Original Research Articles

Electron Inertia Effects on the Gravitational instability Under the Influence of FLR Corrections and Suspended Particles

ABSTRACT

In this paper, we investigate the effects of electron inertia on the gravitational instability of gaseous plasma under the influence of FLR corrections and suspended particles. A general dispersion relation has been derived through relevant linearized perturbation equations. The general dispersion relation is reduced for both longitudinal and transverse mode of propagation. Numerical calculations have been performed to show the effect of various parameters on the growth rate of the gravitational instability. It is found that the simultaneous effect of viscosity, finite conductivity and permeability of the medium does not essentially change the Jeans criterion of instability. From the curves, we find that relaxation time, Stoke drag, viscosity and FLR parameter have a stabilizing effect on the growth rate of instability, but the thermal conductivity and finite electron inertia parameter have a destabilizing effect on the growth rate of instability.

Keywords: Electrical Resistivity, finite Electron-Inertia, Suspended Particles, finite Larmor radius, Magnetic Field, Thermal conductivity and permeability.

1. INTRODUCTION

Nowadays, there has been a great deal of interest in understanding the formation of planetesimals and stars in interstellar media. The gravitational instability of an infinite homogeneous self-gravitating gaseous plasma was first discussed by Jeans [1] and he pointed out that an infinitely extending homogeneous static medium is unstable with respect to the gravitational, sound wave with wave number k less than the critical Jeans wave-number $k_j = \left(\frac{4\pi G\rho}{C_s}\right)^{\frac{1}{2}}$ Where the symbols have their usual meaning. Chandrasekhar [2] has discussed in detail the effect of the magnetic field and rotations on the Jeans criterion of gravitational instability. He inferred that Jeans criterion determines the condition of instability even in the presence of a magnetic field and rotation. A number of researchers [3] – [12] have extended the problem of the gravitational instability of the self-gravitating system under different conditions. Recently, Prajapati et al. [13] have investigated the problem of self-gravitational instability of rotating viscous Hall plasma with arbitrary radiative heat loss functions and electron inertia and have obtained modified Jeans criterion of instability. Ren et al. [14] have studied the electrostatic drift modes in quantum dusty plasma with Jeans terms. Bashir et al. [15] have discussed the problem of self-gravitational electrostatic drift waves for a streaming non-uniform quantum dusty magnetoplasma. Prajapati and Chhajlani [16] have discussed the self-gravitational instability in magnetized finitely conducting viscoelastic fluid. Sharma and Chhajlani [17] have pointed out the modified Jeans instability of magnetized spin $\frac{1}{2}$ Quantum plasma with resistive effects and Hall current. Joshi and Pensia [18] have discussed the effect of radiation on the Jeans instability of quantum plasma under the influence of rotation.

In addition, to this the finite Larmor radius (FLR) effect plays an important role in interstellar gas dynamics, which exhibits itself in the form of a magnetic viscosity in the fluid equations Roberts and Taylor [19], Recently Kaothekar and Chhajlani [20] have discussed the problem of Jeans instability for a self-gravitating, rotating, radiative plasma with finite Larmor radius corrections and point out the stabilizing influence of the finite Larmor radius (FLR) effect.

In all these studies of a gravitational instability of a self-gravitating medium under the combined effects of FLR corrections, finite electron inertia, viscosity, electrical conductivity, magnetic field, permeability and presence of suspended particles have not been investigated. It would, therefore, be of interest to examine the gravitational instability of a self-gravitating gaseous plasma under the influence of finite electron inertia, FLR correction, viscosity, thermal conductivity, permeability, magnetic field, electrical conductivity and presence of suspended particles. In the present work, we have discussed the problem of gravitational instability of a self-gravitating gaseous plasma in the presence of suspended particles and transverse magnetic field, including the simultaneous effects of finite electron inertia, FLR correction, viscosity, thermal conductivity and electrical resistivity. The present study can serve as a theoretical support to understand the astrophysical problems. This problem to the best of our knowledge has not been investigated yet.

2. LINEARIZED PERTURBATION EQUATIONS

We consider an infinite homogeneous, viscous, self-gravitating gaseous plasma composed of gas and the suspended particle mixture with a uniform vertical magnetic field, finite electron inertia and the FLR. Into the unperturbed state, the fluid is assumed to be at rest. Pressure P and the density are constant is space and time. Due to the action of the perturbing field, a small amplitude perturbation induces an oscillatory motion. If the amplitude of these perturbations grows in time, then the system is said to be unstable. The unstable mode well grows when energy transferred to the system exceeds the dissipation. The perturbations in density, velocity, pressure, magnetic field, temperature and the gravitational potential are given as $\delta\rho,\ \vec{v},\ \delta\mathrm{P},\ \overrightarrow{\delta B},\ \delta T\ and\ \delta U$ respectively.

The perturbation state is given by

$\rho = \rho_0 + \delta\rho\,, \quad P = P_0 + \delta P, \quad \vec{B} = \overrightarrow{B_0} + \overrightarrow{\delta B}, \quad T = T_0 + \delta T, \quad U = U_0 + \delta U, \quad \vec{V} = \vec{v}, \quad \vec{u} = \vec{u}$

Suffix ‘0’ is dropped from the equilibrium quantities.

Thus, the linearized perturbation equations with finite Larmor radius and finite electron inertia governing the motion of hydromagnetic electrically conducting fluid plasma having suspended particles are given by.

$$\rho\frac{\delta\vec{v}}{\delta t} = -\vec{\nabla}\delta P - \vec{\nabla}.\boldsymbol{P} + \rho\vec{\nabla}\delta U + K_s N(\vec{u} - \vec{v}) + \frac{1}{4\pi}\left(\vec{\nabla}\times\overrightarrow{\delta B}\right)\times\vec{B} + \rho\vartheta\left(\nabla^2\vec{v} - \frac{1}{k_1}\vec{v}\right) \tag{1}$$

$$\frac{\partial\delta\rho}{\partial t} + \rho\vec{\nabla}.\vec{v} = 0 \tag{2}$$

$$\delta P = C^2\delta\rho \tag{3}$$

$$\nabla^2\delta U + 4\pi G\delta\rho = 0 \tag{4}$$

$$\left(\tau\frac{\partial}{\partial t} + 1\right)\vec{u} = \vec{v} \tag{5}$$

$$\lambda\nabla^2\delta T = \rho C_p\frac{\partial\delta T}{\partial t} - \frac{\partial\delta P}{\partial t} \tag{6}$$

$$\frac{\delta P}{P} = \frac{\delta T}{T} + \frac{\delta\rho}{\rho} \tag{7}$$

$$\frac{\partial \overrightarrow{\delta B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \chi \nabla^2 \overrightarrow{\delta B} + \frac{C^2}{4\pi^2 f_p} \frac{\partial}{\partial t} \nabla^2 \overrightarrow{\delta B} \tag{8}$$

Where,

$\vec{v}(v_x, v_y, v_z), \vec{u}(u_x, u_y, u_z),\ N, \rho, P, U, \vec{B}(0,0,B), T, G, \vartheta, C_p, \lambda, R, k_1, m,\ \rho_s, 4\pi^2 f_p,$

$K_s(6\pi\rho r)$ and $\overrightarrow{\delta B}(\delta B_x, \delta B_y, \delta B_z)$, **P**, denote respectively, the gas velocity, the particle velocity, the number density of the particle, density of the gas, pressure of the gas, Gravitational potential, magnetic field, temperature, Gravitational constant, kinematic viscosity, specific heat at constant pressure, thermal conductivity, gas constant, permeability, mass per unit volume of the particles its density, plasma frequency of electron, the constant in the stokes drag formula, perturbation in magnetic field and stress tensor (Pressure Tensor). The component of pressure tensor **P** taking into account of finite ion Larmor radius, for the magnetic field along Z axis (For the vertical magnetic field) according to Roberts and Taylor [19] are

$P_{xx} = -\rho\upsilon_0\left(\frac{\delta v_y}{\delta x} + \frac{\delta v_x}{\delta y}\right)$, $\quad P_{yy} = \rho\upsilon_0\left(\frac{\delta v_y}{\delta x} + \frac{\delta v_x}{\delta y}\right)$, $\quad P_{zz} = 0$, $\quad P_{xy} = P_{yx} = \rho\upsilon_0\left(\frac{\delta v_x}{\delta x} - \frac{\delta v_y}{\delta y}\right)$,

$P_{xz} = P_{zx} = -2\rho\upsilon_0\left(\frac{\delta v_y}{\delta z} + \frac{\delta v_z}{\delta y}\right)$, $\quad P_{yz} = P_{zy} = 2\rho\upsilon_0\left(\frac{\delta v_z}{\delta x} + \frac{\delta v_x}{\delta z}\right)$,

Where $\rho\upsilon_0 = \frac{N'Tk'}{4W_H}$, and T being the density and temperature of ions and W_H is ion-gyration frequency and k' is Boltzmann's constant.

3. DISPERSION RELATION

We assume that all the perturbed quantities vary as,

$$exp\{i(k_x x + k_z z + \omega t)\} \tag{9}$$

Where k_x, k_z are the wave numbers of perturbation along the x and z-axis so that $k_x^2 + k_z^2 = k^2$ And ω the frequency of harmonic disturbances, Using (2)-(9) in (1), we obtain the following algebraic equations for the components.

$$\xi_1 v_x + \upsilon_0(k_x^2 + 2k_z^2)v_y + \frac{ik_x}{k^2}\Omega_T^2 s = 0 \tag{10}$$

$$\left(-\upsilon_0(k_x^2 + 2k_z^2)\right)v_x + \xi_2 v_y - (2\upsilon_0 k_x k_z)v_z = 0 \tag{11}$$

$$(2\upsilon_0 k_x k_z)v_y + D_1 v_z + \frac{ik_z}{k^2}\Omega_T^2 s = 0 \tag{12}$$

The divergence of (1) with the aid of (2)-(9) gives

$$\frac{ik_x k^2 V^2 a_1}{a_2} v_x + ik_x\upsilon_0(k_x^2 + 4k_z^2)v_y - \left(\sigma D_1 + \Omega_T^2\right)s = 0 \tag{13}$$

Where, $s = \frac{\delta\rho}{\rho}$ is the condensation of the medium, $\gamma = \frac{C_p}{C_v} = \frac{C^2}{C'^2}$ ratio of the specific heat, $V = \frac{B}{\sqrt{4\pi\rho}}$ is the Alfven velocity, $a = \frac{k_s N}{\rho}$ has the dimension of frequency, $\tau = \frac{m}{k_s}$ is the relaxation time, $\tau\Omega_N = \frac{\rho_s}{\rho}$ is the mass conservation, $\sigma = i\omega$, is the growth rate of perturbation, $\theta = \frac{\lambda}{\rho C_p}$ is the thermometric conductivity, $\Omega_m = \chi k^2$ is electrical resistivity, $\Omega_\vartheta = \upsilon\left(k^2 - \frac{1}{k_1}\right)$, $a_1 = (\sigma f + \Omega_m)$, $f = \left(1 + \frac{C^2 k^2}{4\pi^2 f_p}\right)$, C and C' is the adiabatic and isothermal velocities of sound.

$$D_1 = \left(\sigma + \Omega_\vartheta + \frac{a\sigma\tau}{\sigma\tau + 1}\right), \quad D_2 = \left(-\upsilon_0(k_x^2 + 2k_z^2)\right), \quad D_3 = (-\xi_3), \quad D_4 = (2\upsilon_0 k_x k_z), \quad a_2 = a_1^2,$$

$$\Omega_{j'}^2 = \left(C'^2 k^2 - 4\pi G\rho\right), \quad \Omega_j^2 = (C^2 k^2 - 4\pi G\rho), \quad \Omega_T^2 = \left(\frac{\sigma\Omega_j^2 + \theta_k \Omega_{j'}^2}{\sigma + \theta_k}\right), \quad \theta_k = \gamma\theta k^2,$$

$$\xi_1 = \left(D_1 + \frac{V^2 k^2 a_1}{a_2}\right), \quad \xi_2 = \left(D_1 + \frac{a_1 V^2 k_z^2}{a_2}\right), \quad \xi_3 = i k_x \upsilon_0 (k_x^2 + 4k_z^2)$$

The nontrivial solution of the determinant of the matrix obtained from (10)-(13) with $(v_x,\ v_y,\ v_z,)$ having various coefficients, that should vanish is to give the following dispersion relation.

$$\xi_1\,\xi_2 D_1\left(\sigma D_1 + \Omega_T^2\right) + D_4^2\xi_1\left(\sigma D_1 + \Omega_T^2\right) - \xi_1\, D_3 D_4 \frac{ik_z}{k^2}\Omega_T^2 + D_1 D_2^2\left(\sigma D_1 + \Omega_T^2\right) + D_2 D_4\left(\frac{i^2 k_x k_z V^2 a_1}{a_2}\ \Omega_T^2\right) + \frac{ik_x}{k^2} D_1 D_2 D_3\ \Omega_T^2 + (D_1\xi_2 + D_4^2)\left(\frac{i^2 k_x^2 V^2 a_1}{a_2}\ \Omega_T^2\right) = 0 \tag{14}$$

Equation (14) represents the general dispersion relation of the considered problem and it shows the combined influence of finite electron inertia, suspended particles, electrical resistivity, thermal conductivity, viscosity, magnetic field and finite Larmor radius on the self-gravitational instability of a homogeneous gaseous plasma. If we ignore the effect of electron inertia them (equ. 14) is similar to those of Vyas and Chhajlani [4] neglecting the contribution of rotation in that case. If we ignore the suspended particles, finite Larmor radius (equ. 14) reduces to the one similar to obtained by Prajapati et al. [13] excluding the effect of Hall currents, rotation and heat-loss function.

Thus, with these corrections, we find the dispersion relation is modified due to the combined influences of suspended particles, finite electron inertia, viscosity, finite Larmor radius, thermal conductivity and electrical resistivity. The above dispersion relation is very lengthy and to investigate the effects of each parameter we now reduce the dispersion relation (14) for two modes of propagation.

4. ANALYSIS OF THE DISPERSION RELATION

Now we shall discuss the dispersion relation given by equation (14) for the following modes. Longitudinal propagation, i.e. $k_x = 0$, $k_z = k$ and Transverse propagation, i.e. $k_x = k,\ k_z = 0$

4.1 Longitudinal Mode of Propagation $(\boldsymbol{k} \parallel \boldsymbol{B})$

In this case, we assume that all the perturbations are longitudinal to the direction of the magnetic field i.e.$(\boldsymbol{k_x} = \mathbf{0}\,,\ \boldsymbol{k_z} = \boldsymbol{k})$.

Thus, the dispersion relation (14) reduces to the simple form to give

$$D_1[\xi_1^2 + (-2\upsilon_0 k^2)^2]\left(\sigma D_1 + \Omega_T^2\right) = 0 \tag{15}$$

This dispersion relation is the product of three independent factors. These factors show the mode of propagations incorporating different parameters as discussed below. The first factor equated to zero gives,

$$\tau\sigma^2 + \sigma\{1 + \tau(a + \Omega_\vartheta)\} + \Omega_\vartheta = 0 \qquad (16)$$

The dispersion relation (16) shows the combined influence of viscosity and suspended particles on the propagation of the disturbances. This mode is independent of finite electron inertia, finite Larmor radius, magnetic field and self-gravitation. From the root of the dispersion relation (16) the stability of the system may be considered. In (16) there is no term of suspended particles and we get damped mode due to viscosity and it is stable mode. The second factor equated to zero and after simplification, we get

$$\sigma^8\tau^2 f^4 + A_7\sigma^7 + A_6\sigma^6 + A_5\sigma^5 + A_4\sigma^4 + A_3\sigma^3 + A_2\sigma^2 + A_1\sigma + A_0 = 0 \qquad (17)$$

The dispersion relation (17) is a non-gravitating Alfven mode influenced by suspended particles, finite electron inertia, finite Larmor radius, viscosity, thermal conductivity and electrical resistivity. The dispersion relation (17) is eight degree polynomial equations and its coefficient are very long and the constant terms are given as,

$$A_0 = \Omega_\vartheta^2\Omega_m^4 + k^4V^4\Omega_m^2 + 2\Omega_\vartheta k^2V^2\Omega_m^3 + 4\upsilon_0^2k^4\Omega_m^4$$

The third factor equated to zero and after simplification gives

$$\sigma^4\tau + \sigma^3\{1 + \tau(a + \Omega_\vartheta + \theta_k)\} + \sigma^2\left[(\Omega_\vartheta + \theta_k) + \tau\{\Omega_j^2 + \theta_k(a + \Omega_\vartheta)\}\right] + \sigma\left(\Omega_j^2 + \theta_k\Omega_\vartheta + \tau\Omega_\vartheta\Omega_{j'}^2\right) + \Omega_\vartheta\Omega_{j'}^2 = 0 \qquad (18)$$

The dispersion relation (18) is a gravitating mode and shows the combined effect of suspended particles, viscosity and thermal conductivity on the self-gravitational instability of the system for longitudinal propagation. This gravitating mode of propagation is independent of finite electron inertia, magnetic field, finite Larmor radius and electrical resistivity. The dispersion relations (18) are four-degree polynomial equations. If $\sigma_1, \sigma_2, \sigma_3\ and\ \sigma_4$ are the root of the equations, then we have

$\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 = -\left\{\frac{1}{\tau} + a + \Omega_\vartheta + \theta_k\right\}$ And $\sigma_1.\sigma_2.\sigma_3.\sigma_4 = \frac{\theta_k}{\tau}\Omega_{j'}^2$

From the dispersion relation (18) we get the condition of instability for all Jeans length $\lambda > \lambda_{j'}\left\{\left(\frac{\pi}{G\rho}\right)^{\frac{1}{2}} c'\right\}$ Or wave number $k < k_{j'}$. In the absence of thermal conductivity $\theta_k = 0$, the system change from isothermal to behavior to adiabatic behavior. Now we analyze the dynamical stability of the system represented by (18) by applying the Routh-Hurwitz criterion. If $\Omega_j^2 > 0$ and $\Omega_{j'}^2 > 0$, then all the coefficients of (18) positive and the necessary condition for stability satisfied. To obtain the sufficient condition, the principal diagonal minors of the Hurwitz matrix must be positive and we get

$$\Delta_1 = \{1 + \tau(a + \Omega_\vartheta + \theta_k)\} > 0\ ,$$

$$\Delta_2 = \Big[(\Omega_\vartheta + \theta_k) + \tau\{a\theta_k + (\Omega_\vartheta + \theta_k)(a + \Omega_\vartheta + \theta_k)\} + \tau^2\theta_k\left(\Omega_j^2 - \Omega_{j'}^2\right) + \tau^2(\Omega_\vartheta + a)\{\Omega_j^2 + \theta_k(a + \Omega_\vartheta + \theta_k)\}\Big] > 0\ ,$$

$$\Delta_3 = \Big[\!\Big[\Omega_\vartheta\{\Omega_j^2 + \theta_k(\Omega_\vartheta + \theta_k)\} + \theta_k\left(\Omega_j^2 - \Omega_{j'}^2\right) + \tau\left[\Omega_\vartheta\theta_k\{a\theta_k + (\Omega_\vartheta + \theta_k)(a + \Omega_\vartheta + \theta_k)\} + \theta_k a\left(\Omega_j^2 - \Omega_{j'}^2\right)\right] + \tau^2\left[\theta_k\left(\Omega_j^2 - \Omega_{j'}^2\right) + \left(\Omega_j^2 + \tau\theta_k\Omega_{j'}^2\right)\left\{(a + \Omega_\vartheta)\Omega_j^2 + \theta_k\left(\Omega_j^2 - \Omega_{j'}^2\right)\right\} + (a + \Omega_\vartheta)(a + \Omega_\vartheta + \theta_k + \Omega_\vartheta\theta_k)\Omega_j^2 + \theta_k\{a\theta_k + (a + \Omega_\vartheta)(a + \Omega_\vartheta + \theta_k)\}\Omega_{j'}^2 + \Omega_\vartheta\theta_k(\Omega_\vartheta + \theta_k)(a + \Omega_\vartheta + \theta_k)\right]\!\Big]\!\Big] > 0,$$

$$\Delta_4 = \theta_k\Omega_{j'}^2\Delta_3 > 0\ ,$$

All the $\Delta's$ positive, thereby, satisfying the Hurwitz criterion, according to which equation (18) will not admit any positive real root of $\sigma(=i\omega)$ Or a complex root whose real part is positive, hence, it gives a stable mode independent of the finite electron inertia, finite Larmor radius and the magnetic field. To analyze the role of viscosity, suspended particles and thermal conductivity on the growth rate of an unstable mode, we choose the arbitrary values of these parameters in the present problem. We write the dispersion relation (18) is non-dimensional from in term of self-gravitation as,

$$\sigma^{*4}\tau^* + \sigma^{*3}\left[1+\tau^*\left\{k_s^* + \upsilon^*\left(k^{*2}-\frac{1}{K_1^*}\right)+\lambda^*\right\}\right] + \sigma^{*2}\left[\upsilon^*\left(k^{*2}-\frac{1}{K_1^*}\right)+\lambda^*+\tau^*\left\{(k^{*2}-1)+\lambda^*\left(k_s^* + \upsilon^*\left(k^{*2}-\frac{1}{K_1^*}\right)\right)\right\}\right] + \sigma^*\left[(k^{*2}-1)+\lambda^*\upsilon^*\left(k^{*2}-\frac{1}{K_1^*}\right)+\tau^*\upsilon^*\left(k^{*2}-\frac{1}{K_1^*}\right)(k^{*2}-1)\right] + \upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)(k^{*2}-1) = 0 \quad (19)$$

Where the various nondimensional parameters are defined as,

$$\sigma^* = \frac{\sigma}{\sqrt{4\pi G\rho}},\; k_s^* = \frac{k_s N}{\rho\sqrt{4\pi G\rho}},\;\; \lambda^* = \frac{\lambda}{\rho C_p\sqrt{4\pi G\rho}},\;\; k^* = \frac{kC}{\sqrt{4\pi G\rho}},\;\; \upsilon^* = \frac{\upsilon\sqrt{4\pi G\rho}}{C^2},\;\; k_1^* = \frac{k_1\sqrt{4\pi G\rho}}{C^2},\;\; \tau^* = \tau\sqrt{4\pi G\rho} \quad (20)$$

In the present analysis, the expression for dispersion relation and a growth rate of instability are evaluated for the infinitely conducting medium. We have examined the effect of thermal conductivity, relaxation time and Stokes drag parameters on the growth rate of self-gravitational instability. The results are shown in Fig. 1-4 which have depicted the nondimensional growth rate versus the nondimensional wave number of various arbitrary values of the thermal conductivity (λ^*), relaxation time (τ^*), viscosity (υ^*) and Stokes drag parameters (k_s^*).

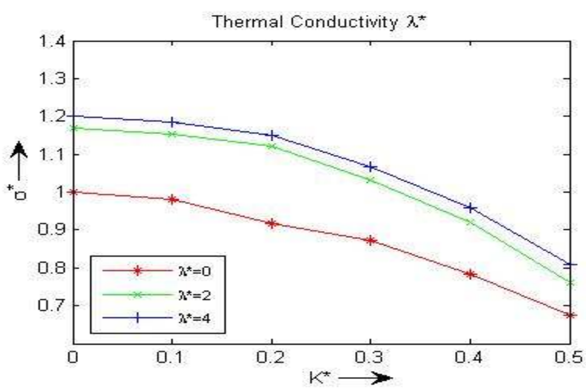

Fig. 1. The growth rate of instability is plotted against the dimensionless wave number k^* with variation in the thermal conductivity λ^*= 0, 2, 4, with taking the values of k_s^*, υ^*, k_1^* and τ^* as unity.

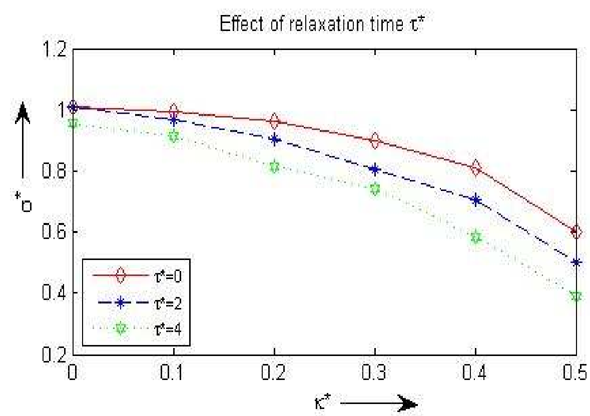

Fig. 2. The growth rate of instability is plotted against the dimensionless wave number k^* with variation in the relaxation time τ^*= 0, 2, 4, with taking the values of k_s^*, υ^*, k_1^* and λ^* as unity.

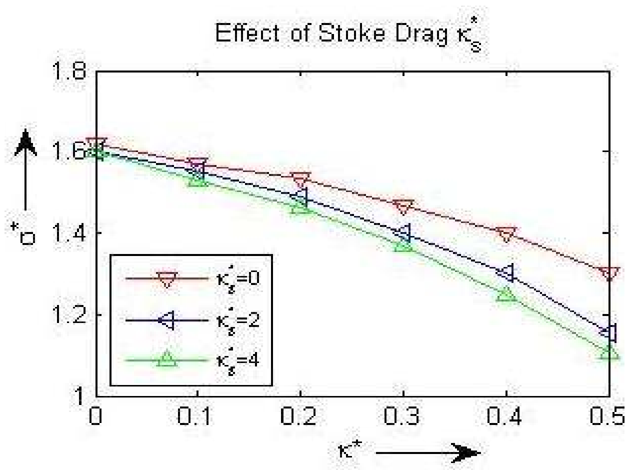

Fig. 3. The growth rate of instability is plotted against the dimensionless wave number k^* with variation in the Stokes' drag constant $\mathbf{k_s^*}$ = 0, 2, 4, with taking the values of λ^*, υ^*, k_1^* and τ^* as unity.

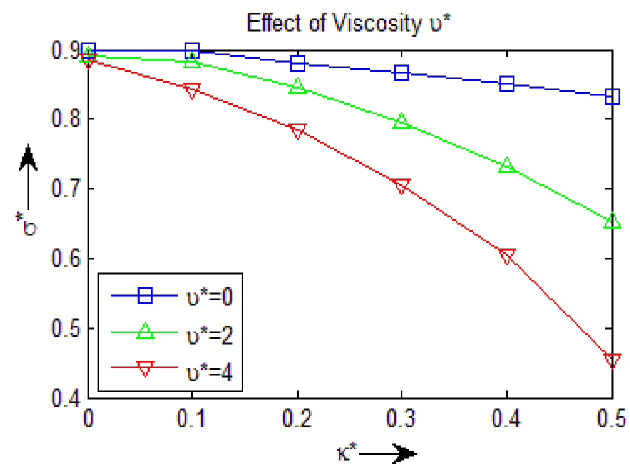

Fig. 4. The growth rate of instability is plotted against the dimensionless wave number k^* with variation in the viscosity υ^* = 0, 2, 4, with taking the values of λ^*, k_1^*, $\mathbf{k_s^*}$ and τ^* as unity.

Fig. 1. Shows the growth rate of an unstable mode $(positive\ real\ root\ of\ \sigma^*)$ against the wave number (k^*) with a variation in the thermal conductivity (λ^*) parameter. We see that the growth rate of the instability increases with increases is (λ^*). The peak value of the growth rate is increased by increasing the thermal conductivity parameters. The present results are different to those of Prajapati et al. [13]. Where the growth rate is unaffected by the presence of the thermal conductivity.

Fig. 2. Shows the variation of growth rate (σ^*) of instability against wave number (k^*) for the different value of relaxation time (τ^*) parameter. It is observed that the relaxation time parameter has a reverse effect on the growth compared to that of the thermal conductivity parameters. In other words, due to an increase in the relaxation time parameter, the growth rate of the instability decreases. Thus, the relaxation time parameter has a damping effect on the growth rate of the system. Also, the peak value of the growth rate is decreased by increasing (τ^*).

Fig. 3. Shows the growth rate of instability against wave number of different values of the Stokes drag (k_s^*) parameter. From the curves, we see that Stokes drag parameter shows the similar effect as shown by relaxation time parameter (τ^*). Thus, the Stokes drag force has a stable influence on the self-gravitational instability of the system.

Fig. 4. Shows the growth rate of instability against wave number of different values of the viscosity (υ^*) parameter. From the curves, we find that viscosity parameter shows the similar effect as shown by relaxation time (τ^*) Stoke drags (k_s^*) parameter. Thus, viscosity has a stable influence on the self-gravitational instability of the system.

4.2 Transverse Mode of Propagation $(\boldsymbol{k} \perp \boldsymbol{B})$

In this case, we assume all the perturbations transverse to the direction of the magnetic field i.e. $(\boldsymbol{k_x} = \boldsymbol{k},\ \ \boldsymbol{k_z} = \boldsymbol{0})$. Thus, the dispersion relation (14) reduces to the simple form to give us,

$$D_1^2\left[\sigma D_1^2 + D_1\left(\Omega_T^2 + \frac{\sigma k^2 V^2}{a_1}\right) + (\upsilon_0 k^2)^2\sigma\right] = 0 \tag{21}$$

This dispersion relation is the product of two independent factors. These factors show the mode of propagations incorporating different parameters as discussed below. The first factor of this dispersion relation is a stable mode as discussed in the previous case. The second factor of the dispersion relations (21) simplification written as

$$
\begin{aligned}
&\sigma^7\tau^2 f + \sigma^6\tau[\tau\{\Omega_m + f(2a + 2\Omega_\vartheta + \theta_k)\} + 2f] \\
&\quad + \sigma^5\big[\tau^2\big\{f\big(\Omega_j^2 + \Omega_\vartheta^2 + a^2\big) + \Omega_m(2a + 2\Omega_\vartheta + \theta_k) + 2af(\Omega_\vartheta + \theta_k) + f\Omega_\vartheta(\Omega_m + \theta_k)\big\} \\
&\quad + 2\tau\{\Omega_m + f(a + 2\Omega_\vartheta + \theta_k)\} + f\big] \\
&\quad + \sigma^4\big[\tau^2\big\{(a + \Omega_\vartheta)\big(2\Omega_m\theta_k + f\Omega_\vartheta\Omega_m + a\Omega_m + \Omega_m\theta_k + f\Omega_j^2\big) + f\Omega_\vartheta(\Omega_\vartheta + a\Omega_m) \\
&\quad + f\theta_k\big(\Omega_{j'}^2 + a^2\big) + \Omega_m\Omega_j^2\big\} \\
&\quad + \tau\big\{(a + \Omega_\vartheta)(2\Omega_m + 2f\Omega_\vartheta + 2f\theta_k) + \big(2f\Omega_j^2 + k^2V^2 + \upsilon_0^2k^4 f\big) + 2\Omega_\vartheta(\Omega_m + f\theta_k) \\
&\quad + 2\Omega_m\theta_k\big\} + \Omega_m + f(2\theta_k + 2\Omega_\vartheta)\big] \\
&\quad + \sigma^3\big[\tau^2(a + \Omega_\vartheta)\big\{f\theta_k\Omega_{j'}^2 + \Omega_m\Omega_j^2 + \Omega_m\theta_k(a + \Omega_\vartheta) + \Omega_m\theta_k\Omega_{j'}^2\big\} \\
&\quad + \tau\big\{\theta_k\big(2f\Omega_{j'}^2 + k^2V^2 + \upsilon_0^2k^4 f\big) + \Omega_m\big(2\Omega_j^2 + \upsilon_0^2k^4\big) \\
&\quad + (a + \vartheta_k)\big(f\Omega_j^2 + k^2V^2 + 2\Omega_\vartheta\Omega_m + f\Omega_\vartheta\theta_k\big) + \theta_k f\big(\Omega_j^2 + a\Omega_m + \theta_k\Omega_\vartheta\big) + a\Omega_m\theta_k\big\} + f\Omega_j^2 \\
&\quad + k^2V^2 + k^4\upsilon_0^2 + f\Omega_\vartheta(\Omega_\vartheta + 2\theta_k)\big] \\
&\quad + \sigma^2\big[\tau^2\big\{\Omega_m\theta_k\Omega_{j'}^2(a + \Omega_\vartheta)\big\} \\
&\quad + \tau\big\{(a + \Omega_\vartheta)\theta_k\big(f\Omega_{j'}^2 + k^2V^2 + \Omega_m\theta_k\big) + (a + \Omega_\vartheta)\big(\Omega_j^2 + \theta_k\Omega_\vartheta\big)\Omega_m + \theta_k\Omega_{j'}^2(2\Omega_m + f\Omega_\vartheta) \\
&\quad + \Omega_m\big(\Omega_j^2\Omega_\vartheta + a\theta_k + \upsilon_0^2k^4\theta_k\big)\big\} + \theta_k\big(f\Omega_{j'}^2 + k^2V^2 + \upsilon_0^2k^4 f\big) + \Omega_\vartheta\big(f\Omega_j^2 + k^2V^2 + f\theta_k\Omega_\vartheta\big) \\
&\quad + \Omega_m\big(\Omega_j^2 + \Omega_\vartheta^2 + \vartheta_0^2k^4\big) + 2\theta_k\Omega_m\Omega_\vartheta\big] \\
&\quad + \sigma\big[\tau\big\{\Omega_m\theta_k\Omega_{j'}^2(a + 2\Omega_\vartheta)\big\} + \theta_k\Omega_\vartheta\big(\Omega_{j'}^2 + k^2V^2\big) + \Omega_m\theta_k\upsilon_0^2\Omega_j^2\Omega_\vartheta\big] + \theta_k\Omega_\vartheta\Omega_m\Omega_{j'}^2 \\
&\quad = 0 \qquad (22)
\end{aligned}
$$

This dispersion relation (22) is self-gravitating Alfven mode and represent the effect of the simultaneous inclusion of the suspended particles, finite Larmor radius, finite electron inertia, thermal conductivity, electrical resistivity, and viscosity on the self-gravitational instability of the system for the transverse propagation. The condition of instability and the expression for the critical Jeans length are obtained from the constant term of (22), which is identical to that for longitudinal propagation. We find that in the dispersion relation (22) some terms are multiplied by terms due to the suspended particles and finite electron inertia, but the constant term is independent of the suspended particles, finite Larmor radius and finite electron inertia. Hence, the conditions of instability will not be affected by the presence of suspended particles, finite Larmor radius and finite electron inertia, but the growth rate of the system will be changed. In the absence of electrical resistivity, the condition of instability is changed and the expression of the critical Jeans wavenumber is given by

$$k < k_j = \left(\frac{4\pi G\rho}{C'^2 + V^2}\right)^{\frac{1}{2}} \qquad (23)$$

From (23) we note that for the electrically infinite conducting system the Jeans criteria of the instability of instability are not affected by finite electron inertia, finite Larmor radius and suspended particles but Jean's condition is modified by a magnetic field.

Now in order to see the effect of suspended particles, finite electron inertia, finite Larmor radius, viscosity and thermal conductivity on self-gravitational instability of the system we reduce the dispersion relation (22) for infinitely conducting mediums. Thus, on putting $\Omega_m = 0$ in (22) we get.

$$\sigma^6\tau^2 f+\sigma^5[\tau^2\{f(2a+2\Omega_\vartheta+\theta_k)\}+2f\tau]+\sigma^4\big[\tau^2\big\{f\big(\Omega_j^2+\Omega_\vartheta^2+a^2\big)+2af(\Omega_\vartheta+\theta_k)+f\Omega_\vartheta\theta_k\big\}+2\tau\{f(a+2\Omega_\vartheta+\theta_k)\}+f\big]+\sigma^3\big[\tau^2\big\{(a+\Omega_\vartheta)\Omega_j^2 f+f\Omega_\vartheta^2+f\theta_k\big(\Omega_{j'}^2+a^2\big)\big\}+\tau\big\{(a+\Omega_\vartheta)(2f\Omega_\vartheta+2f\theta_k)+\big(2f\Omega_j^2+k^2V^2+\upsilon_0^2k^4f\big)+2\Omega_\vartheta f\theta_k\big\}+f(2\theta_k+2\Omega_\vartheta)\big]+\sigma^2\big[\tau^2(a+\Omega_\vartheta)f\theta_k\Omega_{j'}^2+\tau\big\{\theta_k\big(2f\Omega_{j'}^2+k^2V^2+\upsilon_0^2k^4f\big)+(a+\vartheta_k)\big(f\Omega_j^2+k^2V^2+f\Omega_\vartheta\theta_k\big)+\theta_k f\big(\Omega_j^2+\theta_k\Omega_\vartheta\big)\big\}+f\Omega_j^2+k^2V^2+\upsilon_0^2k^4f+f\Omega_\vartheta(\Omega_\vartheta+2\theta_k)\big]+\sigma\big[\tau\big\{(a+\Omega_\vartheta)\theta_k\big(f\Omega_{j'}^2+k^2V^2\big)+\theta_k\Omega_{j'}^2f\Omega_\vartheta\big\}+\theta_k\big(f\Omega_{j'}^2+k^2V^2+\upsilon_0^2k^4f\big)+\Omega_\vartheta\big(f\Omega_j^2+k^2V^2+f\theta_k\Omega_\vartheta\big)\big]+\theta_k\Omega_\vartheta\big(\Omega_{j'}^2+k^2V^2\big)=0 \qquad (24)$$

In order to study the effects of various physical parameters on the growth rate of gravitational instability, we have reduced the dispersion relation (24) in non-dimensional form in terms of self-gravitation as from defined as,

$$\sigma^{*6}\tau^{*2}f^*+\sigma^{*5}\tau^*\left[\tau^*\left\{f^*\left(2k_s^*+2\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)+\lambda^*\right)\right\}+2f^*\right]+\sigma^{*4}\left[\!\left[\tau^{*2}\left[f^*\left\{(k^{*2}-1)+\upsilon^{*2}\left(k^{*2}-\frac{1}{k_1^*}\right)^2+k_s^{*2}\right\}+2k_s^*f^*\left\{\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)+\lambda^*\right\}+f^*\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)\lambda^*\right]+2\tau^*f^*\left\{k_s^*+2\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)+\lambda^*\right\}+f^*\right]\!\right]+\sigma^{*3}\left[\tau^{*2}\left\{\left(k_s^*+\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)\right)(k^{*2}-1)f^*+f^*\upsilon^{*2}\left(k^{*2}-\frac{1}{k_1^*}\right)^2+\lambda^*f^*(k^{*2}-1)+k_s^*\right\}+\tau^*\left\{\left(k_s^*+\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)\right)\left(2f^*\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)+2f^*\lambda^*\right)+2f^*(k^{*2}-1)+k^{*2}V^{*2}+\upsilon_0^{*2}f^*k^{*4}+2\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)f^*\lambda^*\right\}+f^*\left(2\lambda^*+2\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)\right)\right]+\sigma^{*2}\left[\tau^{*2}\lambda^*f^*(k^{*2}-1)\left(k_s^*+\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)\right)+\tau^*\left\{\lambda^*(2f^*(k^{*2}-1)+k^{*2}V^{*2}+\upsilon_0^{*2}f^*k^{*4})+(k_s^*+\upsilon^*)\left(f^*(k^{*2}-1)+k^{*2}V^{*2}+f^*\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)\lambda^*\right)+\lambda^*f^*(k^{*2}-1)+\lambda^*\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)\right\}\right]+\sigma^*\left[\tau^*\left\{\left(k_s^*+\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)\right)\lambda^*(f^*(k^{*2}-1)+k^{*2}V^{*2})+\lambda^*f^*(k^{*2}-1)\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)\right\}+\lambda^*(2f^*(k^{*2}-1)+k^{*2}V^{*2}+\upsilon_0^{*2}f^*k^{*4})+\upsilon^*\left(k^{*2}-\frac{1}{k_1^*}\right)\left(f^*(k^{*2}-1)+k^{*2}V^{*2}+f^*\upsilon^*\lambda^*\left(k^{*2}-\frac{1}{k_1^*}\right)\right)\right]+\upsilon^*\lambda^*\left(k^{*2}-\frac{1}{k_1^*}\right)\left((k^{*2}-1)+k^{*2}V^{*2}\right)=0 \qquad (25)$$

Where the various non dimensional parameters are defined as,

$$\sigma^*=\frac{\sigma}{\sqrt{4\pi G\rho}}, \quad k_s^*=\frac{k_sN}{\rho\sqrt{4\pi G\rho}}, \quad \lambda^*=\frac{\lambda}{\rho C_p\sqrt{4\pi G\rho}}, \quad k^*=\frac{kC}{\sqrt{4\pi G\rho}}, \quad \upsilon^*=\frac{\vartheta\sqrt{4\pi G\rho}}{C^2},$$
$$k_1^*=\frac{k_1\sqrt{4\pi G\rho}}{C^2}, \quad \tau^*=\tau\sqrt{4\pi G\rho}, \quad V^*=\frac{V\sqrt{4\pi G\rho}}{C}, \quad \upsilon_0^*=\frac{\vartheta_0\sqrt{4\pi G\rho}}{C^2} \qquad (26)$$

In order to see the effects of various physical parameters [relaxation time τ^*, stokes drag k_s^*, finite electron inertia f^* and finite Larmor radius υ_0^*] on the growth rate instability, we have performed numerical conclusions of the dispersion relation to locate the positive real roots of the non-dimensional growth σ^* against the non-dimensional wave number k^* for various values of relaxation time τ^*, Stokes drag k_s^*, finite electron inertia f^* and finite Larmor radius υ_0^*. These calculations are presented in figures (5 to 8) to show the variations of the growth rate (σ^*) with wave number (k^*), of the considered system for different values of relaxation time (τ^*), Stokes drags (k_s^*), finite electron inertia (f^*) and finite Larmor radius (υ_0^*) respectively.

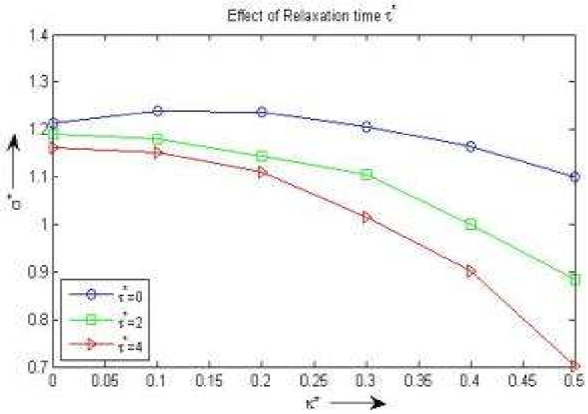

Fig. 5. The growth rate of instability is plotted against the dimensionless wave number k^* with variation in the relaxation time τ^*= 0, 2, 4, with taking the values of k_s^*, V^*, υ_0^*, ϑ^* , f^* and λ^* as unity.

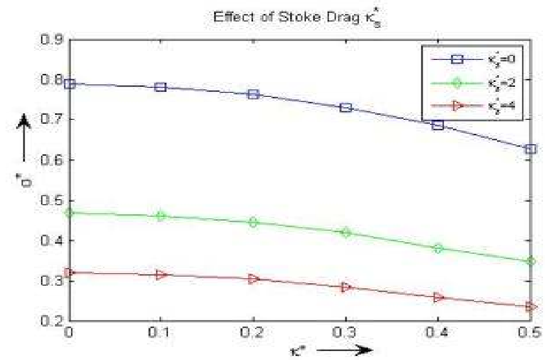

Fig. 6. The growth rate of instability is plotted against the dimensionless wave number k^* with variation in the Stokes' drag constant k_s^* = 0, 2, 4, with taking the values of λ^*, V^*, υ_0^* , f^*, υ^* and τ^* as unity.

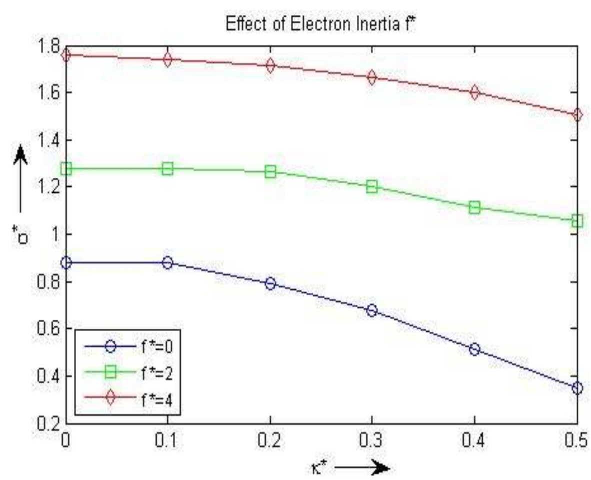

Fig. 7. The growth rate of instability is plotted against the dimensionless wave number k^* with variation in the Electron inertia f^* = 0, 2, 4, with taking the values of k_s^*, υ^* λ^*, V^*, υ_0^* and τ^* as unity.

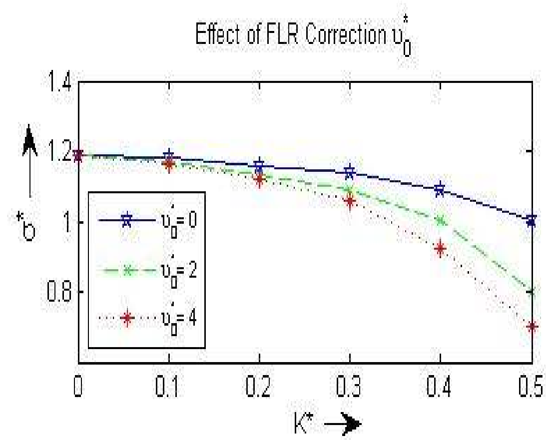

Fig. 8. The growth rate of instability is plotted against the dimensionless wave number k^* with variation in the finite Larmor radius υ_0^* = 0, 2, 4, with taking the values of k_s^*, υ^*, λ^*, V^*, f^* and τ^* as unity.

Fig.5. Shows the variation of the real part of the growth rate (σ^*) with the wave number (k^*) for various values of the relaxation time parameter $\tau^* = 0, 2, 4$ respectively. It is clear from this figure that for any wave number value the growth rate of instability decreases as increasing the relaxation time (τ^*). Hence, the relaxation time has a stabilizing influence on the system.

Fig.6. Shows the variation of the positive real part of growth rate (σ^*) with wave number (k^*) for various values of Stokes drags parameter k_s^* = 0, 2, 4 respectively, if $\tau^* = 1$, $f^* = 1$, υ_0^*= 1, $\upsilon^* = 1$, $\lambda^* = 1$, $V^* = 1$. It is clear from this figure that, for any wave number value, the real positive root of growth rate σ^* decreasing by increasing the Stokes drag parameters by indicates that the Stokes drag k_s^* has a stabilizing effect.

Fig.7. Shows the variation of the positive real part of growth rate (σ^*) with wave number (k^*) for various values of finite electron inertia parameter f^* = 0, 2, 4 respectively, if $\tau^* = 1$, $k_s^* = 1$, $\upsilon^* = 1$, υ_0^*=1, $\lambda^* = 1$, $V^* = 1$. It is clear from this figure that, any wave number value, the real positive root of the growth rate of instability σ^* increases by increasing the finite electron inertia (f^*) parameter which indicates that the finite electron inertia has a destabilizing effect.

Fig.8. Shows the variation of the positive real root of growth rate (σ^*) of unstable mode with the wave number (k^*) for various values of finite Larmor radius υ_0^* = 0, 2, 4 respectively, if $\tau^* = 1$, $\upsilon^* = 1$, $k_s^* = 1$, f^*=1, $\lambda^* = 1$, $V^* = 1$. It is clear from this figure that, any wave number value, the real positive root σ^* Of instability slightly decreases with increasing the finite Larmor radius parameter, which indicates that the finite Larmor radius parameter has to stabilize effect.

5. CONCLUSION

We have studied the gravitational instability of a self-gravitating media under the combined influence of FLR correction, finite electron inertia, suspended particles, viscosity, thermal conductivity and electrical resistivity in the presence of a transverse magnetic field. The general dispersion relation is obtained using normal mode analysis. The analytical expression of the general dispersion relation is obtained with the help of linearized perturbation equations. The general dispersion relation is modified due to the presence of these parameters. The Jeans criterion of instability remains valid, but the critical Jean's wave number is modified. The viscosity parameter has a stabilizing effect on the system in the longitudinal modes of propagation. The Thermal conductivity has a destabilizing influence on the longitudinal wave propagation. The Relaxation time and Stoke drag parameter have a stabilize the system in both the longitudinal and transverse mode of propagation. From the curves, it is found that the thermal conductivity and viscosity show mutually reverse effects on the growth rate of the instability. In other words, the thermal conductivity has a destabilizing influence, while the viscosity has a stabilizing role in the growth rate of the system. The FLR corrections have a stabilizing influence on the transverse wave propagations. In the case of longitudinal propagation, the gravitating mode is influenced by viscosity, thermal conductivity, permeability and suspended particles, but not affected by finite electron inertia, magnetic field and FLR correction. The parameters of the magnetic field, finite electron inertia, FLR (finite Larmor radius) corrections and suspended particles do not change the Jeans condition in this case. The dynamical stability of the system, in this case, is analyzed by applying the Routh-Hurwitz criterion. In the transverse mode of propagation, the self-gravitating Alfven mode is influenced by finite electron inertia, FLR, suspended particles, viscosity, permeability and thermal conductivity of the medium. The Jean's condition of instability is modified by finite electron inertia, thermal conductivity and magnetic field, but not affected by viscosity and suspended particles. In this case, curves depict the effects relaxation time (τ^*), Stokes drag (k_s^*), electron inertia (f^*) and FLR corrections (υ_0^*) parameters. From the curve, it is found that thermal conductivity and FLR correction shows mutually reverse effects on the growth rate of instability. In other words, the thermal conductivity has a destabilizing influence, while the FLR correction has a stabilizing role in the growth rate of the instability. The finite electron inertia has a destabilizing influence on the growth rate of instability. Also, it decreasing the peak value of the growth rate means that the system becomes more and more unstable for higher values of the finite electron inertia parameter. Also, the system becomes more unstable in the presence of finite electron inertia effects and it is more probable to have larger clouds comparing to the ideal system. Despite the vital role of FLR corrections in the very dense interstellar cloud is a key process in the standard theory of star formation, our results show that such non-ideal mechanisms may operate thermally unstable systems such as warm interstellar medium and astrophysical problem.

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