The electrodynamic vacuum field theory approach and the electron inertia problem revisiting

It is a review of some new electrodynamics models of interacting charged point particles and related with them fundamental physical aspects, motivated by the classical A.M.Amper's magnetic and H.Lorentz force laws, as well as O. Jefimenko electromagnetic field expressions. Based on the suitably devised vacuum field theory approach the Lagrangian and Hamiltonian reformulations of some alternative classical electrodynamics models are analyzed in detail~~s~~. A problem closely related to the radiation reaction force is analyzed aiming to explain the Wheeler and Feynman reaction radiation mechanism, well known as the absorption radiation theory, and strongly dependent on the Mach type interaction of a charged point particle in an ambient vacuum electromagnetic medium. There are discussed some relationships between this problem and the one derived within the context of the vacuum field theory approach. The R.Feynman's "heretical" approach to deriving the Lorentz force based Maxwell electromagnetic equations is also revisited, its complete legacy is argued both by means of the geometric considerations and its deep relation with the devised vacuum field theory approach. Based on completely standard reasonings, we reanalyze the Feynman's derivation from the classical Lagrangian and Hamiltonian points of view and construct its nontrivial relativistic generalization compatible with the vacuum field theory approach. The electron inertia problem is reanalyzed within the Lagrangian-Hamiltonian formalisms and the related Feynman proper time paradigm. The validity of the Abraham-Lorentz electromagnetic electron mass origin hypothesis within the shell charged model is argued. The electron stability in the framework of the electromagnetic tension-energy compensation principle is analyzed.

Cet article présente un réexamen de certains nouveaux modèles électrodynamiques d'interaction entre particules chargées ponctuelles, et en lien avec des aspects physiques fondamentales, motivés par les lois magnétiques classiques de A.M. Ampère et par les forces classiques de H.Lorentz, ainsi que par les formulations du champ électromagnétique décrites par O. Jefimenko. Sur la base d'une formulation adéquate de la théorie des champs en vide quantique, les reformulations Lagrangiennes et Hamiltoniennes de certains modèles alternatifs de l'électrodynamique classique sont analysés en profondeur. Un problème étroitement lié à la force de réaction de rayonnement est analysé pour objectif d'expliquer le mécanisme de Wheeler et Feynman de réaction au rayonnement, bien connu comme la théorie d'amortissement de radiation, et dépend fortement de l'interaction de type Mach de particules ponctuelles chargées dans un milieu électromagnétique en vide ambiant. Certains rapports entre ce problème et celui obtenu dans le cadre de l'approche de la théorie des champs en vide quantique sont examinés. L'approche "hérétique" de R.Feynman qui consiste à dériver la force de Lorentz depuis équations électromagnétiques de Maxwell est également revisitée, et son approche est justifiée à la fois par des considérations géométriques et sa relation profonde avec l'approche de la théorie des champs à vide quantique. Sur la base de raisonnements complètement standards, nous réanalysons la dérivation de Feynman des points de vue Lagrangiens et Hamiltoniens classiques et construisons sa généralisation relativiste non triviale compatible avec l'approche de la théorie des champs en vide quantique. Le problème de l'inertie des électrons est réanalysé dans les formalismes de Lagrange et Hamilton et dans le paradigme de Feynman en temps propre correspondant. La validité de l'hypothèse d'Abraham-Lorentz sur l'origine électromagnétique de la masse de l'électron dans le modèle de couche électronique est soutenu. La stabilité de l'électron dans le cadre du principe de compensation tension-énergie électromagnétique est analysée.

Keywords: Amper law, Lorentz type force, Lorenz constraint, Vacuum field theory approach, Maxwell electromagnetic equation,Lagrangian and Hamiltonian formalisms, Fock many-temporal approach, Jefimenko equations, Quantum self-interactifermi model, Radiation theory, Feynman's proper time approach, Abraham-Lorentz electron mass problem.

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1. Classical relativistic electrodynamics models revisiting: Lagrangian and Hamiltonian analysis

1.1. Introductory setting

The Maxwell's equations serve as foundational [1] [2] [3] [4] [5] to the whole modern classical and quantum electromagnetic theory and electrodynamics. They are the cornerstone of a myriad of technologies and are basic to the understanding of innumerable effects. Yet there are a few effects or physical phenomena that cannot be explained [6] [7] [8] [9] [10] [11] [12] [13] within the conventional Maxwell theory. It is important to note here that ~~in~~ [8] [14] [15] [16] [17] ~~there is~~ argued that the Maxwell equations ~~as~~ themselves do not determine causal related ~~to each other~~ relationships between electric and magnetic fields, which prove, in reality, to be generated independently by an external charge and current distributions: "There is a widespread interpretation of Maxwell's equations indicating that spatially varying electric and magnetic fields can cause each other to change in time, thus giving rise to a propagating electromagnetic wave... However, Jefimenko's equations show an alternative point of view [3]. Jefimenko says: "...neither Maxwell's equations nor their solutions indicate an existence of causal links between electric and magnetic fields. Therefore, we must conclude that an electromagnetic field is a *dual entity* always having an electric and a magnetic component simultaneously *created by their common sources*: time-variable electric charges and currents." .... Essential features of these equations are easily observed which are that the right hand sides involve "retarded" time which reflects the "causality" of the expressions. In other words, the left side of each equation is actually "caused" by the right side, unlike the normal differential expressions for Maxwell's equations, where both sides take place simultaneously. In the typical expressions for Maxwell's equations there is no doubt that both sides are equal to each other, but as Jefimenko notes [3], "... since each of these equations connects quantities simultaneous in time, none of these equations can represent a causal relation." The second feature is that the expression for (electric field) E does not depend upon (magnetic field) B and vice versa. Hence, it is impossible for E and B fields to be "creating" each other. Charge density and current density are creating them both." As the Jefimenko's equations for the electric field E and the magnetic field B directly follow from the classical retarded Lienard-Wiechert potentials, generated by physically real external charge and current distributions, one naturally infers that these potentials also present suitably interpreted physical field entities mutually related to their sources. This way of thinking proved to be, from the physical point of view, very fruitful, having brought about a new vacuum field theory approach [18] [19] to alternative explaining the nature of the fundamental Maxwell equations and related electrodynamic phenomena.

We start from detailed revisiting the classical A.M. Ampere's law in electrodynamics and show that main inferences suggested by physicists of the former centuries can be strongly extended for them to agree more exactly with many modern both theoretical achievements and experimental results concerning the fundamental relationship of electrodynamic phenomena with the physical structure of vacuum as their principal carrier.

We discuss ~~The~~ important theoretical physical principles, characterizing the related electrodynamic vacuum field structure, ~~we discuss~~ subject to different charged point particle dynamics, based on the fundamental least action principle. In particular, we obtain the main classical relativistic relationships, characterizing the charge point particle dynamics, ~~we obtain~~ by means of the least action principle within the Feynman's approach to the Maxwell electromagnetic equations and the related Lorentz type force derivation. Moreover, for each of the least action principles constructed in the work, we describe the corresponding Hamiltonian pictures and present the related energy conservation laws. The elementary point charged particle, like electron, mass problem was inspiring many physicists [20] from the past as J. J. Thompson, G.G. Stokes, H.A. Lorentz, E. Mach, M. Abraham, P.A. M. Dirac, G.A. Schott and others. Nonetheless, their studies have not given rise to a clear explanation of this phenomenon that stimulated new researchers to tackle it from different approaches based on new ideas stemming both from the classical Maxwell-Lorentz electromagnetic theory, as in [1] [12] [21] [22] [23] [24] [25] [26] [27] [28] [29] [30] [31] [32] [33] [34] [35] [36] [37] [38] [39], and modern quantum field theories of Yang-Mills and Higgs type, as in [40] [41] [42] [43] and others, whose recent and extensive review is done in [44].

We will mostly concentrate on detail [ed] analysis and consequences of the Feynman proper time paradigm [1] [22] [45] [46] subject to deriving the electromagnetic Maxwell equations and the related Lorentz like force expression considered from the vacuum field theory approach, developed in works [47] [48] [49] [50] [51], and further, on its applications to the electromagnetic mass origin problem. Our treatment of this and related problems, based on the least action principle within the Feynman proper time paradigm [1], has allowed to construct the respectively modified Lorentz type equation for a moving in space and radiating energy charged point particle. Our analysis also elucidates, in particular, the computations of the self-interacting electron mass term in [29], where there was proposed a not proper solution to the well known classical Abraham-Lorentz [52] [53] [54] [55] and Dirac [56] electron electromagnetic "4/3-electron mass" problem. As a result of our scrutinized studying the classical electromagnetic mass problem we have stated that it can be satisfactory solved within the classical H. Lorentz and M. Abraham reasonings augmented with the additional electron stability condition, which was not taken before into account yet appeared to be very important for balancing the related electromagnetic field and mechanical electron momenta. The latter, following recent enough works [31] [35], devoted to analyzing the electron charged shell model, can be realized within the~~se~~ suggested *pressure-energy compensation principle*, suitably applied to the ambient electromagnetic energy fluctuations and the ~~own~~ electrostatic Coulomb electron energy.

In our investigation, we were in part inspired by works [35] [39] [43] [44] [57] [58] [59] to solving the classical problem of reconciling gravitational and electrodynamic charges within the Mach-Einstein ether paradigm. First, we will revisit the classical Mach-Einstein type relativistic electrodynamics of a moving charged point particle, and second, we study the resulting electrodynamic theories associated with our vacuum potential field dynamical equations (31) and (32), making use of the fundamental Lagrangian and Hamiltonian formalisms which were specially devised in [50] [51].

1.2. Classical Maxwell equations and their electromagnetic potentials form revisiting

As the classical Lorentz force expression with respect to an arbitrary inertial reference frame is related with many theoretical and experimental controversies, such as the relativistic potential energy impact into the charged point particle mass, the Aharonov-Bohm effect [60] and the Abraham-Lorentz-Dirac radiation force [2] [5] [6] expression, the analysis of its structure subject to the assumed vacuum field medium structure is a very interesting and important problem, which was discussed by many physicists including E. Fermi, G. Schott, R. Feynman, F. Dyson [1] [45] [46] [61] [62] [63] and many others. To describe the essence of the electrodynamic problems related with the description of a charged point particle dynamics under external electromagnetic field, let us begin with analyzing the classical Lorentz force expression

$$dp/dt = F_L := \xi E + \xi u \times B, \tag{1}$$

where $\xi \in \mathrm{R}$ is a particle electric charge, $u \in T(\mathrm{R}^3)$ is its velocity [47] [64] vector, expressed here in the light speed c units,

$$E := -\partial A/\partial t - \nabla \varphi \tag{2}$$

is the corresponding external electric field and

$$B := \nabla \times A \tag{3}$$

is the corresponding external magnetic field, acting on the charged particle, expressed in terms of suitable vector $A: M^4 \to \mathrm{E}^3$ and scalar $\varphi: M^4 \to \mathrm{R}$ potentials. Here, as before, the sign "∇" is the standard gradient operator with respect to the spatial variable $r \in \mathrm{E}^3$, "$\times$" is the usual vector product in three-dimensional Euclidean vector space $\mathrm{E}^3 := (\mathrm{R}^3, <\cdot,\cdot>)$, which is naturally endowed with the classical scalar product $<\cdot,\cdot>$. These potentials are defined on the Minkowski space M^4; $\mathrm{R} \times \mathrm{E}^3$, which models a chosen laboratory reference frame K_t. Now, it is a well known fact [1] [5] [37] [65] that the force expression (1) does not take into account the dual influence of the charged particle on the electromagnetic field and should be considered valid only if the particle charge $\xi \to 0$. This also means that expression (1) cannot be used for
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studying the interaction between two different moving charged point particles, as was pedagogically demonstrated in classical manuals [1] [5]. As the classical Lorentz force expression (1) is a natural consequence of the interaction of a charged point particle with an ambient electromagnetic field, its corresponding derivation based on the general principles of dynamics, was deeply analyzed by R. Feynman and F. Dyson [1] [45] [46].

Taking this into account, it is natural to reanalyze this problem ~~from the classical~~, taking only into account the Maxwell-Faraday wave theory aspect, specifying the corresponding vacuum field medium. Other questionable inferences from the classical electrodynamics theory, which strongly motivated the analysis in this work, are related both with an alternative interpretation of the well-known *Lorenz condition*, imposed on the four-vector of electromagnetic observable potentials $(\varphi, A): M^4 \to T^*(M^4)$ and the classical Lagrangian formulation [5] of charged particle dynamics under an external electromagnetic field. The Lagrangian approach ~~latter~~ is strongly dependent on ~~an~~ the important Einstein notion of the proper reference frame K_r and the related least action principle, so before explaining it in more detail, we first have to analyze the classical Maxwell electromagnetic theory from a strictly dynamical point of view.

Let us consider, with respect to a laboratory reference frame K, the additional *Lorenz condition*

$$\partial\varphi/\partial t + <\nabla, A> = 0, \tag{4}$$

a priori assum~~ed~~ing the Lorentz invariant wave scalar field equation

$$\partial^2\varphi/\partial t^2 - \nabla^2\varphi = \rho \tag{5}$$

and the charge continuity equation

$$\partial\rho/\partial t + <\nabla, J> = 0, \tag{6}$$

where $\rho: M^4 \to \mathbb{R}$ and $J: M^4 \to \mathbb{E}^3$ are, respectively, the charge and current densities of the ambient matter. Then one can derive [18] [51] that the Lorentz invariant wave equation

$$\partial^2 A/\partial t^2 - \nabla^2 A = J \tag{7}$$

and the classical electromagnetic Maxwell field equations [1] [2] [5] [65] [66]

$$\begin{aligned} \nabla\times E + \partial B/\partial t &= 0, <\nabla, E> = \rho, \\ \nabla\times B - \partial E/\partial t &= J, \ <\nabla, B> = 0, \end{aligned} \tag{8}$$

hold for all $(t,r)\in M^4$ with respect to the chosen laboratory reference frame K_r. As ~~it~~ was shown by O.D. Jefimenko [3] [4], the corresponding solutions to (8) for the electric $E: M^4 \to \mathbb{E}^3$ and magnetic $B: M^4 \to \mathbb{E}^3$ fields can be represented (in the light speed $c=1$ units) by means of the following field expressions that are causally independent to each other

$$
\begin{aligned}
E(t,r) \;&=\; \frac{1}{4\pi}\int_{\mathbb{R}^3}\left[\left(\frac{\rho(t_r,r')}{|r-r'|^3}+\frac{1}{|r-r'|^2}\frac{\partial\rho(t_r,r')}{\partial t}\right)(r-r')-\right.\\
&\left.-\frac{1}{|r-r'|^2}\frac{\partial J(t_r,r')}{\partial t}\right]d^3r', \qquad (9)\\
B(t,r) \;&=\; \frac{1}{4\pi}\int_{\mathbb{R}^3}\left[\frac{J(t_r,r')}{|r-r'|^3}+\frac{1}{|r-r'|^2}\frac{\partial J(t_r,r')}{\partial t}\right]\times(r-r')d^3r',
\end{aligned}
$$

where $(t,r)\in M^4$, and $t_r = t-|r-r'|$ is the retarded time. The result (9) was based on direct derivation from the classical Lienard-Wiechert potentials [2] [3] solving the field equations (5) and (7), causally depending on the corresponding charge and current distributions. Based strongly on this fact in [3] [4] there was argued from physical point of view that related with equations (5) and (7) electric and magnetic potentials really constitute some suitably interpreted physical entities, in contrast to the usual statements [1] [2] [5] about their pure mathematical origin.

It is worth to notice here that, inversely, Maxwell's equations (8) do not directly reduce, via definitions (2) and (3), to the wave field equations (5) and (7) without the Lorenz condition (4). This fact and reasonings presented above are very important: they suggest that, when it comes to choose main governing equations, it proves to be natural replacing the Maxwell's equations (8) with the electric potential field equation (5), the Lorenz condition (4) and the charge continuity equation (6). To make the equivalence statement, claimed above, more transparent we formulate it as the following proposition.

Proposition 1. *The Lorentz invariant wave equation (5) together with the Lorenz condition (4) for the observable potentials* $(\varphi, A): M^4 \to T^*(M^4)$ *and the charge continuity relationship (6) are completely equivalent to the Maxwell field equations (8).*

Proof. Substituting (4), into (5), one easily obtains

$$\partial^2\varphi/\partial t^2 = -<\nabla, \partial A/\partial t> = <\nabla, \nabla\varphi> + \rho, \qquad (10)$$

which implies the gradient expression

$$<\nabla, -\partial A/\partial t - \nabla\varphi> = \rho. \qquad (11)$$

Taking into account the electric field definition (2), expression (11) reduces to

$$<\nabla, E> = \rho, \qquad (12)$$

which is the second of the first pair of Maxwell's equations (8).

Now upon applying $\nabla\times$ to definition (2), we find, owing to definition (3), that

$$\nabla\times E + \partial B/\partial t = 0, \qquad (13)$$

which is the first pair of the Maxwell equations (8). Having differentiated with respect to the temporal variable $t\in\mathbb{R}$ the equation (5) and taken into account the charge continuity equation (6), one finds that

$$<\nabla, \partial^2 A/\partial t^2 - \nabla^2 A - J> = 0. \qquad (14)$$

The latter is equivalent to the wave equation (7) if to observe that the current vector $J: M^4 \to E^3$ is defined by means of the charge continuity equation (6) up to a vector function $\nabla \times S: M^4 \to E^3$. Now applying operation $\nabla \times$ to the definition (3), owing to the wave equation (7) one obtains

$$\begin{aligned} \nabla \times B &= \nabla \times (\nabla \times A) = \nabla < \nabla, A > - \nabla^2 A = \\ &= -\nabla(\partial \varphi / \partial t) - \partial^2 A / \partial t^2 + (\partial^2 A / \partial t^2 - \nabla^2 A) = \\ &= \frac{\partial}{\partial t}(-\nabla \varphi - \partial A / \partial t) + J = \partial E / \partial t + J, \end{aligned} \tag{15}$$

leading directly to

$$\nabla \times B = \partial E / \partial t + J,$$

which is the first of the second pair of the Maxwell equations (8). The final "*no magnetic charge*" equation

$$< \nabla, B > = < \nabla, \nabla \times A > = 0,$$

in (8) follows directly from the elementary identity $< \nabla, \nabla \times > = 0$, thereby completing the proof.

This proposition allows us to consider the observable potential functions $(\varphi, A): M^4 \to T^*(M^4)$ as fundamental ingredients of the ambient *vacuum field medium*, by means of which we can try to describe the related physical behavior of charged point particles imbedded in space-time M^4. As there was still written by J.K. Maxwell [67]: "The conception of such a quantity, on the changes of which, and not on its absolute magnitude, the induction currents depends, occurred to Faraday at an early stage of his researches. He observed that the secondary circuit, when at rest in an electromagnetic field which remains of constant intensity, does not show any electrical effect, whereas, if the same state of the field had been suddenly produced, there would have been a current. Again, if the primary circuit is removed from the field, or the magnetic forces abolished, there is a current of the opposite kind. He therefore recognized in the secondary circuit, when in the electromagnetic field, a 'peculiar electrical condition of matter' to which he gave the name of Electrotonic State." The following observation provides a strong support of this reasonings within this vacuum field theory approach:

Observation. *The Lorenz condition* (4) *actually means that the scalar potential field* $\varphi: M^4 \to R$ *continuity relationship, whose origin lies in some new field conservation law, characterizes the deep intrinsic structure of the vacuum field medium.*

To make this observation more transparent and precise, let us recall the definition [1] [5] [65] [66] of the electric current $J: M^4 \to E^3$ in the dynamical form

$$J := \rho u, \tag{16}$$

where the vector $u \in T(R^3)$ is the corresponding charge velocity. Thus, the following continuity relationship

$$\partial \rho / \partial t + < \nabla, \rho u > = 0 \tag{17}$$

holds, which can easily be rewritten ~~**Error! Reference source not found.**~~ as the integral conservation law

$$\frac{d}{dt}\int_{\Omega_t}\rho(t,r)d^3r=0 \tag{18}$$

for the charge inside of any bounded domain $\Omega_t \subset \mathrm{E}^3$, moving in the space-time M^4 with respect to the natural evolution equation for the moving charge system

$$dr/dt := u. \tag{19}$$

Following the above reasoning, we obtain the following result.

Proposition 2. *The Lorenz condition (4) is equivalent to the integral conservation law*

$$\frac{d}{dt}\int_{\Omega_t}\varphi(t,r)d^3r=0, \tag{20}$$

where $\Omega_t \subset \mathrm{E}^3$ is any bounded domain, moving with respect to the charged point particle ξ evolution equation

$$dr/dt = u(t,r), \tag{21}$$

which represents the velocity vector of the related local potential field changes propagating in the Minkowski space-time M^4. Moreover, for a particle with the distributed charge density $\rho: M^4 \to \mathrm{R}$, the following Umov type local energy conservation relationship

$$\frac{d}{dt}\int_{\Omega_t}\frac{\rho(t,r)\varphi(t,r)}{(1-|u(t,r)|^2)^{1/2}}d^3r=0 \tag{22}$$

holds for any $t \in \mathrm{R}$.

Proof. Consider first the corresponding solutions to the potential field equations (5), taking into account condition (16). Owing to the standard results from [1] [5], one finds that

$$A=\varphi u, \tag{23}$$

which gives rise to the following form of the Lorenz condition (4):

$$\partial\varphi/\partial t+<\nabla,\varphi u>=0, \tag{24}$$

This obviously can be rewritten [68] as the integral conservation law *(20)*, so the expression *(20)* is stated.

To state the local energy conservation relationship *(22)* it is necessary to combine the conditions (17), (24) and find that

$$\partial(\rho\varphi)/\partial t+<u,\nabla(\rho\varphi)>+2\rho\varphi<\nabla,u>=0. \tag{25}$$

Taking into account that the infinitesimal volume transformation $d^3r=\chi(t,r)d^3r_0$, where the Jacobian $\chi(t,r):=|\partial r(t;r_0)/\partial r_0|$ of the corresponding transformation $r:\Omega_{t_0}\to\Omega_t$, induced by the Cauchy problem for the differential relationship *(21)* for any $t\in\mathrm{R}$, satisfies the evolution equation

$$d\chi/dt=<\nabla,u>\chi, \tag{26}$$

easily follow~~ing~~ 3 from *(21)*, and applying to the equality (25) the operator $\int_{\Omega_{t_0}}(\ldots)\chi^2d^3r_0$, one obtains that

$$0=\int_{\Omega_{t_0}}\frac{d}{dt}(\rho\varphi\chi^2)d^3r_0=\frac{d}{dt}\int_{\Omega_{t_0}}(\rho\varphi\chi)Jd^3r_0=$$
$$=\frac{d}{dt}\int_{\Omega_t}(\rho\varphi\chi)d^3r:=\frac{d}{dt}\mathrm{E}(\xi;\Omega_t). \tag{27}$$

Here we denoted the conserved charge $\xi:=\int_{\Omega_t}\rho(t,r)d^3r$ and the local energy conservation quantity $\mathrm{E}(\xi;\Omega_t):=\int_{\Omega_t}(\rho\varphi\chi)d^3r=\mathrm{E}(\xi;\Omega_{t_0}), t\in\mathbb{R}$. The latter quantity can be simplified, owing to the infinitesimal Lorentz invariance four-volume measure relationship $d^3r(t,r_0)\wedge dt=d^3r_0\wedge dt_0$, where variables $(t,r)\in\mathbb{R}_t\times\Omega_t\subset M^4$ are, within the present context, taken with respect to the moving reference frame K_t, related to the infinitesimal charge quantity $d\xi(t,r):=\rho(t,r)d^3r$, and variables $(t_0,r_0)\in\mathbb{R}_{t_0}\times\Omega_{t_0}\subset M^4$ are taken with respect to the laboratory reference frame K_{t_0}, related to the infinitesimal charge quantity $d\xi(t_0,r_0)=\rho(t_0,r_0)d^3r_0$, satisfying the charge conservation invariance $\int_{\Omega_t}d\xi(t,r)=\int_{\Omega_{t_0}}d\xi(t_0,r_0)$. The mentioned above infinitesimal Lorentz invariance relationships make it possible to calculate the local energy conservation quantity $\mathrm{E}(\xi;\Omega_0)$ as

$$\mathrm{E}(\xi;\Omega_t) = \int_{\Omega_t}(\rho\varphi\chi)d^3r=\int_{\Omega_t}(\rho\varphi\frac{d^3r}{d^3r_0})d^3r=$$
$$= \int_{\Omega_t}(\rho\varphi\frac{d^3r\wedge dt}{d^3r_0\wedge dt})d^3r=\int_{\Omega_t}(\rho\varphi\frac{d^3r_0\wedge dt_0}{d^3r_0\wedge dt})d^3r= \tag{28}$$
$$= \int_{\Omega_t}(\rho\varphi\frac{dt_0}{dt})\,d^3r=\int_{\Omega_t}\frac{\rho\varphi d^3r}{(1-|u|^2)^{1/2}},$$

where we took into account that $dt=dt_0(1-|u|^2)^{1/2}$. Thus, owing to (27) and (28) the local energy conservation relationship *(22)* is satisfied, proving the proposition.

The constructed above local energy conservation quantity (28) can be rewritten as

$$\mathrm{E}(\xi;\Omega_t)=\int_{\Omega_t}\frac{d\xi(t,r)\varphi(t,r)}{(1-|u|^2)^{1/2}}=\int_{\Omega_{t_0}}d\xi(t_0,r_0)\varphi(t_0,r_0):=\int_{\Omega_{t_0}}d\mathrm{E}(\xi;r_0)=\mathrm{E}(\xi;\Omega_{t_0}), \tag{29}$$

where $d\mathrm{E}(t_0,r_0)=d\xi(t_0,r_0)\varphi(t_0,r_0)$ is the distributed in vacuum electromagnetic field energy density, related with the electric charge $d\xi(t_0,r_0)$, located initially at point $(t_0,r_0)\in M^4$.

The above proposition suggests a physically motivated interpretation of electrodynamic phenomena in terms of what should naturally be called *the vacuum potential field*, which determines the observable interactions between charged point particles. More precisely, we can *a priori* endow the ambient vacuum medium with a scalar potential energy field density function $W := \xi\varphi : M^4 \to \mathbb{R}$, where $\xi \in \mathbb{R}_+$ is the value of an elementary charge quantity, ~~and~~ satisfying the governing *vacuum field equations*

$$\partial^2 W / \partial t^2 - \nabla^2 W = \rho\xi, \quad \partial W / \partial t + \langle \nabla, \mathrm{A} \rangle = 0,$$
$$\partial^2 \mathrm{A} / \partial t^2 - \nabla^2 \mathrm{A} = \xi\rho v, \quad \mathrm{A} = Wv, \tag{30}$$

taking into account the external charged sources, which possess a virtual capability for disturbing the vacuum field medium. Moreover, this vacuum potential field function $W : M^4 \to \mathbb{R}$ allows the natural potential energy interpretation, whose origin should be assigned not only to the charged interacting medium, but also to any other medium possessing interaction capabilities, including for instance, material particles, interacting through ~~the~~ gravity.

The latter leads naturally to the next important step, consisting in deriving the equation governing the corresponding potential field $\bar{W} : M^4 \to \mathbb{R}$, assigned to a charged point particle moving in the vacuum field medium with velocity $u \in T(\mathbb{R}^3)$ and located at point $r(t) := R(t) \in \mathbb{E}^3$ at time $t \in \mathbb{R}$. As can be readily shown [18] [19] [50] [69], the corresponding evolution equation governing the related potential field function $\bar{W} : M^4 \to \mathbb{R}$, assigned to a moving in the space $\mathbb{E}^3$ charged particle ξ under the stationary distributed field sources, has the form

$$\frac{d}{dt}(-\bar{W}u) = -\nabla \bar{W}, \tag{31}$$

the

where $\bar{W} := W(t,r)|_{r \to R(t)}$, $u(t) := dR(t)/dt$ at point particle location $(t, R(t)) \in M^4$.

Similarly, if there are two interacting charged point particles, located at points $r(t) = R(t)$ and $r_f(t) = R_f(t) \in \mathbb{E}^3$ at time $t \in \mathbb{R}$ and moving, respectively, with velocities $u := dR(t)/dt$ and $u_f := dR_f(t)/dt$, the corresponding potential field function $\bar{W}' : M^4 \to \mathbb{R}$, considered with respect to the reference frame K'_t specified by Euclidean coordinates $(t', r - r_f) \in \mathbb{E}^4$ and moving with the velocity $u_f \in T(\mathbb{R}^3)$ subject to the laboratory reference frame K_t, should satisfy [18] [19] with respect to the reference frame K'_t the dynamical equality

$$\frac{d}{dt'}[-\bar{W}'(u' - u'_f)] = -\nabla \bar{W}', \tag{32}$$

where, by definition, we have denoted the velocity vectors $u' := dr/dt'$, $u'_f := dr_f/dt' \in T(\mathbb{R}^3)$. The latter comes with respect to the laboratory reference frame K_t about the dynamical equality

$$\frac{d}{dt}[-\bar{W}(u - u_f)] = -\nabla \bar{W}(1 - |u_f|^2). \tag{33}$$

The dynamical potential field equations (31) and (32) appear to have important properties and can be used as means for representing classical electrodynamic phenomena.

Consequently, we shall proceed to investigate their physical properties in more detail and compare them with classical results for Lorentz type forces arising in the electrodynamics of moving charged point particles in an external electromagnetic field.

1.2.1. Classical relativistic electrodynamics revisited

The classical relativistic electrodynamics of a freely moving charged point particle in the Minkowski space-time $M^4 ;\ \mathbb{R}\times\mathbb{E}^3$ is based on the Lagrangian approach [1] [5] [65] [66] [70] with Lagrangian function

$$\mathrm{L}_0 := -m_0(1-|u|^2)^{1/2}, \tag{34}$$

where $m_0 \in \mathbb{R}_+$ is the so-called particle rest mass parameter with respect to the so called proper reference frame K_τ, parameterized by means of the Euclidean space-time parameters $(\tau, r) \in \mathbb{E}^4$, and $u \in T(\mathbb{R}^3)$ is its spatial velocity with respect to a laboratory reference frame K_t, parameterized by means of the Minkowski space-time parameters $(t, r) \in M^4$, expressed here and in the sequel in light speed units (with light speed $c = 1$). The least action principle in the form

$$\delta S = 0,\ S := -m_0\int_{t_1}^{t_2}(1-|u|^2)^{1/2}dt \tag{35}$$

for any fixed temporal interval $[t_1, t_2] \subset \mathbb{R}$ gives rise to the well-known relativistic relationships for the mass of the particle

$$m = m_0(1-|u|^2)^{-1/2}, \tag{36}$$

the momentum of the particle

$$p := mu = m_0 u(1-|u|^2)^{-1/2} \tag{37}$$

and the energy of the particle

$$\mathrm{E}_0 = m = m_0(1-|u|^2)^{-1/2}. \tag{38}$$

It follows from [5] [65], that the origin of the Lagrangian (34) can be extracted from the action

$$S := -m_0\int_{t_1}^{t_2}(1-|u|^2)^{1/2}dt = -m_0\int_{\tau_1}^{\tau_2}d\tau, \tag{39}$$

on the suitable temporal interval $[\tau_1, \tau_2] \subset \mathbb{R}$, where, by definition,

$$d\tau := dt(1-|u|^2)^{1/2} \tag{40}$$

and $\tau \in \mathbb{R}$ is the so-called, proper temporal parameter assigned to a freely moving particle with respect to the proper reference frame K_τ. The action (39) is rather questionable from the dynamical point of view, since it is physically defined with respect to the proper reference frame K_τ, giving rise to the constant action $S = -m_0(\tau_2 - \tau_1)$, as the limits of integrations $\tau_1 < \tau_2 \in \mathbb{R}$ were taken to be fixed from the very beginning. Moreover, considering this particle to have charge $\xi \in \mathbb{R}$ and be moving in the Minkowski space-time M^4 under action of an electromagnetic field $(\varphi, A) \in T^*(M^4)$, the corresponding classical (relativistic) action functional is chosen (see [1] [5] [47] [51] [65] [66]) as follows:

$$S := \int_{\tau_1}^{\tau_2} [-m_0 d\tau + \xi < A, \dot{r} > d\tau - \xi\varphi(1-|u|^2)^{-1/2} d\tau], \tag{41}$$

with respect to the *proper reference frame*, parameterized by the Euclidean space-time variables $(\tau, r) \in \mathrm{E}^4$, where we have denoted $\dot{r} := dr/d\tau$ in contrast to the definition $u := dr/dt$. The action (41) can be rewritten with respect to the laboratory reference frame K_t as

$$S = \int_{t_1}^{t_2} \mathrm{L}(r, dr/dt)dt, \mathrm{L}(r, dr/dt) := -m_0(1-|u|^2)^{1/2} + \xi < A, u > -\xi\varphi, \tag{42}$$

on the suitable temporal interval $t_1, t_2] \subset \mathrm{R}$, which gives rise to the following [1] [5] [65] [66] dynamical expressions

$$P = p + \xi A, \; p = mu, m = m_0(1-|u|^2)^{-1/2}, \tag{43}$$

for the particle momentum and

$$\mathrm{E}_0 = (m_0^2 + |P - \xi A|^2)^{1/2} + \xi\varphi \tag{44}$$

for the charged particle ξ energy, where, by definition, $P \in \mathrm{E}^3$ is the common momentum of the particle and the ambient electromagnetic field at a Minkowski space-time point $(t, r) \in M^4$. The related dynamics of the charged particle ξ follows [1] [5] [65] [66] from the Lagrangian equation

$$dP/dt := \nabla \mathrm{L}(r, dr/dt) = -\nabla(\xi\varphi - \xi < A, u >). \tag{45}$$

The expression (44) for the particle energy E_0 also appears to be open to question, since the potential energy $\xi\varphi$, entering additively, has no affect on the particle "inertial" mass $m = m_0(1-|u|^2)^{-1/2}$. This was noticed by L. Brillouin [21], who remarked that the fact that the potential energy has no affect on the particle mass tells us that "... any possibility of existence of a particle mass related with an external potential energy, is completely excluded". Moreover, it is necessary to stress here that the least action principle (42), formulated with respect to the laboratory reference frame K_t time parameter $t \in \mathrm{R}$, appears logically inadequate, for there is a strong physical inconsistency with other time parameters of the Lorentz equivalent reference frames. This was first mentioned by R. Feynman in [1] in his efforts to physically argue the Lorentz force expression with respect to the proper reference frame K_τ. This and other special relativity theory and electrodynamics problems stimulated many prominent physicists of the past [1] [21] [65] [71] [72] and present [7] [23] [24] [25] [26] [44] [57] [59] [60] [73] [74] [75] [76] [77] [78] and [79] [80] [81] [11] [82] [69] [83] [84] [85] [86] [87] to try to develop alternative relativity theories based on completely different space-time and matter structure principles. Some of them prove to be closely related with a virtual relationship between electrodynamics and gravity, based on classical works of H. Lorentz, G. Schott, J. Schwinger, R. Feynman [1] [22] [53] [54] [63] [88] and many others on the so called "electrodynamic mass" of elementary particles. Arguing this way of this mass, one can readily come to a certain paradox: the well-known energy-mass relationship for the particle mass suitably determines the energy of its gravitational field. Yet this energy should lead to an increase in the mass of the particle that in turn should lead to increased gravitational field and so on. In the limit, for instance, an electron must have infinite mass and energy, what we do not really observe. There also is another controversial inference from the action expression (42). As one can easily show, owing to (45), the corresponding expression for the Lorentz force

$$dp/dt = F_L := \xi E + \xi u \times B \tag{46}$$

holds, where we have defined here, as before,

$$E := -\partial A/\partial t - \nabla\varphi \tag{47}$$

the corresponding electric field and

$$B := \nabla \times A \tag{48}$$

the related magnetic field, acting on the charged point particle ξ. The expression (46), in particular, means that the Lorentz force F_L depends linearly on the particle velocity vector $u \in T(\mathbb{R}^3)$, and so there is a strong dependence on the reference frame with respect to which the charged particle ξ moves. Attempts to reconcile this and some related controversies [21] [1] [89] [11] [69] [13] forced Einstein to devise his special relativity theory and proceed further to creating his general relativity theory trying to explain the gravity by means of geometrization of space-time and matter in the Universe. Here we must mention that the classical Lagrangian function L in (42) is written in terms of a combination of terms expressed by means of both the Euclidean proper reference frame variables $(\tau, r) \in \mathrm{E}^4$ and arbitrarily chosen Minkowski reference frame variables $(t, r) \in M^4$.

These problems were recently analyzed using a completely different " *no-geometry*" approach [18] [19] [69], where new dynamical equations were derived, which were free of the controversial elements mentioned above. Moreover, this approach avoided the introduction of the well known Lorentz transformations of the space-time reference frames with respect to which the action functional (42) is invariant. From this point of view, there are interesting ~~for discussion~~ conclusions from [90] [91] [92] [93], in which some electrodynamic models, possessing intrinsic Galilean and Poincaré-Lorentz symmetries, were reanalyzed from diverse geometrical points of view. From a completely different point of view the related electrodynamics of charged particles was reanalyzed in [3] [4] [8] [14] [15], where all relativistic relationships were successfully inferred from the classical Lienard-Wiechert potentials, solving the corresponding electromagnetic equations. Subject to a possible geometric space-type structure and the related vacuum field background, exerting the decisive influence on the particle dynamics, we need to mention here recent works [79] [85] [13] and the closely related ~~with their ideas the~~ classical articles [94] [95]. Next, we shall revisit the results obtained in [18] [19] from the classical Lagrangian and Hamiltonian formalisms [47] [64] [66] [96] in order to shed new light on the physical underpinnings of the vacuum field theory approach to the study of combined electromagnetic and gravitational effects.

1.3. The vacuum field theory electrodynamics equations: Lagrangian analysis

1.3.2. A moving in vacuum point charged particle - an alternative electrodynamic

model

In the vacuum field theory approach to combining electromagnetism and the gravity, devised in [18] [19], the main vacuum potential field function $\bar{W}: M^4 \to \mathbb{R}$, related to a charged point particle ξ under the external stationary distributed field sources, satisfies the dynamical equation (30), namely

$$\frac{d}{dt}(-\bar{W}u) = -\nabla \bar{W} \tag{49}$$

in the case when the external charged particles are at rest, where, as above, $u := dr/dt$ is the particle velocity with respect to some reference system.

To analyze the dynamical equation (49) from the Lagrangian point of view, we write the corresponding action functional as

$$S := -\int_{t_1}^{t_2} \bar{W} dt = -\int_{\tau_1}^{\tau_2} \bar{W}(1+|\dot{r}|^2)^{1/2} d\tau, \tag{50}$$

expressed with respect to the proper reference frame K_τ. Fixing the proper temporal parameters $\tau_1 < \tau_2 \in \mathbb{R}$, one finds from the least action principle ($\delta S = 0$) that

$$\begin{aligned} p &:= \partial \mathrm{L}/\partial \dot{r} = -\bar{W}\dot{r}(1+|\dot{r}|^2)^{-1/2} = -\bar{W}u, \\ \dot{p} &:= dp/d\tau = \partial \mathrm{L}/\partial r = -\nabla \bar{W}(1+|\dot{r}|^2)^{1/2}, \end{aligned} \tag{51}$$

where, owing to (50), the corresponding Lagrangian function is

$$\mathrm{L} := -\bar{W}(1+|\dot{r}|^2)^{1/2}. \tag{52}$$

Recalling now the definition of the particle mass

$$m := -\bar{W} \tag{53}$$

and the relationships

$$d\tau = dt(1-|u|^2)^{1/2}, \dot{r}d\tau = udt, \tag{54}$$

from (51) we easily obtain exactly the dynamical equation (49). Moreover, one now readily finds that the dynamical mass, defined by means of expression (53), is given as

$$m = m_0(1-|u|^2)^{-1/2},$$

which coincides with the equation (36) of the preceding section. Now one can formulate the following proposition using the above results.

Proposition 3. *The alternative freely moving point particle electrodynamic model (49) allows the least action formulation (50) with respect to the "rest" reference frame variables, where the Lagrangian function is given by expression (52). Its electrodynamics is completely equivalent to that of a classical relativistic freely moving point particle, described in Subsection 1.2.1 .*

1.3.3. A moving in vacuum interacting two charge system - an alternative electrodynamic model

We proceed now to the case when our charged point particle ξ moves in the space-time with velocity vector $u \in T(\mathbb{R}^3)$ and interacts with another external charged point particle ξ_f, moving with velocity vector $u_f \in T(\mathbb{R}^3)$ with respect to a common reference frame K_t. As was shown in [18] [19], the respectively modified dynamical equation for the vacuum potential field function $\bar{W}' : M^{4,'} \to \mathbb{R}$ subject to the moving reference frame K_t' is given by equality (32), or

$$\frac{d}{dt'}[-\bar{W}'(u'-u_f')] = -\nabla \bar{W}', \qquad (55)$$

where, as before, the velocity vectors $u' := dr/dt', u_f' := dr_f/dt' \in T(\mathbb{R}^3)$. Since the external charged particle ξ_f moves in the space-time M^4, it generates the related magnetic field $B := \nabla \times A$, whose magnetic vector potentials $A : M^4 \to \mathbb{E}^3$ and $A' : M^{4,'} \to \mathbb{E}^3$ are defined, owing to the results of [18] [19] [69], as

$$\xi A := \bar{W} u_f, \xi A' := \bar{W}' u_f', \qquad (56)$$

Whence, taking into account that the field potential

$$\bar{W} = \bar{W}'(1-|u_f|^2)^{-1/2} \qquad (57)$$

and the particle momentum $p' = -\bar{W}' u' = -\bar{W} u$, equality (55) becomes equivalent to

$$\frac{d}{dt'}(p' + \xi A') = -\nabla \bar{W}', \qquad (58)$$

if considered with respect to the moving reference frame K_t', or to the Lorentz type force equality

$$\frac{d}{dt}(p + \xi A) = -\nabla \bar{W}(1-|u_f|^2), \qquad (59)$$

if considered with respect to the laboratory reference frame K_t, owing to the classical Lorentz invariance relationship (57), as the corresponding magnetic vector potential, generated by the external charged point test particle ξ_f with respect to the reference frame K_t', is identically equal to zero. To imbed the dynamical equation (59) into the classical Lagrangian formalism, we start from the following action functional, which naturally generalizes the functional (50):

$$S := -\int_{\tau_1}^{\tau_2} \bar{W}'(1+|\dot{r}-\dot{r}_f|^2)^{1/2} d\tau. \qquad (60)$$

Here, as before, $\bar{W}'$ is the respectively calculated vacuum field potential $\bar{W}$ subject to the moving reference frame K_t', $\dot{r} = u' dt'/d\tau, \dot{r}_f = u_f' dt'/d\tau$, $d\tau = dt'(1-|u'-u_f'|^2)^{1/2}$, which take into account the relative velocity of the charged point particle ξ subject to the reference frame K_t', specified by the Euclidean coordinates $(t', r-r_f) \in \mathbb{R}^4$, and moving simultaneously with velocity vector $u_f \in T(\mathbb{R}^3)$ with respect to the laboratory reference frame K_t, specified by the Minkowski coordinates $(t, r) \in M^4$ and related to those of the reference frame K_t' and K_τ by means of the following infinitesimal relationships:

$$dt^2 = (dt')^2 + |dr_f|^2, (dt')^2 = d\tau^2 + |dr - dr_f|^2 . \tag{61}$$

So, it is clear in this case that our charged point particle ξ moves with the velocity vector $u' - u'_f \in T(\mathbb{R}^3)$ with respect to the reference frame K'_t in which the external charged particle ξ_f is at rest. Thereby, we have reduced the problem of deriving the charged point particle ξ dynamical equation to that before solved in Subsection 1.2.1.

Now we can compute the least action variational condition $\delta S = 0$, taking into account that, owing to (60), the corresponding Lagrangian function with respect to the proper reference frame K_τ is given as

$$\mathrm{L} := -\bar{W}' (1 + |\dot{r} - \dot{r}_f|^2)^{1/2} . \tag{62}$$

As a result of simple calculations, the generalized momentum of the charged particle ξ equals

$$\begin{aligned} P &:= \partial \mathrm{L} / \partial \dot{r} = -\bar{W}' (\dot{r} - \dot{r}_f)(1 + |\dot{r} - \dot{r}_f|^2)^{-1/2} = \\ &= -\bar{W}' \dot{r}(1 + |\dot{r} - \dot{r}_f|^2)^{-1/2} + \bar{W}' \dot{r}_f (1 + |\dot{r} - \dot{r}_f|^2)^{-1/2} = \\ &= m' u' + \xi A' := p' + \xi A' = p + \xi A, \end{aligned} \tag{63}$$

where, owing to (57) the vectors $p' := -\bar{W}' u' = -\bar{W} u = p \in \mathbb{E}^3$, $A' = \bar{W}' u'_f = \bar{W} u_f = A \in \mathbb{E}^3$, and giving rise to the dynamical equality

$$\frac{d}{d\tau}(p' + \xi A') = -\nabla \bar{W}' (1 + |\dot{r} - \dot{r}_f|^2)^{1/2} \tag{64}$$

with respect to the proper reference frame K_τ. As $dt' = d\tau(1 + |\dot{r} - \dot{r}_f|^2)^{1/2}$ and $(1 + |\dot{r} - \dot{r}_f|^2)^{1/2} = (1 - |u' - u'_f|^2)^{-1/2}$, we obtain from (64) the equality

$$\frac{d}{dt'}(p' + \xi A') = -\nabla \bar{W}' , \tag{65}$$

exactly coinciding with equality (58) subject to the moving reference frame K'_t. Now, making use of expressions (61) and (57), one can rewrite (65) as that with respect to the laboratory reference frame K_t :

$$\begin{aligned}
&\frac{d}{dt'}(p'+\xi A') = -\nabla \bar{W}' \Rightarrow \\
&\Rightarrow \frac{d}{dt'}\left(\frac{-\bar{W}u'}{(1+|u'_f|^2)^{1/2}} + \frac{\xi \bar{W}u'_f}{(1+|u'_f|^2)^{1/2}}\right) = -\frac{\nabla \bar{W}}{(1+|u'_f|^2)^{1/2}} \Rightarrow \\
&\Rightarrow \frac{d}{dt'}\left(\frac{-\bar{W}dr}{(1+|u'_f|^2)^{1/2}dt'} + \frac{\xi \bar{W}dr_f /}{(1+|u'_f|^2)^{1/2}dt'}\right) = -\frac{\nabla \bar{W}}{(1+|u'_f|^2)^{1/2}} \Rightarrow \\
&\Rightarrow \frac{d}{dt}\left(-\bar{W}\frac{dr}{dt} + \xi \bar{W}\frac{dr_f}{dt}\right) = -\nabla \bar{W}(1-|u_f|^2),
\end{aligned} \tag{66}$$

exactly coinciding with (59):

$$\frac{d}{dt}(p+\xi A) = -\nabla \bar{W}(1-|u_f|^2). \tag{67}$$

Remark 1. *The equation (67) allows to infer the following important and physically reasonable phenomenon: if the test charged point particle velocity* $u_f \in T(\mathbb{R}^3)$ *tends to the light velocity* $c=1$, *the corresponding acceleration force* $F_m := -\nabla \bar{W}(1-|u_f|^2)$ *is vanishing. Thereby, the electromagnetic fields, generated by such rapidly moving charged point particles, have no influence on the dynamics of charged objects if observed with respect to an arbitrarily chosen laboratory reference frame* K_t.

The latter equation (67) can be easily rewritten as

$$\begin{aligned}
dp/dt &= -\nabla \bar{W} - \xi dA/dt + \nabla \bar{W}|u_f|^2 = \\
&= \xi(-\xi^{-1}\nabla \bar{W} - \partial A/\partial t) - \xi < u, \nabla > A + \xi \nabla < A, u_f >,
\end{aligned} \tag{68}$$

or, using the well-known **Error! Reference source not found.** identity

$$\nabla < a,b > = < a,\nabla > b + < b,\nabla > a + b\times(\nabla\times a) + a\times(\nabla\times b), \tag{69}$$

where $a,b \in \mathbb{E}^3$ are arbitrary vector functions, in the standard Lorentz type form

$$dp/dt = \xi E + \xi u\times B - \nabla < \xi A, u-u_f >. \tag{70}$$

The result (70), being before found and written down with respect to the moving reference frame K'_t in [18] [19] [69] makes it possible to formulate the next important proposition.

Proposition 4. *The alternative classical relativistic electrodynamic model (58) allows the least action formulation based on the action functional (60) with respect to the proper reference* *frame K_τ, where the Lagrangian function is given by expression (62). The resulting Lorentz type force expression equals (70), being modified by the additional force component $F_c := -\nabla < \xi A, u - u_f >$, important for explanation* [97] [98] [99] *of the well known Aharonov-Bohm effect.*

1.3.4. A moving charged point particle dynamics formulation dual to the classical relativistic invariant alternative electrodynamic model

It is easy to see that the action functional (60) is written utilizing the classical Galilean transformations of reference frames. If we now consider the action functional (50) for a charged point particle moving with respect the reference frame K_τ, and take into account its interaction with an external magnetic field generated by the vector potential $A:\ M^4 \to \mathrm{E}^3$, it can be naturally generalized as

$$S := \int_{t_1}^{t_2} (-\bar{W} dt + \xi < A, dr >) = \int_{\tau_1}^{\tau_2} [-\bar{W}(1 + |\dot{r}|^2)^{1/2} + \xi < A, \dot{r} >] d\tau, \tag{71}$$

where $d\tau = dt(1 - |u|^2)^{1/2}$.

Thus, the corresponding common particle-field momentum takes the form

$$\begin{aligned} P \ &:= \partial \mathrm{L} / \partial \dot{r} = -\bar{W}\dot{r}(1 + |\dot{r}|^2)^{-1/2} + \xi A = \\ &= mu + \xi A := p + \xi A, \end{aligned} \tag{72}$$

and satisfies

$$\begin{aligned} \dot{P} \ &:= dP / d\tau = \partial \mathrm{L} / \partial r = -\nabla \bar{W}(1 + |\dot{r}|^2)^{1/2} + \xi \nabla < A, \dot{r} >= \\ &= -\nabla \bar{W}(1 - |u|^2)^{-1/2} + \xi \nabla < A, u > (1 - |u|^2)^{-1/2}, \end{aligned} \tag{73}$$

where

$$\mathrm{L} := -\bar{W}(1 + |\dot{r}|^2)^{1/2} + \xi < A, \dot{r} > \tag{74}$$

is the corresponding Lagrangian function. Since $d\tau = dt(1 - |u|^2)^{1/2}$, one easily finds from (73) that

$$dP / dt = -\nabla \bar{W} + \xi \nabla < A, u >. \tag{75}$$

Upon substituting (72) into (75) and making use of the identity (69), we obtain the classical expression for the Lorentz force F, acting on the moving charged point particle ξ:

$$dp / dt := F_L = \xi E + \xi u \times B, \tag{76}$$

where, by definition,

$$E := -\xi^{-1} \nabla \bar{W} - \partial A / \partial t \tag{77}$$

is its associated electric field and

$$B := \nabla \times A \tag{78}$$

is the corresponding magnetic field. This result can be summarized as follows.

Proposition 5. *The classical relativistic Lorentz force (76) allows the least action formulation (71)* *with respect to the proper reference frame variables, where the Lagrangian function is given by formula (74). Yet its electrodynamics, described by the Lorentz force (76), is not equivalent to the classical relativistic moving point particle electrodynamics, described by means of the Lorentz force (46), as the inertial mass expression* $m=-\bar{W}$ *does not coincide with that of (36).*

Expressions (76) and (70) are equal up to the gradient like term $F_c := -\nabla < \xi A, u-u_f >$, which reconciles the Lorentz forces acting on a charged moving particle ξ with respect to different reference frames. This fact is important for our vacuum field theory approach since it uses no special geometry and makes it possible to analyze both electromagnetic and gravitational fields simultaneously by employing the new definition of the dynamical mass by means of the Mach-Einstein type expression (53).

1.4. The A.M. Ampere's law in electrodynamics - the classical and modified Lorentz force derivations

The classical ingenious Andre-Marie Ampere's analysis of magnetically interacting ~~to each other~~ two electric currents in thin conductors~~, as is well known,~~ was based [1] [5] [65] [66] on the following experimental fact: the force between two electric currents depends on the distance between conductors, their mutual spatial orientation and the ~~quantitative values of~~ currents. Having additionally accepted the infinitesimal superposition principle ~~of~~ A.M. Ampere ~~had~~ derived a general analytical expression for the force between two infinitesimal elements of currents ~~under regard~~:

$$df(r,r') = II' \frac{(r-r')}{|r-r'|^2} \alpha(s,s';n) dl dl', \qquad (79)$$

where vectors $r, r' \in \mathrm{E}^3$ point at infinitesimal currents $dr = s dl, dr' = s' dl'$ with normalized orientation vectors $s, s' \in \mathrm{E}^3$ of two closed conductors l and l' carrying currents $I \in \mathrm{R}$ and $I' \in \mathrm{R}$, respectively and the unit vector $n := (r-r')/|r-r'|$, fixing the spatial orientations of these infinitesimal elements, and the function $\alpha : (\mathrm{S}^2)^2 \times \mathrm{S}^2 \to \mathrm{R}$ being some real-valued smooth mapping. Taking further into account the mutual symmetry between the infinitesimal elements of currents dl and dl', belonging respectively to these two electric conductors, the infinitesimal force (79) was assumed by A.M. Ampere *to satisfy locally the third Newton's law*:

$$df(r,r') = -df(r',r) \qquad (80)$$

with the mapping

$$\alpha(s,s';n) = \frac{\mu_0}{4\pi}(3k_1 < s,n >< s',n > + k_2 < s,s' >),. \qquad (81)$$

where $<\cdot,\cdot>$ is the natural scalar product in E^3 and $k_1, k_2 \in \mathrm{R}$ are some still undetermined real and dimensionless parameters. The assumption (80) is evidently looking very restrictive and can be considered as reasonable only subject to a stationary system of conductors under regard, when the mutual action at a distance principle [1] [5] can be applied. According to ~~Owing to himself~~ J.C. Maxwell [67]: "*... we may draw the conclusions, first, that action and reaction are not always equal and opposite, and second, that apparatus may be constructed to generate any amount of work from its own resources. For let two oppositely electrified bodies A and B travel along the line joining them with equal velocities in the direction AB, then if either the potential or the attraction of the bodies at a given time is that due to their position at some former time (as these authors suppose), B, the foremost body, will attract A forwards more than B attracts A backwards. Now let A and B be kept asunder by a rigid rod. The combined system, if set in motion in the direction AB, will pull in that direction with a force which may either continually augment the velocity, or may be used as an inexhaustible source of energy.*"

Based on the fact that there is no possibility to measure the force between two infinitesimal current elements, A.M. Ampere took into account (80), (81) and calculated the corresponding force exerted by the whole conductor l' on an infinitesimal current element of the other conductor under regard:

$$
\begin{aligned}
dF(r) &:= \oint_{l'} df(r,r') = \\
&= \frac{II'\mu_0}{4\pi}\oint_{l'} \frac{(r-r')}{|r-r'|^2}\left(3k_1 \left\langle dr, \frac{r-r'}{|r-r'|}\right\rangle\left\langle dr', \frac{r-r'}{|r-r'|}\right\rangle + k_2 \frac{r-r'}{|r-r'|}\langle dr, dr'\rangle\right) = \\
&= \frac{II'\mu_0}{4\pi}\oint_{l'} \nabla_{r'}\left(\frac{1}{|r-r'|}\right)\left(3k_1 \langle dr, r-r'\rangle\langle dr', r-r'\rangle + k_2 \langle dr, dr'\rangle\right),
\end{aligned}
\tag{82}
$$

which can be equivalently transformed as

$$
\begin{aligned}
dF(r) &= \frac{II'\mu_0}{4\pi}\oint_{l'} \nabla_{r'}\left(\frac{1}{|r-r'|}\right)\left(3k_1 \langle dr, r-r'\rangle\langle dr', r-r'\rangle + k_2 \langle dr, dr'\rangle\right) = \\
&= \frac{II'\mu_0}{4\pi}\oint_{l'} \nabla_{r'}\left(\frac{1}{|r-r'|}\right)[k_1(3\langle dr, r-r'\rangle\langle dr', r-r'\rangle - \\
&\quad - \langle dr, dr'\rangle) + (k_1+k_2)\langle dr, dr'\rangle] = \\
&= -k_1 \frac{\mu_0 I}{4\pi}\left\langle dr, \nabla \oint_{l'}\left(\frac{I' dr'}{|r-r'|}\right)\right\rangle - (k_1+k_2)\left\langle \nabla, \oint_{l'}\left\langle dr, \frac{I' dr'}{|r-r'|}\right\rangle\right\rangle,
\end{aligned}
\tag{83}
$$

owing to the integral identity