum_{ij\mathbf{N}(\mathbf{k})} c_{ij}^{(2)}\left(t,\mathbf{N}(\mathbf{k})\right)\hat{b}_i^+\hat{b}_j^+\left|\mathbf{N}(\mathbf{k})\right\rangle . \qquad (2)$$

The summation over $\mathbf{N}(\mathbf{k})$ means the summation over all possible photon field conglomerates. We are interested in the probability of exciting of atom (1) at a moment of time $t > t_0$. According to Dirac theory [2] the condition probability of such event by the transition at the same time of atom (2) at its ground state and at the absence of free photons in space is $\left|c^{(1)}_{i_{ex}j_g}(t)\right|^2$. The condition probability of exciting (1) atom at a presence in space photons in state $\left|\mathbf{N}(\mathbf{k})\right\rangle$ is $\left|c^{(2)}_{i_{ex}j_g}(t,\mathbf{N}(\mathbf{k}))\right|^2$. The total probability $P_{i_{ex}}(t)$ of the exciting of test atom (1) is the sum of condition probabilities

$$P_{i_{ex}}(t)=\sum_j\left|c^{(1)}_{i_{ex}j}(t)\right|^2+\sum_{j\mathbf{N}(\mathbf{k})}\left|c^{(2)}_{i_{ex}j}(t,\mathbf{N}(\mathbf{k}))\right|^2 \qquad (3)$$

One may use the other way and look for probability under consideration as a mean number of excited atoms in the state with energy $\varepsilon_{i_{ex}}$ if in system only one atom is present

$$P_{i_{ex}}(t)=\left\langle\tilde{\Psi}(t)\left|\hat{b}^{+}_{i_{ex}}\hat{b}_{i_{ex}}\right|\tilde{\Psi}(t)\right\rangle. \qquad (4)$$

If Fermi had used [1] the formula (3) then in paper [6] one utilizes the formula (4). Both calculations have to lead to one and the same result since the acquaintance (3) follows from acquaintance (4) after introduction in it of the expression (2). The reason of results discrepancy in papers [2] and [6] is another. It is analyzed further.

Let us say that the square of matrix element $\left|c^{(1)}_{i_{ex}j_g}(t)\right|^2$ describes the probability of the excitation of (1) atom in coherent channel of atoms interaction. In this channel as a result of coherent process of reaction in space the free photons do not appear. Let us name the other channels of (1) atom excitation as no coherent. It follows from (3) that coherent channel of (1) atom excitation gives opportunity to estimate from the low value the total excitation probability of (1) atom

$$P_{i_{ex}}(t)\geq\left|c^{(1)}_{i_{ex}j_g}(t)\right|^2.$$

In Fermi’s paper [1] the right side of this inequality is calculated. As it has shown in [5] the result of such calculation includes inside it the superluminal signal. Such signal can’t be compensated by more precisely calculations.

If the probability of (1) atom excitation is calculated using formula (4) and interaction representation is used then one comes across the formula (3) describing the presence of superluminal forerunner. On the other words the interaction representation with necessity predicts the appearance of superluminal forerunner. According to the paper [5] in Heisenberg representation the superluminal signals never appear. We state the none-identity of Heisenberg and Schrödinger representations in quantum electrodynamics of nonstationary processes. The reason of such none- identity is investigated later.

Later we shall use the other arguments which also lead to the conclusion on none-identity of these representations and permit at the same time to clean the reason of none-identity appearance.

In order to solve such problem let us calculate the scalar product (4) in both interaction and Heisenberg representations. At the same time we shall pay attention on the reason of the discrepancy in such calculation results.

3.Interaction representation

The probability of (1) atom excitation in a form of scalar product (4) permits to calculate of such product in any arbitrary quantum electrodynamics representation. In this paragraph we use the interaction representation. The Schrödinger equation in Schrödinger representation using the Hamiltonian (1) has a view

$$i\hbar\frac{\partial\Psi(t)}{\partial t}=\hat{H}\Psi(t).$$

In interaction representation the same equation has a form

$$i\hbar\frac{\partial\tilde{\Psi}(t)}{\partial t}=\hat{H}'(t)\tilde{\Psi}(t), \qquad (5)$$

where

$$\hat{H}'(t)=-\frac{e}{mc}\int\hat{\psi}_1^+(x_1)\hat{p}_{\mathbf{r}_1}^{\nu_1}\hat{A}^{\nu_1}(x_1)\hat{\psi}_1(x_1)d\mathbf{r}_1-\frac{e}{mc}\int\hat{\psi}_2^+(x_2)\hat{p}_{\mathbf{r}_2}^{\nu_2}\hat{A}^{\nu_2}(x_2)\hat{\psi}_2(x_2)d\mathbf{r}_2\theta(t-t_0), \qquad (6)$$

$$\hat{\psi}_1(x_1)=\sum_i\psi_i(\mathbf{r}_1-\mathbf{R}_1)\hat{b}_i e^{-i\frac{\varepsilon_i}{\hbar}t}, \quad \hat{\psi}_2(x_2)=\sum_j\psi_j(\mathbf{r}_2-\mathbf{R}_2)\hat{b}_j e^{-i\frac{\varepsilon_j}{\hbar}t},$$

$$\hat{A}^{\nu}(x)=\sum_{\mathbf{k}\lambda}\sqrt{\frac{\hbar c}{2kV}}e^{\nu}_{\mathbf{k}\lambda}\left(\hat{\alpha}_{\mathbf{k}\lambda}e^{i\mathbf{kr}-ikct}+\hat{\alpha}^+_{\mathbf{k}\lambda}e^{-i\mathbf{kr}+ikct}\right).$$

Here ε_i and ε_j are the atom internal energies in consequence quantum states, $x=\{\mathbf{r},t\}$. The solution of equation (5) has a view

$$\tilde{\Psi}(t)=\hat{S}(t)\tilde{\Psi}^0,\quad \hat{S}(t)=\hat{T}\left(\frac{1}{i\hbar}\int_{-\infty}^{t}\hat{H}'(t')dt'\right),$$

$\hat{T}$ being chronological operator. The transformation of excitation from one atom to another in the lowest order of perturbation theory is defined by the forth order $\propto e^4$. For such goal due to (4) the matrix $\hat{S}(t)$ has to be evaluated in the third order

$$\hat{S}(t)=1+\hat{S}^{(1)}(t)+\hat{S}^{(2)}(t)+\hat{S}^{(3)}(t)\,, \tag{7}$$

$$\hat{S}^{(1)}(t)=\frac{1}{i\hbar}\int_{-\infty}^{t}\hat{H}'(t')dt'\,,\hat{S}^{(2)}(t)=\frac{\hat{T}}{2!}\left(\frac{1}{i\hbar}\int_{-\infty}^{t}\hat{H}'(t')dt'\right)^2,\hat{S}^{(3)}(t)=\frac{\hat{T}}{3!}\left(\frac{1}{i\hbar}\int_{-\infty}^{t}\hat{H}'(t')dt'\right)^3. \tag{8}$$

The operators $\hat{S}^{(1)}(t)$ and $\hat{S}^{(3)}(t)$ describe no-coherent channels of reactions in which in space the excited atom (1) and free photons are present. The coherent channel of atom (1) excitation is described by operator $\hat{S}^{(2)}(t)$. The introduction (7) into (4) shows that

$$P_{i_{ex}}(t)=\left\langle \hat{S}^{(2)}(t)\right|\hat{b}^+_{i_{ex}}\hat{b}_{i_{ex}}\left|\hat{S}^{(2)}(t)\right\rangle+\left\langle \hat{S}^{(1)}(t)+\hat{S}^{(3)}(t)\right|\hat{b}^+_{i_{ex}}\hat{b}_{i_{ex}}\left|\hat{S}^{(1)}(t)+\hat{S}^{(3)}(t)\right\rangle.$$

Let us calculate $\hat{S}^{(2)}(t)$. The introduction (6) into (8) leads to

$$\hat{S}^{(2)}(t)=\left(\frac{e}{i\hbar mc}\right)^2\int\hat{\psi}_1^+(x_1)\,\hat{p}^{\nu_1}_{\mathbf{r}_1}\hat{\psi}_1(x_1)\hat{\psi}_2^+(x_2)\,\hat{p}^{\nu_2}_{\mathbf{r}_2}\hat{\psi}_2(x_2)\cdot$$

$$\cdot\left[i\hbar D^{\nu_1\nu_2}(x_1,x_2)+\hat{N}\hat{A}^{\nu_1}(x_1)\hat{A}^{\nu_2}(x_2)\right]dx_1dx_2\,. \tag{9}$$

Here we omitted the terms described the atoms self-action, $\hat{N}$ is the normal product operator, $dx=d\mathbf{r}dt$. They used the conventional identity

$$\hat{T}\hat{A}^{\nu_1}(x_1)\hat{A}^{\nu_2}(x_2)=i\hbar D^{\nu_1\nu_2}(x_1,x_2)+\hat{N}\hat{A}^{\nu_1}(x_1)\hat{A}^{\nu_2}(x_2)\,.$$

In its turn

$$D^{\nu_1\nu_2}(x_1,x_2)=D_r^{\nu_1\nu_2}(x_1,x_2)+\Delta^{\nu_1\nu_2}(x_1,x_2)\,, \tag{10}$$

where $D_r^{\nu_1\nu_2}(x_1,x_2)$ is the retarded Green function

$$D_r^{\nu_1\nu_2}(x_1,x_2)=\frac{1}{i\hbar}\left[\hat{A}^{\nu_1}(x_1);\hat{A}^{\nu_2}(x_2)\right]\theta(t_1-t_2)=-\frac{\delta_{\nu_1\nu_2}-n^{\nu_1}n^{\nu_2}}{4\pi\left|\mathbf{r}_1-\mathbf{r}_2\right|}\delta\left(t_1-t_2-\frac{\left|\mathbf{r}_1-\mathbf{r}_2\right|}{c}\right). \qquad (11)$$

One supposes that the points $\mathbf{r}_1$ and $\mathbf{r}_2$ are divided by the wave radiation zone, $n^{\nu}=\left(\mathbf{r}_1-\mathbf{r}_2\right)^{\nu}/\left|\mathbf{r}_1-\mathbf{r}_2\right|$. Further

$$\Delta^{\nu_1\nu_2}(x_1,x_2)=\frac{1}{ih}\left\langle 0\right|\hat{A}^{\nu_1}(x_1)\hat{A}^{\nu_2}(x_2)\left|0\right\rangle=-\frac{ic}{4\pi^2}\frac{\delta_{\nu_1\nu_2}-n^{\nu_1}n^{\nu_2}}{\left|\mathbf{r}_1-\mathbf{r}_2\right|}\int_0^{\infty}e^{ikc(t_1-t_2)}\sin k\left|\mathbf{r}_1-\mathbf{r}_2\right|dk\,. \qquad (12)$$

The term in (9) containing the operator $\hat{N}$ describes the no-coherent channel of reaction. In this channel besides an excited atom (1) the two free photons appear in space. The probability of such reaction is described by one of terms in the late sum in (3). This process we omit. In coherent channel according to (9)

$$\hat{S}^{(2)}(t)=\hat{S}_1^{(2)}(t)+\hat{S}_2^{(2)}(t)\,. \qquad (13)$$

The first term contains function $D_r^{\nu_1\nu_2}$ while the second one contains the function $\Delta^{\nu_1\nu_2}$. The introduction (11) and (12) and (9) yields

$$\hat{S}_1^{(2)}(t)=\frac{1}{i\hbar}\left(\frac{e}{mc}\right)^2\int_{-\infty}^{t}p_{i_{ex}i_g}^{\nu_1}\exp\left(i\frac{\varepsilon_{i_{ex}}-\varepsilon_{i_g}}{\hbar}t_1\right)\cdot$$

$$\cdot\hat{b}_{i_{ex}}^{+}\hat{b}_{i_g}\hat{b}_{j_g}^{+}\hat{b}_{j_{ex}}\int_{-\infty}^{t}p_{j_g j_{ex}}^{\nu_2}\exp\left(-i\frac{\varepsilon_{j_{ex}}-\varepsilon_{j_g}}{\hbar}t_2\right)D_r^{\nu_1\nu_2}\left(\mathbf{R}_1,\mathbf{R}_2,t_1,t_2\right)\theta(t_2-t_0)dt_1dt_2\,,$$

$$\hat{S}_2^{(2)}(t)=-\frac{1}{\hbar}\left(\frac{e}{mc}\right)^2\frac{c}{4\pi^2}\int_{-\infty}^{t}p_{i_{ex}i_g}^{\nu_1}\exp\left(\frac{\varepsilon_{i_{ex}}-\varepsilon_{i_g}}{\hbar}t_1\right)\cdot$$

$$\cdot\hat{b}_{i_{ex}}^{+}\hat{b}_{i_g}\hat{b}_{j_g}^{+}\hat{b}_{j_{ex}}\int_{-\infty}^{t}p_{j_g j_{ex}}^{\nu_2}\exp\left(-i\frac{\varepsilon_{j_{ex}}-\varepsilon_{j_g}}{\hbar}t_2\right)\int_0^{\infty}\frac{\delta_{\nu_1\nu_2}-n^{\nu_1}n^{\nu_2}}{\left|\mathbf{R}_1-\mathbf{R}_2\right|}\sin k\left|\mathbf{R}_1-\mathbf{R}_2\right|e^{ikc(t_1-t_2)}dk\,\theta(t_2-t_0)dt_1dt_2$$

,(14)

$$p_{i_g i_{ex}}^{\nu}=\int\psi_{i_g}^{*}(\boldsymbol{\rho})\hat{p}_{\boldsymbol{\rho}}^{\nu}\psi_{i_{ex}}(\boldsymbol{\rho})d\boldsymbol{\rho}\,.$$

The introduction (14) in (4) shows that

$$P_{i_{ex}}(t)=\left\langle \hat{S}_1^{(2)}+\hat{S}_2^{(2)}\left|\hat{b}_{i_{ex}}^{+}\hat{b}_{i_{ex}}\right|\hat{S}_1^{(2)}+\hat{S}_2^{(2)}\right\rangle_0+\left\langle \hat{S}^{(1)}+\hat{S}^{(3)}\left|\hat{b}_{i_{ex}}^{+}\hat{b}_{i_{ex}}\right|\hat{S}^{(1)}+\hat{S}^{(3)}\right\rangle_0 . \qquad (15)$$

The quantum averaging process in this equality is performed over initial state of system. The operator $\hat{S}_1^{(2)}(t)$ does not contain superluminal forerunner while in operator $\hat{S}_2^{(2)}(t)$ such forerunner is present.

4. Heisenberg representation

The transposition from Schrödinger representation to the Heisenberg representation is performed by operator $\hat{U}(t)$ satisfying the equation

$$i\hbar\frac{\partial\hat{U}(t)}{\partial t}=\left(\hat{H}^0+\hat{H}'\right)\hat{U}(t). \qquad (16)$$

The field operators in Heisenberg representation have a view

$$\check{\psi}(x)=\hat{U}^{+}(t)\hat{\psi}(\mathbf{r})\hat{U}(t),\ \check{A}^{\nu}(x)=\hat{U}^{+}(t)\hat{A}^{\nu}(\mathbf{r})\hat{U}(t),\ \check{b}_{i_{ex}}(t)=\hat{U}^{+}(t)\hat{b}_{i_{ex}}\hat{U}(t).$$

The differential equation (16) may be transformed to the integral one

$$\hat{U}(t)=\hat{U}^0(t)+\frac{1}{i\hbar}\hat{U}^0(t)\int_{-\infty}^{t}\hat{U}^0(t')\hat{H}'(t')\hat{U}(t')dt',\ \hat{U}^0(t)=e^{-i\frac{\hat{H}^0}{\hbar}t}.$$

By using twice the iterative procedure we obtain [21] the following expression for the operator $\check{b}_{i_{ex}}(t)$

$$\check{b}_{i_{ex}}(t)=\hat{b}_{i_{ex}}(t)+\frac{1}{i\hbar}\int_{-\infty}^{t}\left[\hat{b}_{i_{ex}}(t);\hat{H}'(t')\right]dt'+\frac{1}{(i\hbar)^2}\int_{-\infty}^{t}\int_{-\infty}^{t}\theta(t'-t'')\left[\left[\hat{b}_{i_{ex}}(t);\hat{H}'(t')\right]\hat{H}'(t'')\right]dt'dt''+o(e^3)$$

,

where

$$\hat{b}_{i_{ex}}(t)=\hat{b}_{i_{ex}}e^{-i\frac{\hat{H}^0}{\hbar}t}.$$

By using the explicit form of operators $\hat{H}'(t)$, $\hat{\psi}(x)$ and $\hat{\psi}^{+}(x)$ in dipole approximation one yields

$$\check{b}_{i_{ex}}(t)=\hat{b}_{i_{ex}}\, e^{-i\frac{\varepsilon_{i_{ex}}}{\hbar}t}-\frac{e}{i\hbar mc}e^{-i\frac{\varepsilon_{i_{ex}}}{\hbar}t}\int_{-\infty}^{t} p^{\nu_1}_{i_{ex}i_g}\exp\left(i\frac{\varepsilon_{i_{ex}}-\varepsilon_{i_g}}{\hbar}t'\right)\hat{A}^{\nu_1}(\mathbf{R}_1,t')dt'\hat{b}_{i_g}+\left(\frac{e}{i\hbar mc}\right)^2 e^{-i\frac{\varepsilon_{i_{ex}}}{\hbar}t}\cdot$$

$$\cdot\int_{-\infty}^{t}\int_{t_0}^{t} p^{\nu_1}_{i_{ex}i_g}\exp\left(i\frac{\varepsilon_{ex}-\varepsilon_{i_g}}{\hbar}t'\right)p^{\nu_2}_{j_g j_{ex}}\exp\left(-i\frac{\varepsilon_{j_{ex}}-\varepsilon_{j_g}}{\hbar}t''\right)D^{\nu_1\nu_2}_r\left(\mathbf{R}_1,t',\mathbf{R}_2,t''\right)dt'dt''\hat{b}_{i_g}\hat{b}^+_{j_g}\hat{b}_{j_{ex}}+o(e^3).$$

Now it is evident that

$$P_{i_{ex}}(t)=\left\langle \check{b}^+_{i_{ex}}(t)\left[-\frac{e}{i\hbar mc}e^{-i\frac{\varepsilon_{i_{ex}}}{\hbar}t}\int_{-\infty}^{t} p^{\nu_1}_{i_{ex}i_g}\exp\left(i\frac{\varepsilon_{i_{ex}}-\varepsilon_{i_g}}{\hbar}t'\right)\hat{A}^{\nu_1}(\mathbf{R}_1,t')dt'\hat{b}_{i_g}+\left(\frac{e}{i\hbar mc}\right)^2 e^{-i\frac{\varepsilon_{i_{ex}}}{\hbar}t}\cdot\right.\right.$$

$$\left.\left.\cdot\int_{-\infty}^{t}\int_{t_0}^{t} p^{\nu_1}_{i_{ex}i_g}\exp\left(i\frac{\varepsilon_{ex}-\varepsilon_{i_g}}{\hbar}t'\right)p^{\nu_2}_{j_g j_{ex}}\exp\left(-i\frac{\varepsilon_{j_{ex}}-\varepsilon_{j_g}}{\hbar}t''\right)D^{\nu_1\nu_2}_r\left(\mathbf{R}_1,t',\mathbf{R}_2,t''\right)dt''dt'\hat{b}_{i_g}\hat{b}^+_{j_g}\hat{b}_{j_{ex}}+o(e^3)\right]\right\rangle_0$$

.(17)

Here the quantum averaging is performed over initial state of system.

5. The discussing of the results

The formulae (15) and (17) being calculated in different representations describe one and the same probability $P_{i_{ex}}(t)$. If in (15) the omitted term containing $\hat{N}$ is reconstructed then in $\propto e^4$ approximation (15) and (17) evidentially would be equal. But in the present forms they are senseless since they contain in infinite limits the integrals from oscillated functions. It is necessary to use the adiabatic hypothesis [22]. We stress that for the equality (15) and (17) expressions it is necessary to take into account all the terms proportional to $\propto e^4$, and among them the terms following from the product of first order term on the third one. If such terms are neglected in such sum, that is necessary for coinciding with adiabatic hypothesis, then the results will be different.

The detail analysis we began from formula (17) obtained in Heisenberg representation. The first term in this formula which is proportional to $\propto e^2$ describes the (1) atom excitation due to its interaction with electromagnetic vacuum. Such fact of not equality to zero the probability in question contradicts to the initial condition $\hat{b}^+_{i_g}\hat{b}^+_{j_{ex}}\left|0\right\rangle$. Besides this fact the electromagnetic vacuum cannot excite the atom being in its ground state according to the physical understanding. The probability of such processes has to be equal to zero. In conventional quantum electrodynamics such excitation is absent since it contradicts the low of energy conservation. The low of energy conservation follows from the adiabatic hypothesis that is additionally putted on the solutions of quantum electrodynamics. Mathematically this hypothesis is expresses by the equality

$$\delta(\omega-\omega_0)=\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(\omega-\omega_0)t}dt\,,$$

$\delta(\omega-\omega_0)$ being Dirac function. In its turn this equality demands the integration over the time in infinite limits. Only the additional using of adiabatic hypothesis turns the set of perturbation theory to the physically sense. But in the problem under consideration the using of adiabatic hypothesis in its usual form is impossible since the variable t is finite. On the other hand the atom (1) before the interaction with excited atom (2) was in its ground state the infinitely long time interval $\left(-\infty \div t\right)$ permanently interacting with electromagnetic vacuum. The time length of the interaction interval from the physically point of view is infinitely long. We use this circumstance to investigate of the problematic right side term in (17)

$$\int_{-\infty}^{t}\hat{A}^{\nu}(\mathbf{R}_1,t')\exp\left(i\frac{\varepsilon_{i_{ex}}-\varepsilon_{i_g}}{\hbar}t'\right)dt'=$$

$$=\sum_{\mathbf{k}\lambda}\int_{-\infty}^{t}\sqrt{\frac{\hbar c}{2kV}}e^{\nu}_{\mathbf{k}\lambda}\exp\left(i\frac{\varepsilon_{i_{ex}}-\varepsilon_{i_g}}{\hbar}t'\right)\left(\hat{\alpha}_{\mathbf{k}\lambda}e^{i\mathbf{k}\mathbf{R}_1-ikct'}+\hat{\alpha}^{+}_{\mathbf{k}\lambda}e^{-i\mathbf{k}\mathbf{R}_1+ikct'}\right)dt'\,. \tag{18}$$

It is necessary to pay attention to the fact that the probability of excitation transposition between (2) and (1) atoms does not depend on the time t but only on the time difference $t-t_0$. Taken into account that the interaction of the atom (1) with electromagnetic field up to the time t_0 has the infinitely long duration it necessary to pose that the physical mining the expression (17) has only in the limit $t\to\infty$. At the same time the difference $t-t_0$ rests constant (general adiabatic hypothesis). Now from (18) yields

$$\lim_{t\to\infty}\int_{-\infty}^{t}\hat{A}^{\nu}(\mathbf{R}_1,t')\exp\left(i\frac{\varepsilon_{i_{ex}}-\varepsilon_{i_g}}{\hbar}t'\right)dt'=$$

$$=2\pi\sum_{\mathbf{k}\lambda}\sqrt{\frac{\hbar c}{2kV}}e^{\nu}_{\mathbf{k}\lambda}\left(\hat{\alpha}_{\mathbf{k}\lambda}e^{i\mathbf{k}\mathbf{R}_1}\delta\left(\frac{\varepsilon_{i_{ex}}-\varepsilon_{i_g}}{\hbar}-kc\right)+\hat{\alpha}^{+}_{\mathbf{k}\lambda}e^{-i\mathbf{k}\mathbf{R}_1}\delta\left(\frac{\varepsilon_{i_{ex}}-\varepsilon_{i_g}}{\hbar}+kc\right)\right).$$

This expression carries in the result the zero contribution since the free photons are absent in space. The vacuum term transforms into zero due to the energy conservation low. Now it is evident that the product of the first term of perturbation theory by the third one also turns into zero. In approximation $\propto e^4$ only one term rests

$$P_{i_{ex}}(t)=$$

$$=\frac{1}{\hbar^2}\left(\frac{e}{mc}\right)^4\left|\int_{-\infty}^{t}\int_{t_0}^{t} p^{\nu_1}_{i_{ex}i_g}\exp\left(i\frac{\varepsilon_{ex}-\varepsilon_{i_g}}{\hbar}t'\right)p^{\nu_2}_{j_g j_{ex}}\exp\left(-i\frac{\varepsilon_{j_{ex}}-\varepsilon_{j_g}}{\hbar}t''\right)D^{\nu_1\nu_2}_r\left(\mathbf{R}_1,t',\mathbf{R}_2,t''\right)dt''dt'\right|^2 . \quad (19)$$

This result found in Heisenberg representation being equal to the result of paper [6] does not contain of the superluminal forerunners. This result may be explained as the one photon radiation by the atom (2) at time moment t'' and its absorption by the atom (1) at a time moment t'. The propagator

$$D^{\nu_1\nu_2}_r(\mathbf{R}_1,\mathbf{R}_2,t',t'')\sim\delta\left(t'-t''-\frac{\left|\mathbf{R}_1-\mathbf{R}_2\right|}{c}\right)$$

points out the condition $c\left(t'-t''\right)=\left|\mathbf{R}_1-\mathbf{R}_2\right|$.

In the interaction representation we came across the same mathematical problem by calculation the operator (8)

$$\hat{S}^{(1)}(t)=-\frac{e}{i\hbar mc}\int\hat{\psi}^+_1(x_1)\hat{p}^{\nu}_{\mathbf{r}_1}\hat{A}^{\nu}(\mathbf{R}_1,t_1)\hat{\psi}_1(x_1)d\mathbf{r}_1dt_1=$$

$$=-\frac{e}{i\hbar mc}\int_{-\infty}^{t}p^{\nu}_{i_{ex}i_g}\exp\left(i\frac{\varepsilon_{i_{ex}}-\varepsilon_{i_g}}{\hbar}t'\right)\hat{A}^{\nu}(\mathbf{R}_1,t')dt' .$$

In the limit $t\to\infty$ by condition $t-t_0=const$ one gets $\hat{S}^{(1)}(t)\to 0$ if in space the free photons are absent. Now in (15) one gets $\left\langle\hat{S}^{(1)}(t)\hat{b}^+_{i_{ex}}\hat{b}_{i_{ex}}\hat{S}^{(3)}(t)\right\rangle_0=0$.

Let us consider now the operator $\hat{S}^{(2)}_2(t)$. In this operator according (14) integration over intermedia variables t_1 captures the area $t_1<t_0$. Let us divide the integral over t_1 in (14) by the sum of two integrals

$$\int_{-\infty}^{t_0} \exp\left(i\frac{\varepsilon_{i_{ex}} - \varepsilon_{i_g} + kc\hbar}{\hbar} t_1 \right) dt_1 + \int_{t_0}^{t} \exp\left(i\frac{\varepsilon_{i_{ex}} - \varepsilon_{i_g} + kc\hbar}{\hbar} t_1 \right) dt_1 .$$

But the limit transition $t \to \infty$ if $t - t_0 = const$ demands the limit transition $t_0 \to \infty$. In this case the first integral transforms in Dirac δ-function $\delta\left(kc + \left(\varepsilon_{i_{ex}} - \varepsilon_{i_g}\right)/\hbar\right)$ which is equal to zero due to the positive value of its argument. The expression (15) describing the probability of atom (1) excitation in approximation $\propto e^4$ is now rewritten in the following view

$$P_{i_{ex}}(t) = \frac{1}{\hbar^2}\left(\frac{e}{mc}\right)^4 \Bigg| \int_{t_0}^{t} p^{\nu_1}_{i_{ex} i_g} \exp\left(i\frac{\varepsilon_{i_{ex}} - \varepsilon_{i_g}}{\hbar} t_1 \right) \int_{t_0}^{t} p^{\nu_2}_{j_g j_{ex}} \exp\left(-i\frac{\varepsilon_{j_{ex}} - \varepsilon_{j_g}}{\hbar} t_2 \right) \cdot \tag{20}$$

$$\cdot \left[D^{\nu_1\nu_2}_r(\mathbf{R}_1, \mathbf{R}_2, t_1, t_2) + \frac{i}{8\pi^2}\frac{\delta_{\nu_1\nu_2} - n^{\nu_1}n^{\nu_2}}{|\mathbf{R}_1 - \mathbf{R}_2|}\left(\frac{1}{t_1 - t_2 - \frac{|\mathbf{R}_1 - \mathbf{R}_2|}{c} + i0} - \frac{1}{t_1 - t_2 + \frac{|\mathbf{R}_1 - \mathbf{R}_2|}{c} + i0} \right) \right] dt_1 dt_2 \Bigg|^2$$

.

Here the first term coincides with the result (19) obtained in Heisenberg representation. The second one describes the signals placed in superluminal zone at a distance of the order of one wave length that coincides with corrective Fermi calculations. In the limit $t_0 \to -\infty$, $t \to \infty$ the second term turns into zero due to integrands analytical properties. By this reason in stationary problems the representations Schrödinger and Heisenberg are identical. In nonstationary conditions formulae (20) and (19) calculated in different representations are not coincide.

Other words the using of the general adiabatic hypothesis leads to non-equivalency of Schrödinger and Heisenberg representations in non-stationary quantum electrodynamics. We stress that the Schrödinger representation permits the appearance of superluminal forerunners.

The existence of the superluminal signals does not break [23] the causality principle. It is necessary the causality principle to understand in the following form: the consequences can't act on their reasons. The Lorentz invariance of quantum electrodynamics equations is not the obstacle for superluminal signals appearance.

6. Conclusion

In this work the non-stationary processes of transformation excitation from one atom to another is considered. The result of Fermi work in which the matrix element for such process was calculated permits to think about the principal presence in nature the superluminal signals. The repeated calculation of this process probability performed by using Heisenberg representation leaded to the conclusion of the absence of superluminal signals in quantum theory. In the same work they postulated the no corrections of quantum transposition calculation as a square of corresponded matrix element. The other words they doubt about the Dirac theory of quantum transpositions.

It is shown in present work that the calculation of quantum transposition probability as matrix elements squared (Dirac's method) or as quantum average of corresponded quantum operators lead to identity results if last calculations are performed in Schrödinger representation.

Different results mentioned above are not the consequences of different probabilities definition. The results different is the consequences of non-identity Schrödinger and Heisenberg representation in quantum electrodynamics of nonstationary processes. As a proof of non-identity representations in present work the probability of test atom excitation by spontaneous radiation of another atom expressed through quantum averaging of corresponded operators is calculated. The calculations of such quantum averaging are performed by both Schrödinger and Heisenberg representations leading to the different results. The representations none-identity follows finely from the no correct definition of scattering matrix $\hat{S}(t)$ creating the connection of interaction (Schrödinger) and Heisenberg representations. Since the product $\hat{S}(t)\left|\Phi\right\rangle$ where Φ is arbitrary wave function in quantum electrodynamic is represented as a divergent set then is non astonishing that the different summation set methods lead to different results. By using of the formal properties of $\hat{S}(t)$ operator the sets of perturbation theory obtained in Schrödinger and Heisenberg representations at first glance are equal. But such sets do not represent meaningful solutions of quantum electrodynamics. In order to put them the physical sense it is necessary to use the adiabatic hypothesis which supposes switching and shutting off the interaction at $t \to \pm\infty$ .This hypothesis mathematically expressed by using the following equality

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(\omega-\omega_0)t'}dt' = \delta(\omega-\omega_0) \qquad (21).$$

By investigating of quantum transitions at finite time intervals it is not possible to use the conventional adiabatic hypothesis. Instead this hypothesis it is necessary to use its generation in the form

$$\lim_{t\to\infty} \frac{1}{2\pi}\int_{-\infty}^{t} e^{i(\omega-\omega_0)t'}dt' = \delta(\omega-\omega_0)\,.$$

At the same time as in conventional quantum electrodynamics it is necessary to watch the order of carry out the mathematical operations. First of all it is necessary to carry out the limit transition (21) and only then to carry out the quantum operation of summation $\langle ... \rangle$. After using the general adiabatic hypothesis the sets of perturbation theory lead to reasonable results. But such results obtained in Schrödinger and Heisenberg representations are different. The difference may appear already in the terms proportional to $\propto e^4$.

The none-identity of representations is worth in practical aspect. As is shown above the Schrödinger representation predicts the presents in the nature of specific quantum superluminal signals. The Heisenberg representation cannot describe the superluminal processes at all. In connection with experimentally observed superluminal phenomena such property of Schrödinger representation possesses the real interest. Due to none-identity of Schrödinger and Heisenberg representations the theories using these representations have to be considered as two mutual non-connecting theories. The physical systems in which the matrix $\hat{S}(t)$ is well definite are quasi-classical in the sense of non-possibility inside them the superluminal signals. The Schrödinger and Heisenberg representations for such systems are identical. In general case the choice of one of these representations only the experiment may show. At present time only one such experiment is known which shows on Schrödinger representation and predicts at the same time the existence in quantum electrodynamics the superluminal signals.

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