Bouncing Behavior of Kaluza-Klein Cosmological Model in General Relativity

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ABSTRACT

Kaluza-Klein cosmological model has been obtained in the general theory of relativity. The source for energy-momentum tensor is a perfect fluid. The field equations have been solved by using a special form of the average scale factor $R(t)=\left((t-t_0)^2+\frac{t_0}{1-\beta}\right)^{\frac{1}{1-\beta}}$ proposed by Scheerer R. J.. The physical properties and the bouncing behavior of the model are also discussed.

Keywords: Kaluza-Klein space time, Bouncing Universe.

1. INTRODUCTION

According to recent cosmological observations in terms of Supernovae Ia [1-2], large scale structure [3-4] with the baryon acoustic oscillations [5], cosmic microwave background radiations [6-8], and weak lensing [9], the current expansion of the universe is accelerating and homogeneous. At the present time, the cosmic acceleration is explained by two ways: One is the introduction of the so called dark energy with negative pressure in general relativity and the other is the modification of gravity like f(R) gravity, f(t) gravity, f(R,T) gravity etc. on the large distances.

The solution of the singularity problem of the standard Big Bang cosmology is known as bouncing universe. A bouncing universe with an initial contraction to a non-vanishing minimal radius and then subsequent an expanding phase provides a possible solution to the singularity problem of the standard Big Bang cosmology. Moreover, for the universe entering into the hot Big Bang era after the bouncing, the equation of state (EoS) of the matter content ω in the universe must transit from $\omega<-1$ to $\omega>-1$. In the contracting phase, the scale factor $R(t)$ is decreasing, this means $\dot{R}(t)<0$, and in the expanding phase, scale factor $\dot{R}(t)>0$. Finally at the bouncing point, $\dot{R}(t)=0$ and near this point $\ddot{R}(t)>0$, for a period of time. It is also discussed with other view that in the bouncing cosmology, the Hubble parameter H passes across zero $(H=0)$ from $H<0$ to $H>0$. Cai *et al.* have investigated bouncing universe with quintom matter. He showed that a bouncing universe has an initial narrow state by a minimal radius and then develops to an expanding phase [10]. This means for the universe arriving to the Big-bang era after the bouncing, the EoS parameter should crossing from $\omega<-1$ to $\omega>-1$. Sadatian [11] have studied rip singularity scenario and bouncing universe in a Chaplygin gas dark energy model. Recently, Bamba *et al.* [12] have investigated bounce cosmology from $f(R)$ gravity and $f(R)$ bi-gravity. Astashenok [13] has studied effective energy models and dark energy models with bounce

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in frames of $f(T)$ gravity. Solomans *et al.* [14] have investigated bounce behavior in Kantowski-Sach and Bianchi cosmology. Silva *et al.* [15] have studied bouncing solutions in Rastall's theory with a barotropic fluid. Brevik and Timoshkin [16] have obtained inhomogeneous dark fluid and dark matter leading to a bounce cosmology. Singh *et al.* [17] have studied k-essence cosmologies in Kantowski-Sachs and Bianchi space times.

The Kaluza-Klein theory [18-19] was introduced to unify Maxwell's theory of electromagnetism and Einstein's gravity theory by adding the fifth dimension. Due to its potential function to unify the fundamental interaction, Kaluza-Klein theory has been regarded as a candidate of fundamental theory. Ponce [20], Chi [21], Fukui [22], Liu and Wesson [23], Coley [24] have studied Kaluza-Klein cosmological models with different contexts. Adhav et al. [25] have obtained Kaluza-Klein inflationary universe in general theory of relativity. Reddy *et al.* [26] have discussed a five dimensional Kaluza- Klein cosmological model in the presence of perfect fluid in $f(R,T)$ gravity. Ranjeet *et al.* [27] have studied variable modified Chaplygin gas in anisotropic universe with Kaluza- Klein metric. Katore *et al.* [28] have obtained Kaluza-Klein cosmological model for perfect fluid and dark energy. Ram and Priyanka [29] have presented some Kaluza-Klein cosmological models in $f(R,T)$ gravity theory. Sahoo *et al.* [30] have investigated Kaluza-Klein cosmological model in $f(R,T)$ gravity with $\lambda(T)$. Recenty, Reddy *et al.* [31] have studied Kaluza-Klein minimally interacting holographic dark energy model in a scalar tensor theory of gravitation. Ghate and Mhaske [32] have investigated Kaluza-Klein barotropic cosmological model with varying gravitational constant G in creation field theory of gravitation.

In this paper, Bouncing behavior of Kaluza-Klein cosmological model has been studied in the general theory of relativity. This work is organized as follows: In section 2, the metric and field equations have been presented. The field equations have been solved in section 3 by using the physical condition that the expansion scalar θ is proportional to shear scalar σ and the special form of average scale factor $R(t)=\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{1}{1-\beta}}$ proposed by Scheerer [33]. The physical and geometrical behavior of the model have been discussed in section 4 . In the last section 5, concluding remarks have been expressed.

2. METRIC AND FIELD EQUATIONS

Five dimensional Kaluza-Klein metric is considered in the form

$$ds^2 = dt^2 - A(t)^2(dx^2+dy^2+dz^2) - B(t)^2 d\psi^2 \quad (1)$$

where $A(t)$ and $B(t)$ are functions of cosmic time t and the fifth coordinate ψ is taken to be space-like.

The energy-momentum tensor when the source for energy is perfect fluid is given by

$$T_i^j = (\rho+p)u_i u^j - p\delta_i^j, \quad (2)$$

where u^i is the flow vector satisfying $g_{ij}u^i u^j = 1$. Here ρ is the total energy density of perfect fluid and p is the corresponding pressure. For the perfect fluid, p and ρ are related by and equation of state

$$p = \omega\rho, \quad 0 \le \omega \le 1. \quad (3)$$

In co-moving system of coordinates, using equation (2), one can find

$$T_0^0 = \rho \text{ and } T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p. \quad (4)$$

The Einstein's field equations are given by

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$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j . \quad (5)$$

Using equation (2), for the metric (1), the field equations (5) are given by

$$3\frac{\dot{A}^2}{A^2} + 3\frac{\dot{A}\dot{B}}{AB} = \rho , \quad (6)$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}^2}{A^2} + 2\frac{\dot{A}\dot{B}}{AB} = -\omega\rho \quad (7)$$

$$3\frac{\ddot{A}}{A} + 3\frac{\dot{A}^2}{A^2} = -\omega\rho \ , \quad (8)$$

where an overhead dot represents differentiation with respect to t.
The average scalar factor R and volume scalar V are given by

$$R^4 = V = A^3 B . \quad (9)$$

The generalized mean Hubble parameter H is defined by

$$H = \frac{\dot{R}}{R} = \frac{1}{4}\left(H_x + H_y + H_z + H_\phi\right), \quad (10)$$

where the directional Hubble parameters H_x, H_y, H_z and H_ϕ are given by

$$H_x = H_y = H_Z = \frac{\dot{A}}{A}, \qquad H_\phi = \frac{\dot{B}}{B} . \quad (11)$$

The expansion scalar θ and shear scalar σ are given by

$$\theta = 4H = \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right), \quad (12)$$

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{4} H_i^2 - 4H^2\right] . \quad (13)$$

The deceleration parameter (DP) q is defined by

$$q = -1 + \frac{d}{dt}(H) . \quad (14)$$

3. SOLUTION OF FIELD EQUATIONS

The field equations (6) to (8) are a system of three highly non-linear differential equations in four unknowns A, B, ρ and ω. The system is thus initially undetermined. We need one extra condition for solving the field equations completely.
We assume that the expansion (θ) is proportional to shear (σ) This condition leads to

$$\frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \alpha_0 \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)$$

which yields

$$\frac{\dot{B}}{B} = m\frac{\dot{A}}{A} ,$$

where α_0 and m are arbitrary constants.

Above equation, after integration, reduces to

$$B = \eta (A)^m ,$$

where η is an integration constant.

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Here, for simplicity and without loss of generality, we assume that $\eta = 1$.
Hence we have

$$B = (A)^m, \ (m \neq 1). \tag{15}$$

Collins *et al.* [34] have pointed out that for spatially homogeneous metric, the normal congruence to the homogeneous expansion satisfies that the condition $\frac{\sigma}{\theta}$ is constant.

In cosmology, The constant deceleration parameter is commonly used by several researchers [35-39], as it duly gives a power law for metric function or corresponding quantity.

The motivation to choose time dependent deceleration parameter (DP) is behind the fact that the expansion of the universe was decelerating in the past and accelerating at present as observed by recent observations of Type Ia supernova [1, 2, 40-42] and CMB anisotropies [43-44]. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. Now for a Universe which was decelerating in past and accelerating at the present time, the DP must show signature flipping [45-47]. So, in general, the DP is not a constant but time variable. The motivation to choose the following scale factor is that it provides a time-dependent DP.

Under above motivations, we choose the scale factor which is time dependent given by

$$R(t) = \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{1}{1-\beta}} \tag{16}$$

where t_0 is initial time and $\beta < 1$ is constant.

Solving equations $A = B^m$ and $R(t) = (A^3 B)^{\frac{1}{4}}$, and using (16) we get

$$B = \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{4m}{(1-\beta)(m+3)}}. \tag{17}$$

With the help of equation (17), equation (15) takes the form

$$A = \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{4}{(1-\beta)(m+3)}}. \tag{18}$$

Using above two equations (17) and (18), the metric (1) takes the form

$$ds^2 = dt^2 - \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{8}{(1-\beta)(m+3)}} \left(dx^2 + dy^2 + dz^2\right) - \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{8m}{(1-\beta)(m+3)}} d\psi^2. \tag{19}$$

Equation (19) represents Kaluza-Klein cosmological model with time dependent scale factor.

4. PHYSICAL PROPERTIES OF THE MODEL

The physical quantities such as spatial volume V, Hubble parameter H, expansion scalar θ, mean anisotropy A_m, shear scalar σ^2, energy density ρ, equation of state parameter ω are obtained as follows:

The average scale factor is

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$$R(t) = \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{1}{1-\beta}}.$$

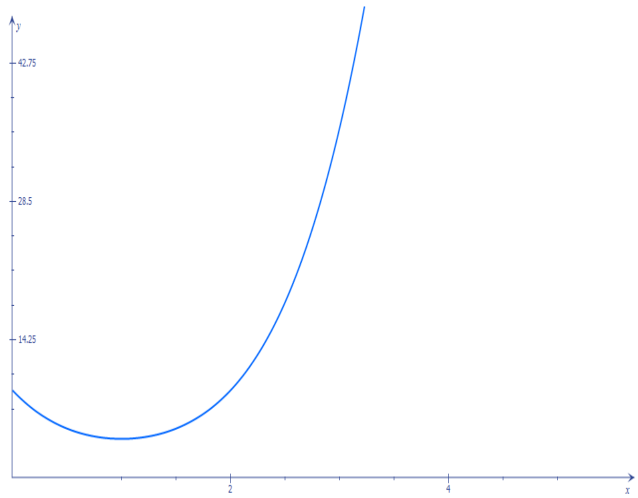

Fig. 1 Plot of Average scale factor versus time for $\beta = 0.5, t_0 = 1$

From fig. 1, in the earlier stage, the scale factor is slightly decreasing ($\dot{R}(t) < 0$) and in the expanding phase the scale factor increases rapidly ($\dot{R}(t) > 0$). Hence our model is bouncing at $t = t_0$ ($\dot{R}(t) = 0$).

The spatial volume is given by

$$V = R^4 = \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{4}{1-\beta}}. \tag{20}$$

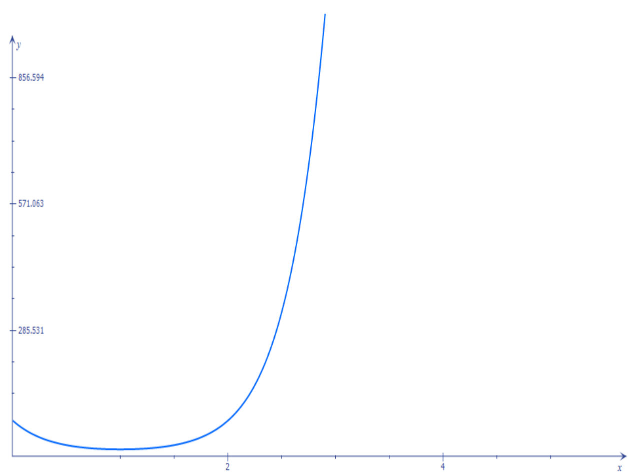

Fig. 1 Plot of Volume versus Time for $\beta = 0.5, t_0 = 1$

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The spatial volume is finite at time $t=0$ and increases with increasing value of time hence the model starts expanding with finite volume.

The Hubble parameter is given by

$$H=\frac{2(t-t_0)}{(1-\beta)(t-t_0)^2+t_0}. \tag{21}$$

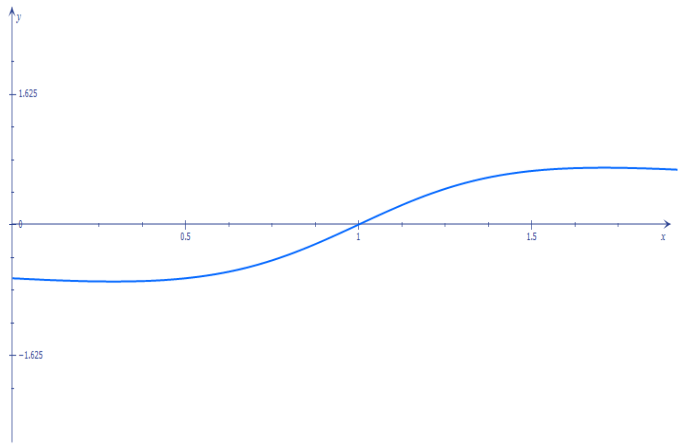

Fig. 2 Plot of Hubble Parameter versus Time for $\beta=0.5, t_0=1$

From fig. 2, the Hubble parameter $H<0$, for $t<1$ and $H>0$, for $t>1$ indicating that H passes across zero $(H=0)$ at $t=1$, which represents that the universe is bouncing at $t=1$.

The expansion scalar is

$$\theta=\frac{32(t-t_0)}{(1-\beta)(t-t_0)^2+\frac{t_0}{1-\beta}}. \tag{22}$$

The mean anisotropy parameter A_m is

$$Am=3\frac{(m-1)^2}{(m+3)^2}=cons\tan t\neq 0, \quad \text{for } m\neq 1 \tag{23}$$

The shear scalar is

$$\sigma^2=24\frac{(m-1)^2}{(m+3)^2(1-\beta)^2}\frac{(t-t_0)^2}{\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^2}. \tag{24}$$

We observe that

$$\lim_{t\to\infty}\frac{\sigma^2}{\theta^2}=\frac{3}{128}\frac{\left(m-1^2\right)}{\left(m+3^2\right)}\neq 0, \text{ for } (m\neq 1) \quad . \tag{25}$$

The mean anisotropy parameter A_m is constant and $\lim_{t\to\infty}\frac{\sigma^2}{\theta^2}(\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe except at $m=1$ *i.e.* the model does not approach isotropy.

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The matter energy density is given by

$$\rho = \frac{192m(m+1)(t-t_0)^2}{(1-\beta)^2(m+3)^2\left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]^2}. \quad (26)$$

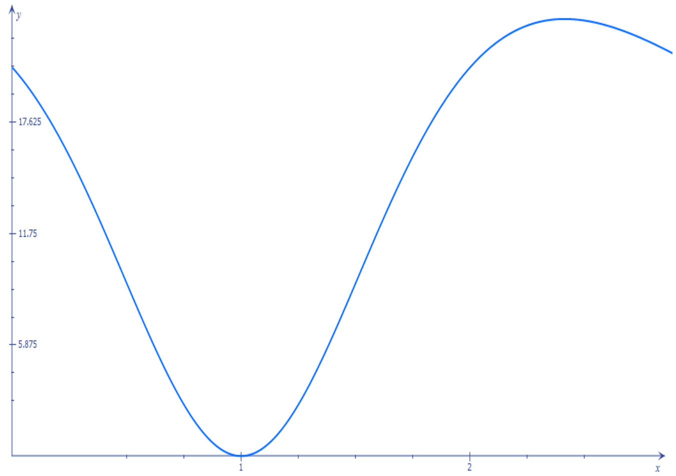

Fig. 3 Plot of Energy Density versus Time $\beta = 0.5, t_0 = 1, m = 2$

From fig. 3, the energy density decreases at the early stage of evolution when $t < 1$ and goes into the hot Big-bang era. The model bounces at $t = 1$ and after bouncing the energy density rapidly increases for $t > 1$.

The equation of state (Eos) parameter ω is given by

$$\omega = \frac{-2}{m+1} + \frac{(1-\beta)(m+3)}{4(m+1)} - \frac{(1-\beta)(m+3)}{24(m+1)(t-t_0)^2}\left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]. \quad (27)$$

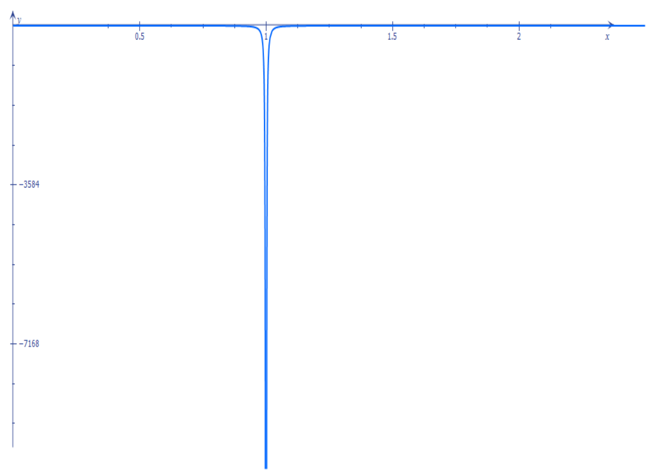

Fig. 4 Plot of EoS parameter versus Time for $\beta = 0.5, t_0 = 1, m = 2$

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A bouncing universe model has an initial narrow state by a non-zero minimal radius and then develops to an expanding phase. For the universe going into the hot Big Bang era after the bouncing, the equation of state parameter of the universe crosses from $\omega < -1$ to $\omega > -1$. From fig. 4, before bouncing point at $t = 1$, we see that the skewness parameter $\omega < -1$ and after the bounce, the universe enter into the hot Big Bang era and occurs the big rip singularity. Further the Eos parameter $\omega > -1$, for $t > 1$. Hence our model is bouncing at $t = 1$.

CONCLUSION

Kaluza-Klein cosmological model has been investigated in the general theory of relativity. The source for energy momentum tensor is a perfect fluid. The field equations have been solved by using time dependent deceleration parameter. The mean anisotropy parameter A_m is constant and $\lim_{t\to\infty}\frac{\sigma^2}{\theta^2}(\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe except at $m = 1$ *i.e.* the model does not approach isotropy. It is interesting to note that the behavior of the model is bouncing as the Hubble parameter H passes across zero $(H = 0)$ from $H < 0$ to $H > 0$, for some finite time $t = t_0$. Also the energy density decreases at the early stage of evolution and rapidly increases showing big bounce $t = t_0$. The Hubble parameter $H < 0$, for $t < t_0$ and $H > 0$, for $t > t_0$ indicating that H passes across zero $(H = 0)$ at $t = t_0$, ($t_0 \neq 0$) which represents the model is bouncing at $t = t_0$. The skewness parameter $\omega < -1$ before the bounce at $t = t_0$ and $\omega > -1$ after the bounce. ,

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