Bouncing Behavior of Kaluza-Klein Cosmological Model in General Relativity

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ABSTRACT

Kaluza-Klein cosmological model has been obtained in the general theory of relativity. The source for energy-momentum tensor is assumed a perfect fluid. The field equations have been solved by using a special form of the average scale factor $R(t) = \left((t-t_0)^2 + \frac{t_0}{1-\beta} \right)^{\frac{1}{1-\beta}}$ proposed by Cai *et al.* The physical properties and the bouncing behavior of the model are also discussed.

1. INTRODUCTION

According to recent cosmological observations in terms of Supernovae Ia [1-2], large scale structure [3-4] with the baryon acoustic oscillations [5], cosmic microwave background radiations [6-8], and weak lensing [9], the current expansion of the universe is accelerating and homogeneous. At the present time, the cosmic acceleration is explained by two ways: One is the introduction of the so called dark energy with negative pressure in general relativity and the other is the modification of gravity like f(R) gravity, f(t) gravity, f(R,T) gravity etc. on the large distances.

The solution of the singularity problem of the standard Big Bang cosmology is known as bouncing universe. A bouncing universe with an initial contraction to a non-vanishing minimal radius and then subsequent an expanding phase provides a possible solution to the singularity problem of the standard Big Bang cosmology. Moreover, for the universe entering into the hot Big Bang era after the bouncing, the equation of state (EoS) of the matter content ω in the universe must transit from $\omega < -1$ to $\omega > -1$. In the contracting phase, the scale factor $R(t)$ is decreasing, this means $\dot{R}(t) < 0$, and in the expanding phase, scale factor $\dot{R}(t) > 0$. Finally at the bouncing point, $\dot{R}(t) = 0$ and near this point $\ddot{R}(t) > 0$, for a period of time. It is also discussed with other view that in the bouncing cosmology, the Hubble parameter H passes across zero $(H = 0)$ from $H < 0$ to $H > 0$. Cai *et al.* have investigated bouncing universe with quintom matter. He showed that a bouncing universe has an initial narrow state by a minimal radius and then develops to an expanding phase [10]. This means for the universe arriving to the Big-bang era after the bouncing, the EoS parameter should crossing from $\omega < -1$ to $\omega > -1$. Sadatian [11] have studied rip singularity scenario and bouncing universe in a Chaplygin gas dark energy model. Recently, Bamba *et al.* [12] have investigated bounce cosmology from $f(R)$ gravity and $f(R)$ bi-gravity. Astashenok [13] has studied effective energy models and dark energy models with bounce

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42   in frames of $f(T)$ gravity. Solomans *et al.* [14] have investigated bounce behavior in
43   Kantowski-Sach and Bianchi cosmology. Silva *et al.* [15] have studied bouncing solutions in
44   Rastall's theory with a barotropic fluid. Brevik and Timoshkin [16] have obtained
45   inhomogeneous dark fluid and dark matter leading to a bounce cosmology. Singh *et al.* [17]
46   have studied k-essence cosmologies in Kantowski-Sachs and Bianchi space times.

47   The Kaluza-Klein theory [18-19] was introduced to unify Maxwell's theory of
48   electromagnetism and Einstein's gravity theory by adding the fifth dimension. Due to its
49   potential function to unify the fundamental interaction, Kaluza-Klein theory has been
50   regarded as a candidate of fundamental theory. Ponce [20], Chi [21], Fukui [22], Liu and
51   Wesson [23], Coley [24] have studied Kaluza-Klein cosmological models with different
52   contexts. Adhav et al. [25] have obtained Kaluza-Klein inflationary universe in general theory
53   of relativity. Reddy *et al.* [26] have discussed a five dimensional Kaluza- Klein cosmological
54   model in the presence of perfect fluid in $f(R,T)$ gravity. Ranjeet *et al.* [27] have studied
55   variable modified Chaplygin gas in anisotropic universe with Kaluza- Klein metric. Katore *et*
56   *al.* [28] have obtained Kaluza-Klein cosmological model for perfect fluid and dark energy.
57   Ram and Priyanka [29] have presented some Kaluza-Klein cosmological models in $f(R,T)$
58   gravity theory. Sahoo *et al.* [30] have investigated Kaluza-Klein cosmological model in
59     $f(R,T)$ gravity with $\lambda(T)$. Recenty, Reddy *et al.* [31] have studied Kaluza-Klein minimally
60   interacting holographic dark energy model in a scalar tensor theory of gravitation. Ghate
61   and Mhaske [32] have investigated Kaluza-Klein barotropic cosmological model with varying
62   gravitational constant G in creation field theory of gravitation.
63   In this paper, Bouncing behavior of Kaluza-Klein cosmological model has been studied in the
64   general theory of relativity. This work is organized as follows: In section 2, the metric and
65   field equations have been presented. The field equations have been solved in section 3 by
66   using the physical condition that the expansion scalar θ is proportional to shear scalar σ

67   and the special form of average scale factor $R(t) = \left[(t-t_0)^2 + \dfrac{t_0}{1-\beta} \right]^{\frac{1}{1-\beta}}$ proposed by Cai *et al.*

68   [10]. The physical and geometrical behavior of the model have been discussed in section 4 .
69   In the last section 5, concluding remarks have been expressed.
70

71   **2. METRIC AND FIELD EQUATIONS**

72   Five dimensional Kaluza-Klein metric is considered in the form
73     $$ds^2 = dt^2 - A(t)^2 (dx^2 + dy^2 + dz^2) - B(t)^2 d\psi^2 , \tag{1}$$
74   where $A(t)$ and $B(t)$ are functions of cosmic time t and the fifth coordinate ψ is taken to be
75   space-like.
76   The energy-momentum tensor when the source for energy is assumed a perfect fluid given
77   by
78     $$T_i^j = (\rho + p)u_i u^j - p\delta_i^j , \tag{2}$$
79   where u^i is the flow vector satisfying $g_{ij}u^i u^j = 1.$ Here ρ is the total energy density of perfect
80   fluid and p is the corresponding pressure. For the perfect fluid, p and ρ are related by and
81   equation of state
82     $$p = \omega\rho , \quad 0 \le \omega \le 1 . \tag{3}$$
83   In co-moving system of coordinates, using equation (2), one can find
84     $$T_0^0 = \rho \text{ and } T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p . \tag{4}$$
85   The Einstein's field equations are given by

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86 $$R_i^j - \frac{1}{2}R g_i^j = -T_i^j .$$ (5)

87 Using equation (2), for the metric (1), the field equations (5) are given by

88 $$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = \rho ,$$ (6)

89 $$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = -\omega\rho$$ (7)

90 $$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} = -\omega\rho ,$$ (8)

91 where an overhead dot represents differentiation with respect to t.

92 The average scalar factor R and volume scalar V are given by

93 $$R^4 = V = A^3 B .$$ (9)

94 The generalized mean Hubble parameter H is defined by

95 $$H = \frac{\dot{R}}{R} = \frac{1}{4}\left(H_x + H_y + H_z + H_\phi\right),$$ (10)

96 where the directional Hubble parameters H_x, H_y, H_z and H_ϕ are given by

97 $$H_x = H_y = H_Z = \frac{\dot{A}}{A}, \qquad H_\phi = \frac{\dot{B}}{B} .$$ (11)

98 The expansion scalar θ and shear scalar σ are given by

99 $$\theta = 4H = \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right),$$ (12)

100 $$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{4} H_i^2 - 4H^2\right].$$ (13)

101 The deceleration parameter (DP) q is defined by

102 $$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right).$$ (14)

103

104 3. SOLUTION OF FIELD EQUATIONS

105 The field equations (6) to (8) are a system of three highly non-linear differential equations in
106 four unknowns A, B, ρ and ω. The system is thus initially undetermined. We need one extra
107 condition for solving the field equations completely.

108 We assume that the expansion (θ) is proportional to shear (σ) This condition leads to

109 $$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \alpha_0\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)$$

110 which yields

111 $$\frac{\dot{B}}{B} = m\frac{\dot{A}}{A} ,$$

112 where α_0 and m are arbitrary constants.

113

114 Above equation, after integration, reduces to

115 $$B = \eta(A)^m ,$$

116 where η is an integration constant.

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118   Here, for simplicity and without loss of generality, we assume that $\eta = 1$.

119   Hence we have

120
$$B = (A)^m, \ (m \neq 1).$$
(15)

121   Collins *et al.* [33] have pointed out that for spatially homogeneous metric, the normal

122   congruence to the homogeneous expansion satisfies that the condition $\dfrac{\sigma}{\theta}$ is constant.

123   In cosmology, The constant deceleration parameter is commonly used by several
124   researchers [34-38], as it duly gives a power law for metric function or corresponding
125   quantity.
126   The motivation to choose time dependent deceleration parameter (DP) is behind the fact that
127   the expansion of the universe was decelerating in the past and accelerating at present as
128   observed by recent observations of Type Ia supernova [1, 2, 39-41] and CMB anisotropies
129   [42-43]. Also, the transition redshift from deceleration expansion to accelerated expansion is
130   about 0.5. Now for a Universe which was decelerating in past and accelerating at the
131   present time, the DP must show signature flipping [44-46]. So, in general, the DP is not a
132   constant but time variable. The motivation to choose the following scale factor is that it
133   provides a time-dependent DP.
134   Under above motivations, we use a special form of deceleration parameter as

135
$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = -1 + \frac{1}{2}\left[(1-\beta) - \frac{t_0}{(t-t_0)^2}\right], \ \beta < 1$$
(16)

136   where R is average scale factor of the universe.
137   This form is proposed by Cai *et al.* [10] and then modified by Sadatian [11].
138   After integration of (16) , we obtain the Hubble parameter as

139
$$H = \frac{\dot{R}}{R} = \frac{2(t-t_0)}{(1-\beta)(t-t_0)^2 + t_0}.$$

140   Integrating twice equation (16), we get the average scale factor which is time dependent
141   given by

142
$$R(t) = \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{1}{1-\beta}}$$
(17)

143   Where t_0 is an initial time and $\beta < 1$ is constant.

144   Solving equations $A = B^m$ and $R(t) = (A^3 B)^{\frac{1}{4}}$, and using (17) we get

145
$$B = \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{4m}{(1-\beta)(m+3)}}.$$
(18)

146   With the help of equation (17), equation (15) takes the form

147
$$A = \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{4}{(1-\beta)(m+3)}}.$$
(19)

148   Using above two equations (18) and (19), the metric (1) takes the form

149
$$ds^2 = dt^2 - \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{8}{(1-\beta)(m+3)}}(dx^2 + dy^2 + dz^2) - \left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^{\frac{8m}{(1-\beta)(m+3)}} d\psi^2.$$
(20)

150   Equation (20) represents Kaluza-Klein cosmological model with time dependent scale
151   factors.

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154 **4. PHYSICAL PROPERTIES OF THE MODEL**

155 The physical quantities such as spatial volume V, Hubble parameter H, expansion scalar

156 θ, mean anisotropy A_m, shear scalar σ^2, energy density ρ, equation of state parameter

157 ω are obtained as follows:

158 The average scale factor is

159
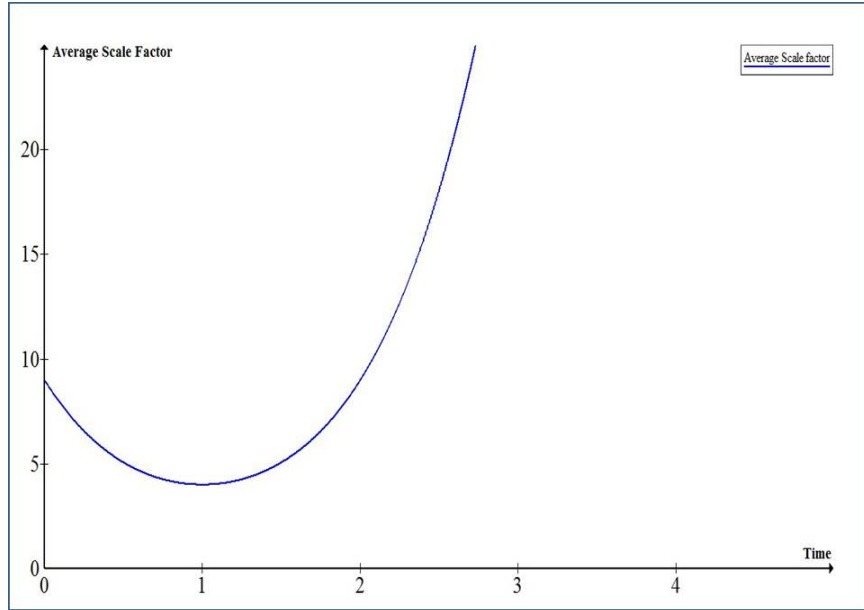$$R(t) = \left[(t - t_0)^2 + \frac{t_0}{1 - \beta} \right]^{\frac{1}{1-\beta}} .$$

160

161
162
163 Fig. 1 Plot of Average scale factor versus time for $\beta = 0.5, t_0 = 1$

164

165 From fig. 1, in the earlier stage, the scale factor is slightly decreasing ($\dot{R}(t) < 0$) and in the

166 expanding phase the scale factor increases rapidly ($\dot{R}(t) > 0$). Hence our model is bouncing

167 at $t = t_0$ ($\dot{R}(t) = 0$).

168 The spatial volume is given by

169
$$V = R^4 = \left[(t - t_0)^2 + \frac{t_0}{1 - \beta} \right]^{\frac{4}{1-\beta}} . \qquad (21)$$

170 The spatial volume is finite at time $t = 0$ and increases with increasing value of time hence
171 the model starts expanding with finite volume.
172 The Hubble parameter is given by

173
$$H = \frac{2(t - t_0)}{(1 - \beta)(t - t_0)^2 + t_0} . \qquad (22)$$

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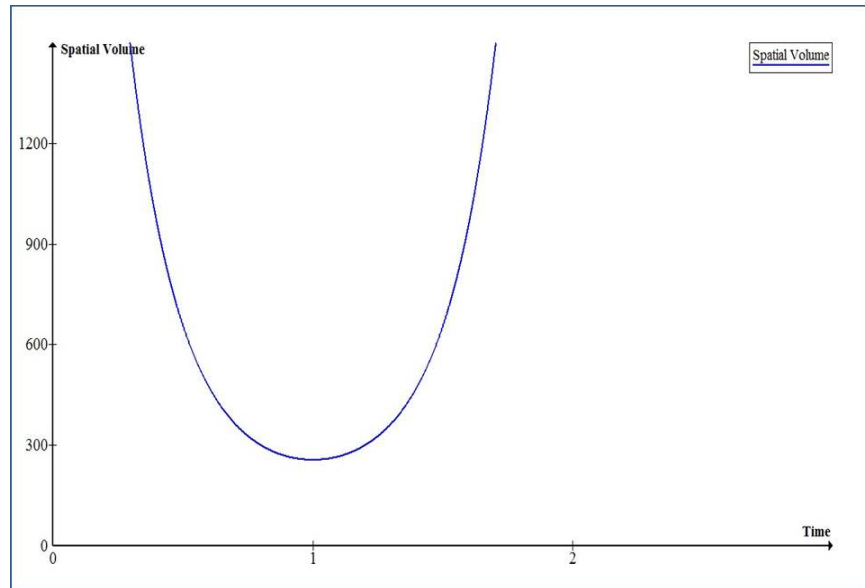* Tel.: +xx 8983268071.
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180 Fig. 2 Plot of Volume versus Time for $\beta = 0.5, t_0 = 1$

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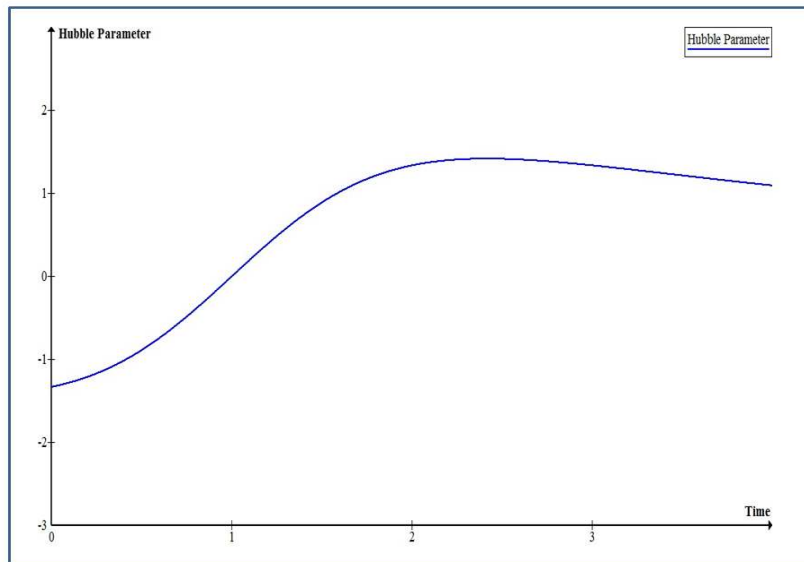
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187 Fig. 3 Plot of Hubble Parameter versus Time for $\beta = 0.5, t_0 = 1$

188

189 From fig. 2, the Hubble parameter $H < 0$, for $t < 1$ and $H > 0$, for $t > 1$ indicating that H
190 passes across zero $(H = 0)$ at $t = 1$, which represents that the universe is bouncing at $t = 1$.
191 The expansion scalar is

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192
$$\theta = \frac{32(t - t_0)}{(1 - \beta)(t - t_0)^2 + \dfrac{t_0}{1 - \beta}} \; . \tag{23}$$

193 The mean anisotropy parameter A_m is

194
$$Am = 3\frac{(m - 1)^2}{(m + 3)^2} = cons\tan t \neq 0 , \quad \text{for } m \neq 1 \tag{24}$$

195 The shear scalar is

196
$$\sigma^2 = 24\frac{(m - 1)^2}{(m + 3)^2(1 - \beta)^2} \frac{(t - t_0)^2}{\left[(t - t_0)^2 + \dfrac{t_0}{1 - \beta}\right]^2} \; . \tag{25}$$

197 We observe that

198
$$\lim_{t \to \infty} \frac{\sigma^2}{\theta^2} = \frac{3}{128}\frac{(m - 1^2)}{(m + 3^2)} \neq 0 , \text{ for } (m \neq 1) \qquad . \tag{26}$$

199 The mean anisotropy parameter A_m is constant and $\lim\limits_{t \to \infty} \dfrac{\sigma^2}{\theta^2}(\neq 0)$ is also constant, hence

200 the model is anisotropic throughout the evolution of the universe except at $m = 1$ *i.e.* the
201 model does not approach isotropy.
202 The matter energy density is given by

203
$$\rho = \frac{192m(m + 1)(t - t_0)^2}{(1 - \beta)^2(m + 3)^2\left[(t - t_0)^2 + \dfrac{t_0}{1 - \beta}\right]^2} \; . \tag{27}$$

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205
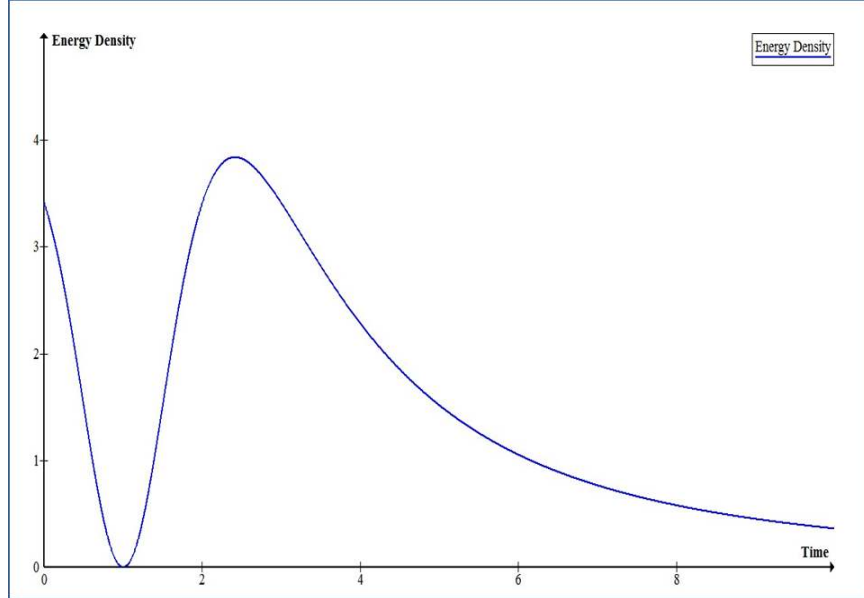
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208 Fig. 4 Plot of Energy Density versus Time $\beta = 0.5, t_0 = 1, \; m = 2$
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210 From fig. 3, the energy density decreases at the early stage of evolution when $t<1$ and goes
211 into the hot Big-bang era. The model bounces at $t=1$ and after bouncing the energy density
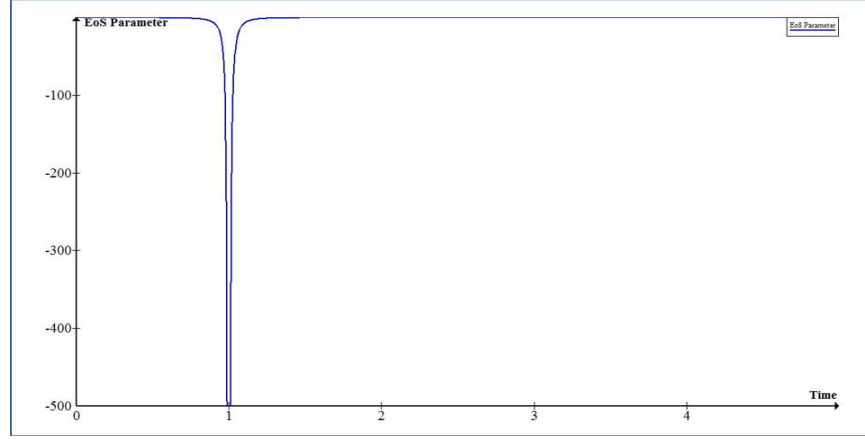212 rapidly increases for $t>1$.
213

214
215
216 Fig. 5 Plot of EoS parameter versus Time for $\beta=0.5, t_0=1, m=2$
217
218 The equation of state (Eos) parameter ω is given by

$$\omega = \frac{-2}{m+1} + \frac{(1-\beta)(m+3)}{4(m+1)} - \frac{(1-\beta)(m+3)}{24(m+1)(t-t_0)^2}\left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right].\tag{28}$$

220 A bouncing universe model has an initial narrow state by a non-zero minimal radius and then
221 develops to an expanding phase. For the universe going into the hot Big Bang era after the
222 bouncing, the equation of state parameter of the universe crosses from $\omega<-1$ to $\omega>-1$.
223 From fig. 4, before bouncing point at $t=1$, we see that the skew-ness parameter $\omega<-1$
224 and after the bounce, the universe enter into the hot Big Bang era and occurs the big rip
225 singularity. Further the Eos parameter $\omega>-1$, for $t>1$. Hence our model is bouncing at
226 $t=1$.
227

CONCLUSION

229 Kaluza-Klein cosmological model has been investigated in the general theory of relativity.
230 The source for energy momentum tensor is a perfect fluid. The field equations have been
231 solved by using time dependent deceleration parameter. The mean anisotropy parameter
232 A_m is constant and $\lim_{t\to\infty}\frac{\sigma^2}{\theta^2}(\neq 0)$ is also constant, hence the model is anisotropic throughout
233 the evolution of the universe except at $m=1$ *i.e.* the model does not approach isotropy. It is
234 interesting to note that the behavior of the model is bouncing as the Hubble parameter
235 H passes across zero $(H=0)$ from $H<0$ to $H>0$, for some finite time $t=t_0$. Also the
236 energy density decreases at the early stage of evolution and rapidly increases showing big
237 bounce $t=t_0$. The Hubble parameter $H<0$, for $t<t_0$ and $H>0$, for $t>t_0$ indicating that
238 H passes across zero $(H=0)$ at $t=t_0$, $(t_0\neq 0)$ which represents the model is bouncing at
239 $t=t_0$. The skew-ness parameter $\omega<-1$ before the bounce at $t=t_0$ and $\omega>-1$ after the
240 bounce. ,
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