$$\oint_{l'} \nabla_{r'}\left(\frac{1}{|r-r'|}\right)(3<dr,r-r'><dr',r-r'>-<dr,dr'>)=<dr,\nabla>\oint_{l'}\frac{dr'}{|r-r'|}, \tag{84}$$

which can be easily checked by means of integration by parts. ~~If to~~ By introduc~~e~~ing the vector potential

$$A(r):=\frac{\mu_0 I'}{4\pi}\oint_{l'}\frac{dr'}{|r-r'|}, \tag{85}$$

generated by the conductor $l'$ at point $r\in \mathrm{E}^3$, belonging to the infinitesimal element $dl$ of the conductor $l$, the resulting infinitesimal force (83) gives rise to the following expression:

$$\begin{aligned} dF(r) &= k_1(-I<dr,\nabla)A(r)+I\nabla<dr,A(r)>)-(2k_1+k_2)I\nabla<dr,A(r)>= \\ &= k_1 I dr\times(\nabla\times A(r))-(2k_1+k_2)I\nabla<dr,A(r)>= \\ &= k_1 J(r)d^3r\times B(r)-(2k_1+k_2)\nabla<Jd^3r,A(r)>, \end{aligned} \tag{86}$$

where we have taken into account the standard magnetic field definition

$$B(r):=\nabla\times A(r) \tag{87}$$

and the corresponding current density relationship

$$J(r)d^3r:=Idr. \tag{88}$$

There are, evidently, many different possibilities to choose the dimensionless parameters $k_1,k_2\in \mathrm{R}$. In his analysis A.M. Ampere had chosen the case when $k_1=1, k_2=-2$ and obtained the well known ~~nowadays~~ *magnetic force* expression

$$dF(r)=J(r)d^3r\times B(r), \tag{89}$$

which easily reduces to the *classical Lorentz expression*

$$df_L(r)=\xi u\times B(r) \tag{90}$$

~~charged~~ point particle

for a force exerted by an external magnetic field on a moving with a velocity $u\in T(\mathrm{R}^3)$ ~~point particle~~ with an electric charge $\xi\in \mathrm{R}$.

If to take *an alternative choice* and put $k_1=1, k_2=-1$, the expression (86) yields *a modified magnetic Lorentz type force*, exerted by an external magnetic field generated by a moving charged particle with a velocity $u'\in T(\mathrm{R}^3)$ on a point particle, endowed with the electric charge $\xi\in \mathrm{R}$ and moving with a velocity $u\in T(\mathrm{R}^3)$:

$$dF(r)=J(r)d^3r\times B(r)-\nabla<J(r)d^3r,A(r)>, \tag{91}$$

which ~~was before~~ has occasionally been discussed in different works [9] [10] [11] [69] [100] and recently

been                                                                                                    the
~~enough strongly obtained and~~ analyzed in detail from the Lagrangian point of view in works [18] [19] [50] [51] in the following equivalent to (70) infinitesimal form~~;~~

$$\delta f_L(r) = \xi u \times (\nabla \times \xi \delta A(r)) - \xi \nabla < u - u_f, \delta A(r) >, \tag{92}$$

H
~~w~~here $\delta A(r) \in T^*(\mathbb{R}^3)$ denotes the magnetic potential generated by an external charged point particle moving with velocity $u_f \in T(\mathbb{R}^3)$ and exerting the magnetic force $\delta f_L(r)$ on the charged particle located at point $r \in \mathbb{R}^3$ and moving with velocity $u \in T(\mathbb{R}^3)$ with respect to a common reference system $\mathrm{K}_t$. We also need to mention here that the modified Lorentz force expression (91) does not take naturally into account the resulting pure ly ~~every weak~~ electric force, as the conductors $l$ and $l'$ are considered to be electrically neutral. Simultaneously, we see that the magnetic potential has a physical significance in its own right [6] [9] [11] [50] [69] and has meaning in a way that extends beyond the calculation of force fields.

Really, to obtain the Lorentz type force (91) exerted by the external magnetic field generated by *the whole conductor* $l'$ on an infinitesimal current element $dl$ of the conductor $l$, it is necessary to integrate the expression (92) along this conductor loop $l'$ :

$$\begin{aligned}
dF_L(r) &:= \oint_{l'} \delta f_L(r) = J(r)dr \times (\nabla \times \oint_{l'} \delta A(r)) - \nabla < J(r)dr, \oint_{l'} \delta A(r) > + \\
&+ \nabla \oint_{l'} < u', \xi \delta A(r) > = J(r)dr \times (\nabla \times A(r)) - \nabla < J(r)dr, \oint_{l'} \delta A(r) > + \\
&+ \nabla \oint_{l'} < dr', \xi \delta A(r)/dt > = J(r)dr \times B(r) - \nabla < J(r)dr, \oint_{l'} \delta A(r) > + \\
&+ \nabla \int_{S(l')} < dS(l'), \nabla \times \xi \delta A(r)/dt > = J(r)dr \times B(r) - \nabla < J(r)dr, \oint_{l'} \delta A(r) > + \\
&+ \nabla \oint_{l'} < dS(l'), \xi \delta B(r)/dt > = J(r)dr \times B(r) - \nabla < J(r)dr, \oint_{l'} \delta A(r) > + \\
&+ \xi \nabla (d\Phi(r)/dt) = J(r)dr \times B(r) - \nabla < J(r)dr, A(r) > - \rho(r)d^3r \nabla \overline{W} = \\
&= J(r)dr \times B(r) - \nabla < J(r)dr, \oint_{l'} \delta A(r) > + \rho(r)d^3r(-\nabla \overline{W} - \partial A(r)/\partial t) = \\
&= J(r)dr \times B(r) - \nabla < J(r)dr, \oint_{l'} \delta A(r) > + \rho(r)d^3rE(r),
\end{aligned} \tag{93}$$

that is the equality

$$dF(r) = \rho(r)d^3rE(r) + J(r)d^3r \times B(r) - \nabla < J(r)d^3r, A(r) >, \tag{94}$$

where, by definition, the electric field $E(r) := -\nabla \overline{W} - \partial A(r)/\partial t$. Now one can easily derive from (94) the searched for *Lorentz type force* expression (91), if ~~to~~ takes into account that the whole
one

electric field $E(r)$; 0 owing to the neutrality of the conductors.

The ~~presented~~ above analysis of ~~the~~ A.M. Ampere's derivation of the magnetic force expression (86), as well as its consequences (91) and (92), make it possible to suppose that the missed modified Lorentz type force expression (91) could also be embedded into the classical relativistic Lagrangian and related Hamiltonian formalisms, giving rise to eventually new aspects and interpretations of many observed ~~during the past centuries looking "strange"~~ experimental phenomena.

### 1.5. The vacuum field theory electrodynamics equations: Hamiltonian analysis

Any Lagrangian theory has an equivalent canonical Hamiltonian representation via the classical Legendre transformation [64] [66] [96] [101] [102]. As we have already formulated our vacuum field theory of a moving charged particle $\xi$ in Lagrangian form, we proceed now to its Hamiltonian analysis making use of the action functionals (50), (62) and (71).

Take, first, the Lagrangian function (52) and the momentum expression (51) for defining the corresponding Hamiltonian function with respect to the moving reference frame $K_\tau$ :

$$\begin{aligned} H &:=< p,\dot{r} > -\mathrm{L} = \\ &= -< p,p > \bar{W}^{-1}(1-|p|^2/\bar{W}^2)^{-1/2} + \bar{W}(1-|p|^2/\bar{W}^2)^{-1/2} = \\ &= -|p|^2\,\bar{W}^{-1}(1-|p|^2/\bar{W}^2)^{-1/2} + \bar{W}^2\bar{W}^{-1}(1-|p|^2/\bar{W}^2)^{-1/2} = \\ &= -(\bar{W}^2-|p|^2)(\bar{W}^2-|p|^2)^{-1/2} = -(\bar{W}^2-|p|^2)^{1/2}. \end{aligned} \tag{95}$$

Consequently, it is easy to show [64] [96] [102] [66] that the Hamiltonian function (95) ~~is~~ expresses a conservation law of the dynamical field equation (49), that is for all $\tau, t \in \mathrm{R}$

$$dH/d\tau = dH/dt = 0, \tag{96}$$

which naturally leads to an energy interpretation of $H$. Thus, we can represent the particle energy as

$$\mathrm{E} = (\bar{W}^2 - |p|^2)^{1/2}. \tag{97}$$

Accordingly the Hamiltonian equivalent to the vacuum field equation (49) can be written as

$$\begin{aligned} \dot{r} &:= dr/d\tau = \partial H/\partial p = p(\bar{W}^2-|p|^2)^{-1/2} \\ \dot{p} &:= dp/d\tau = -\partial H/\partial r = \bar{W}\nabla\bar{W}(\bar{W}^2-|p|^2)^{-1/2}, \end{aligned} \tag{98}$$

and we have the following result.

**Proposition 6.** *The alternative freely moving point particle electrodynamic model (49) allows the canonical Hamiltonian formulation (98) with respect to the "rest" reference frame variables, where the Hamiltonian function is given by expression (95). Its electrodynamics is completely* *equivalent to the classical relativistic freely moving point particle electrodynamics described in Subsection 1.2.1.*

In the analogous manner, one can now use the Lagrangian (62) to construct the Hamiltonian function for the dynamical field equation (58), describing the motion of charged particle $\xi$ in an external electromagnetic field in the canonical Hamiltonian form:

$$\dot r := dr/d\tau = \partial H/\partial P,\ \dot P := dP/d\tau = -\partial H/\partial r, \tag{99}$$

where

$$\begin{aligned} H\ &:= \langle P, \dot r\rangle - \mathrm{L} = \\ &= \langle P, \dot r_f - P\bar W^{',-1}(1-|P|^2/\bar W^{',2})^{-1/2}\rangle + \bar W^{'}[\bar W^{',2}(\bar W^{',2}-|P|^2)^{-1}]^{1/2} = \\ &= \langle P, \dot r_f\rangle + |P|^2(\bar W^{',2}-|P|^2)^{-1/2} - \bar W^{',2}(\bar W^{',2}-|P|^2)^{-1/2} = \\ &= -(\bar W^{',2}-|P|^2)(\bar W^{',2}-|P|^2)^{-1/2} + \langle P, \dot r_f\rangle = \\ &= -(\bar W^{',2}-|P|^2)^{1/2} - \xi\langle A^{'}, P\rangle(\bar W^{',2}-|P|^2)^{-1/2} = \\ &= -(\bar W^{2}-|\xi A|^2-|P|^2)^{1/2} - \xi\langle A, P\rangle(\bar W^{2}-|\xi A|^2-|P|^2)^{-1/2}, \end{aligned} \tag{100}$$

being written with respect to the laboratory reference frame $\mathrm{K}_\tau$. Here we took into account that, owing to definitions (56), (57) and (63),

$$\begin{aligned} \xi A^{'}\ &:= \bar W^{'} u_f^{'} = \bar W^{'} dr_f/dt^{'} = \xi A = \\ &= \bar W^{'}\frac{dr_f}{d\tau}\cdot\frac{d\tau}{dt^{'}} = \bar W^{'}\dot r_f(1-|u-u_f|)^{1/2} = \\ &= \bar W^{'}\dot r_f(1+|\dot r-\dot r_f|^2)^{-1/2} = \\ &= -\bar W^{'}\dot r_f(\bar W^{',2}-|P|^2)^{1/2}\bar W^{',-1} = -\dot r_f(\bar W^{',2}-|P|^2)^{1/2}, \end{aligned} \tag{101}$$

and, in particular,

$$\dot r_f = -\xi A(\bar W^{',2}-|P|^2)^{-1/2}, \bar W = \bar W^{'}(1-|u_f|^2)^{-1/2}, \tag{102}$$

where $A: M^4 \to \mathrm{R}^3$ is the related magnetic vector potential generated by the moving external charged particle $\xi_f$. Equations (99) can be rewritten with respect to the laboratory reference frame $\mathrm{K}_\tau$ in the form

$$dr/dt = u,\ dp/dt = \xi E + \xi u\times B - \xi\nabla\langle A, u-u_f\rangle, \tag{103}$$

which coincides with the result (70).

Whence, we see that the Hamiltonian function (100) satisfies the energy conservation conditions

$$dH / d\tau = dH / dt' = dH / dt = 0, \tag{104}$$

for all $\tau, t'$ and $t \in \mathbb{R}$, and that the suitable energy expression is

$$\mathrm{E} = (\bar{W}^2 - \xi^2 |A|^2 - |P|^2)^{1/2} + \xi < A, P > (\bar{W}^2 - \xi^2 |A|^2 - |P|^2)^{-1/2}, \tag{105}$$

where the generalized momentum $P = p + \xi A$. The result (105) differs essentially from that obtained in [5], which makes use of the Einstein's Lagrangian for a moving charged point particle $\xi$ in an external electromagnetic field. Thus, we obtain the following proposition.

**Proposition 7.** *The alternative classical relativistic electrodynamic model (103), which is intrinsically compatible with the classical Maxwell equations (6), allows the Hamiltonian formulation (99) with respect to the proper reference frame variables, where the Hamiltonian function is given ~~by~~ the expression (100).*

The inference above is a natural candidate for experimental validation of our theory. It is strongly motivated by the following remark.

**Remark 2.** *It is necessary to mention here that the Lorentz force expression (103) uses the particle momentum $p = mu$, where the dynamical "mass" $m := -\bar{W}$ satisfies condition (105). The latter gives rise to the following crucial relationship between the particle energy $\mathrm{E}_0$ and its rest mass $m_0 = -\bar{W}_0$ (for the velocity $u = 0$ at the initial time moment $t = 0$):*

$$\mathrm{E}_0 = m_0 \frac{(1 - |\xi A_0 / m_0|^2)}{(1 - 2|\xi A_0 / m_0|^2)^{1/2}}, \tag{106}$$

*or, equivalently, at the condition $|\xi A_0 / m_0|^2 < 1/2$*

$$m_0 = \mathrm{E}_0 \left( \frac{1}{2} \pm \frac{1}{2} \sqrt{1 - 4|\xi A_0 / \mathrm{E}_0|^2} + |\xi A_0 / \mathrm{E}_0|^2 \right)^{1/2}, \tag{107}$$

*where $A_0 := A|_{t=0} \in \mathbb{E}^3$, which strongly differs from the classical expression $m_0 = \mathrm{E}_0 - \xi \varphi_0$, following from (44) and is not depending a priori on the external potential energy $\xi \varphi_0$. As the quantity $|\xi A_0 / \mathrm{E}_0| \to 0$, the following asymptotical mass values follow from (107):*

$$m_0 ; \ \mathrm{E}_0 - \frac{|\xi A_0|^4}{2|\mathrm{E}_0|^3 \mathrm{E}_0}, \quad m_0^{(\pm)} ; \ \pm\sqrt{2} |\xi A_0|. \tag{108}$$

*The first mass value $m_0 ; \ \mathrm{E}_0 - \frac{|\xi A_0|^4}{2|\mathrm{E}_0|^3 \mathrm{E}_0}$ is physically reasonable from the classic relativistic point of view, giving rise at weak enough magnetic potential to the charged particle energy $\mathrm{E}_0$, yet the second mass values $m_0^{(\pm)} ; \ \pm\sqrt{2}|\xi A_0|$ still need their physical interpretation, as they may describe both matter and anti-matter states, consisting, at a very huge energy modulus $|\mathrm{E}_0| \to \infty$, of some charged particle excitations of the vacuum. It is also worth ~~of~~ mentioning that the sign of the mass $m_0$ coincides with that of the energy $\mathrm{E}_0$ if only the* *inequality* $1-|\xi A_0/m_0|^2 \geq 0$ *holds.*

To make this difference more clear, we now analyze the Lorentz force (76) from the Hamiltonian point of view based on the Lagrangian function (74). Thus, we obtain ~~that~~ the corresponding Hamiltonian function

$$\begin{aligned} H \; &:= <P,\dot r> - \mathrm{L} = <P,\dot r> + \bar W(1+|\dot r|^2)^{1/2} - \xi <A,\dot r> = \\ &= <P-\xi A,\dot r> + \bar W(1+|\dot r|^2)^{1/2} = \\ &= -<p,p>\bar W^{-1}(1-|p|^2/\bar W^2)^{-1/2} + \bar W(1-|p|^2/\bar W^2)^{-1/2} = \\ &= -(\bar W^2-|p|^2)(\bar W^2-|p|^2)^{-1/2} = -(\bar W^2-|p|^2)^{1/2}. \end{aligned} \tag{109}$$

Since $p = P-\xi A$, the expression (109) assumes the final "*no interaction*" [5] [65] [103] [104] form

$$H = -(\bar W^2 - |P-\xi A|^2)^{1/2}, \tag{110}$$

which is conserved with respect to the evolution equations (72) and (73), that is

$$dH/d\tau = dH/dt = 0 \tag{111}$$

for all $\tau, t \in \mathrm{R}$. These equations ~~latter~~ are equivalent to the following Hamiltonian system

$$\begin{aligned} \dot r \; &= \partial H/\partial P = (P-\xi A)(\bar W^2-|P-\xi A|^2)^{-1/2}, \\ \dot P \; &= -\partial H/\partial r = (\bar W\nabla \bar W - \nabla <\xi A,(P-\xi A)>)(\bar W^2-|P-\xi A|^2)^{-1/2}, \end{aligned} \tag{112}$$

as one can readily check by direct calculations. Actually, the first equation

$$\begin{aligned} \dot r \; &= (P-\xi A)(\bar W^2-|P-\xi A|^2)^{-1/2} = p(\bar W^2-|p|^2)^{-1/2} = \\ &= mu(\bar W^2-|p|^2)^{-1/2} = -\bar W u(\bar W^2-|p|^2)^{-1/2} = u(1-|u|^2)^{-1/2}, \end{aligned} \tag{113}$$

holds, owing to the condition $d\tau = dt(1-|u|^2)^{1/2}$ and definitions $p := mu,\ m = -\bar W$, postulated from the very beginning. Similarly we obtain that

$$\begin{aligned} \dot P \; &= -\nabla \bar W(1-|p|^2/\bar W^2)^{-1/2} + \nabla <\xi A,u>(1-|p|^2/\bar W^2)^{-1/2} = \\ &= -\nabla \bar W(1-|u|^2)^{-1/2} + \nabla <\xi A,u>(1-|u|^2)^{-1/2}, \end{aligned} \tag{114}$$

coincides with equation (75) in the evolution parameter $t \in \mathrm{R}$. This can be formulated as the next result.

**Proposition 8.** *The dual to the classical relativistic electrodynamic model (76) allows the canonical Hamiltonian formulation (112) with respect to the proper reference frame variables, where the Hamiltonian function is given by expression (110). Moreover, this formulation circumvents the "mass-potential energy" controversy attached to the classical electrodynamic model (42).*

The modified Lorentz force expression (76) and the related rest energy relationship are characterized by the following remark.

**Remark 3.** *If we make use of the modified relativistic Lorentz force expression (76) as an alternative to the classical one of (46), the corresponding charged particle $\xi$ energy expression (110) also gives rise to a true physically reasonable energy expression (at the velocity $u := 0 \in \mathrm{E}^3$ at the initial time moment $t = 0$); namely, $\mathrm{E}_0 = m_0$ instead of the physically controversial classical expression $\mathrm{E}_0 = m_0 + \xi\varphi_0$, where $\varphi_0 := \varphi|_{t=0}$, corresponding to the case (44).*

### 1.6. Conclusions

All ~~of~~ the dynamical field equations discussed above are canonical Hamiltonian systems with respect to the corresponding proper reference frames $\mathrm{K}_\tau$, parameterized by suitable time parameters $\tau \in \mathrm{R}$. Upon passing to the basic laboratory reference frame $\mathrm{K}_t$ with the time parameter $t \in \mathrm{R}$, naturally the related Hamiltonian structure is lost, giving rise to a new interpretation of the real particle motion. Namely, one that has an absolute sense only with respect to the proper reference system, and otherwise being completely relative with respect to all other reference frames. As for the Hamiltonian expressions (95), (100) and (110), one observes that they all depend strongly on the vacuum potential energy field function $\bar{W} : M^4 \to \mathrm{R}$, thereby avoiding the mass problem of the classical energy expression pointed out by L. Brillouin **~~Error! Reference source not found.~~**. It should be noted that the canonical Dirac quantization procedure can be applied only to the corresponding dynamical field systems considered with respect to their proper reference frames.

**Remark 4.** *Some comments are in order concerning the classical relativity principle. We have obtained our results relying only on the natural notion of the proper reference frame and its suitable Lorentz parametrization with respect to any other moving reference frames. It seems reasonable then that the true state changes of a moving charged* particle $\xi$ *are exactly realized only with respect to its proper reference system. Then the only remaining question would be about the physical justification of the corresponding relationship between time parameters of moving and proper reference frames.*

The relationship between reference frames that we have used through is expressed as

$$d\tau = dt(1 - |u|^2)^{1/2}, \tag{115}$$

where $u := dr/dt \in \mathrm{E}^3$ is the velocity vector with which the proper reference frame $\mathrm{K}_\tau$ moves with respect to another arbitrarily chosen reference frame $\mathrm{K}_t$. Expression (115) implies, in particular, that

$$dt^2 - |dr|^2 = d\tau^2, \tag{116}$$

which is identical to the classical infinitesimal Lorentz invariant. This is not a coincidence, since all our dynamical vacuum field equations were derived in turn [18][19] from the governing equations of the vacuum potential field function $W : M^4 \rightarrow \mathbb{R}$ in the form

$$\partial^2 W / \partial t^2 - \nabla^2 W = \xi\rho, \partial W / \partial t + \nabla(vW) = 0, \partial\rho / \partial t + \nabla(v\rho) = 0, \tag{117}$$

which is *a priori* Lorentz invariant. Here $\rho \in \mathbb{R}$ is the charge density and $v := dr / dt$ the associated local velocity of the vacuum field potential evolution. Consequently, the dynamical infinitesimal Lorentz invariant (116) reflects this intrinsic structure of equations (117). If it is rewritten in the following nonstandard Euclidean form:

$$dt^2 = d\tau^2 + |dr|^2 \tag{118}$$

it gives rise to a completely different relationship between the reference frames $\mathrm{K}_t$ and $\mathrm{K}_\tau$, namely

$$dt = d\tau(1 + |\dot{r}|^2)^{1/2}, \tag{119}$$

where $\dot{r} := dr / d\tau$ is the related particle velocity with respect to the proper reference system. Thus, we observe that all our Lagrangian analysis in this Section is based on the corresponding functional expressions written in these "Euclidean" space-time coordinates and with respect to which the least action principle was applied. So we see that there are two alternatives - the first is to apply the least action principle to the corresponding Lagrangian functions expressed in the Minkowski space-time variables with respect to an arbitrarily chosen reference frame $\mathrm{K}_t$, and the second is to apply the least action principle to the corresponding Lagrangian functions expressed in Euclidean space-time variables with respect to the proper reference frame $\mathrm{K}_\tau$.

This leads us to a slightly amusing but thought-provoking observation: It follows from our analysis that all of the results of classical special relativity related with the electrodynamics of charged point particles can be obtained (in a one-to-one correspondence) using ~~of~~ our new definitions of the dynamical particle mass and the least action principle with respect to the associated Euclidean space-time variables in the proper reference system.

An additional remark concerning the quantization procedure of the proposed electrodynamics models is in order: If the dynamical vacuum field equations are expressed in canonical Hamiltonian form, as we have done in this paper, only straightforward technical details are required to quantize the equations and obtain the corresponding Schrödinger evolution equations in suitable Hilbert spaces of quantum states. There is another striking implication from our approach: the Einstein equivalence principle [1] [5] [65] [89] is rendered superfluous for our vacuum field theory of electromagnetism and gravity.

Using the canonical Hamiltonian formalism devised here for the alternative charged point particle electrodynamics models, we found it rather easy to treat the Dirac quantization. The results obtained compared favorably with classical quantization, but it must be admitted that we still have not given a compelling physical motivation for our new models. This is something that we plan to revisit in future investigations. Another important aspect of our vacuum field theory no-geometry (geometry-free) approach to combining the electrodynamics with the gravity, is the manner in which it singles out the decisive role of the proper reference frame $\mathrm{K}_\tau$. More precisely, all of our electrodynamics models allow both the Lagrangian and Hamiltonian formulations with respect to the proper reference system evolution parameter $\tau \in \mathbb{R}$, which are well suited the to canonical quantization. The physical nature of this fact remains ~~is~~ as yet not quite clear. In fact, as far as we know [5] [65] [75] [76] [89], there is no physically reasonable explanation of this decisive role of the proper reference system, except for that given by R. Feynman who argued in [1] that the relativistic expression for the classical Lorentz force (46) has physical sense only with respect to the proper reference frame variables $(\tau, r) \in \mathrm{R} \times \mathrm{E}^3$. In future research we plan to analyze the quantization scheme in more detail and begin work on formulating a vacuum quantum field theory of infinitely many particle systems.

## 2. The Lorentz type force analysis within the Feynman proper time paradigm and the radiation theory

### 2.1. Introductory setting

The elementary point charged particle, like electron, mass problem was inspiring many physicists [20] from the past as J. J. Thompson, G.G. Stokes, H.A. Lorentz, E. Mach, M. Abraham, P.A. M. Dirac, G.A. Schott and others. Nonetheless, their studies have not given rise to a clear explanation of this phenomenon that stimulated new researchers to tackle it from different approaches based on new ideas stemming both from the classical Maxwell-Lorentz electromagnetic theory, as in [1] [12] [21] [22] [24] [25] [26] [27] [28] [29] [30] [31] [32] [33] [34] [35] [36] [37] [39] [74] [105] [106] [107], and modern quantum field theories of Yang-Mills and Higgs type, as in [40] [41] [43] [108] and others, whose recent and extensive review is done in [44].

In the present work I will mostly concentrate on detail analysis and consequences of the Feynman proper time paradigm [1] [22] [45] [46] subject to deriving the electromagnetic Maxwell equations and the related Lorentz like force expression considered from the vacuum field theory approach, developed in works [49] [50] [51], and further, on its applications to the electromagnetic mass origin problem. Our treatment of this and related problems, based on the least action principle within the Feynman proper time paradigm [1], has allowed to construct the respectively modified Lorentz type equation for a moving in space and radiating energy charged point particle. Our analysis also elucidates, in particular, the computations of the self-interacting electron mass term in [29], where there was proposed a not proper solution to the well known classical Abraham-Lorentz [52] [53] [54] [55] and Dirac [56] electron electromagnetic "4/3-electron mass" problem. As a result of our scrutinized studying the classical electromagnetic mass problem we have stated that it can be satisfactory solved within the classical H. Lorentz and M. Abraham reasonings augmented with the additional electron stability condition, which was not taken before into account yet appeared to be very important for balancing the related electromagnetic field and mechanical electron momenta. The latter, following recent ~~enough~~ works [31] [35], devoted to analyzing the electron charged shell model, can be realized within there suggested *pressure-energy compensation principle,* suitably applied to the ambient electromagnetic energy fluctuations and the own electrostatic Coulomb electron energy.

### 2.2. Feynman proper time paradigm geometric analysis

In this section, we will develop further the vacuum field theory approach within the Feynman proper time paradigm, devised before in [49] [51], to the electromagnetic J.C. Maxwell and H. Lorentz electron theories and show that they should be suitably modified: namely, the basic Lorentz force equations should be generalized following the Landau-Lifschitz least action recipe [5], taking also into account the pure electromagnetic field impact. When appl~~ied~~ying ~~the~~ devised vacuum field theory approach to the classical electron shell model, the resulting Lorentz force expression appears to satisfactorily explain the electron inertial mass term exactly coinciding with the electron relativistic mass, thus confirming the well known assumption [2] [109] by M. Abraham and H. Lorentz.

As was reported by F. Dyson [45] [46], the original Feynman approach derivation of the electromagnetic Maxwell equations was based on an *a priori* general form of the classical Newton type force, acting on a charged point particle moving in three-dimensional space $\mathbb{R}^3$ endowed with the canonical Poisson brackets on the phase variables, defined on the associated tangent space $T(\mathbb{R}^3)$. As a result of this approach ~~there~~ only the first part of the Maxwell equations were derived, as the second part, owing to F. Dyson [45], is related with the charged matter nature, which appeared to be hidden. Trying to complete this Feynman approach to the derivation of Maxwell's equations more systematically we have observed [49] that the original Feynman's calculations, based on Poisson brackets analysis, were performed on the *tangent space* $T(\mathbb{R}^3)$, which is, subject to the problem posed, not physically proper. The true Poisson brackets can be correctly defined only on the *coadjoint phase space* $T^*(\mathbb{R}^3)$, as seen from the classical Lagrangian equations and the related Legendre transformation [47] [64] [96] [110] from $T(\mathbb{R}^3)$. to $T^*(\mathbb{R}^3)$. Moreover, within this observation, the corresponding dynamical Lorentz type equation for a charged point particle should be written for the particle momentum, not for the particle velocity, whose value is well defined only with respect to the proper relativistic reference frame, associated with the charged point particle owing to the fact that the Maxwell equations are Lorentz invariant.

Thus, from the very beginning, we shall reanalyze the structure of the Lorentz force exerted on a moving charged point particle with a charge $\xi \in \mathbb{R}$ by another point charged particle with a charge $\xi_f \in \mathbb{R}$, making use of the classical Lagrangian approach, and rederive the corresponding electromagnetic Maxwell equations. The latter appears to be strongly related to the charged point mass structure of the electromagnetic origin as was suggested by R. Feynman and F. Dyson.

Consider a charged point particle moving in an electromagnetic field. For its description, it is convenient to introduce a trivial fiber bundle structure $\pi : \mathrm{M} \to \mathbb{R}^3, \mathrm{M} = \mathbb{R}^3 \times G$, with the abelian structure group $G := \mathbb{R}\setminus\{0\}$, equivariantly acting on the canonically symplectic coadjoint space $T^*(\mathrm{M})$ endowed both with the canonical symplectic structure

$$\begin{aligned} \omega^{(2)}(p,y;r,g) \; &:= dpr^*\alpha^{(1)}(r,g) = < dp, \wedge dr > + \\ &+ \; < dy, \wedge g^{-1}dg >_{\mathrm{G}} + < ydg^{-1}, \wedge dg >_{\mathrm{G}} \end{aligned} \qquad (120)$$

for all $(p,y;r,g) \in T^*(\mathrm{M})$, where $\alpha^{(1)}(r,g) := < p, dr > + < y, g^{-1}dg >_{\mathrm{G}} \in T^*(\mathrm{M})$ is the corresponding Liouville form on $\mathrm{M}$, and with a connection one-form $\mathrm{A} : M \to T^*(M) \times \mathrm{G}$ as

$$\mathrm{A}(r,g) := g^{-1} < \xi A(r), dr > g + g^{-1}dg, \tag{121}$$

with $\xi \in \mathrm{G}^*, (r,g) \in \mathrm{R}^3 \times G$, and $< \cdot, \cdot >$ being the scalar product in $\mathrm{E}^3$. The corresponding curvature 2-form $\Sigma^{(2)} \in \Lambda^2(\mathrm{R}^3) \otimes \mathrm{G}$ is

$$\Sigma^{(2)}(r) := d\mathrm{A}(r,g) + \mathrm{A}(r,g) \wedge \mathrm{A}(r,g) = \xi \sum_{i,j=1}^{3} F_{ij}(r) dr^i \wedge dr^j, \tag{122}$$

where

$$F_{ij}(r) := \frac{\partial A_j}{\partial r^i} - \frac{\partial A_i}{\partial r^j} \tag{123}$$

for $i,j = \overline{1,3}$ is the electromagnetic tensor with respect to the reference frame $\mathrm{K}_t$, characterized by the phase space coordinates $(r,p) \in T^*(\mathrm{R}^3)$. As an element $\xi \in \mathrm{G}^*$ is still not fixed, it is natural to apply the standard [47] [64] [96] [110] invariant Marsden-Weinstein-Meyer reduction to the orbit factor space $\tilde{P}_\xi := P_\xi / G_\xi$ subject to the related momentum mapping $l : T^*(\mathrm{M}) \to \mathrm{G}^*$, constructed with respect to the canonical symplectic structure (120) on $T^*(\mathrm{M})$, where, by definition, $\xi \in \mathrm{G}^*$ is constant, $P_\xi := l^{-1}(\xi) \subset T^*(\mathrm{M})$ and $G_\xi = \{g \in G : Ad_G^* \xi\}$ is the isotropy group of the element $\xi \in \mathrm{G}^*$.

As a result of the Marsden-Weinstein-Meyer reduction, one finds that $G_\xi$ ; $G$, the factor-space $\tilde{P}_\xi$ ; $T^*(\mathrm{R}^3)$ is endowed with a suitably reduced symplectic structure $\overline{\omega}_\xi^{(2)} \in T^*(\tilde{P}_\xi)$ and the corresponding Poisson brackets on the reduced manifold $\tilde{P}_\xi$ are

$$\begin{aligned} \{r^i, r^j\}_\xi &= 0, \{p_j, r^i\}_\xi = \delta^i_j, \\ \{p_i, p_j\}_\xi &= \xi F_{ij}(r) \end{aligned} \tag{124}$$

for $i,j = \overline{1,3}$, considered with respect to the reference frame $\mathrm{K}_t$. Introducing a new momentum variable

$$\tilde{\pi} := p + \xi A(r) \tag{125}$$

on $\tilde{P}_\xi$, it is easy to verify that $\overline{\omega}_\xi^{(2)} \to \tilde{\omega}_\xi^{(2)} :=< d\tilde{\pi}, \wedge dr >$, giving rise to the following *"minimal interaction"* canonical Poisson brackets:

$$\{r^i, r^j\}_{\tilde{\omega}_\xi^{(2)}} = 0, \{\tilde{\pi}_j, r^i\}_{\tilde{\omega}_\xi^{(2)}} = \delta^i_j, \{\tilde{\pi}_i, \tilde{\pi}_j\}_{\tilde{\omega}_\xi^{(2)}} = 0 \tag{126}$$

for $i,j = \overline{1,3}$ with respect to some new reference frame $\tilde{\mathrm{K}}_{t'}$, characterized by the phase space coordinates $(r, \tilde{\pi}) \in \tilde{P}_\xi$ and a new evolution parameter $t' \in \mathrm{R}$ if and only if the Maxwell field compatibility equations

$$\partial F_{ij} / \partial r_k + \partial F_{jk} / \partial r_i + \partial F_{ki} / \partial r_j = 0 \tag{127}$$

are satisfied on $\mathrm{R}^3$ for all $i,j,k = \overline{1,3}$ with the curvature tensor (123).

Now we proceed to a dynamic description of the interaction between two moving charged point particles $\xi$ and $\xi_f$, moving respectively, with the velocities $u := dr/dt$ and $u_f := dr_f / dt$ subject to the reference frame $\mathrm{K}_t$. Unfortunately, there is a fundamental problem in correctly formulating a physically suitable action functional and the related least action condition. There are clearly possibilities such as

$$S_p^{(t)} := \int_{t_1}^{t_2} dt \mathrm{L}_p^{(t)}[r; dr/dt] \tag{128}$$

on a temporal interval $[t_1, t_2] \subset \mathbb{R}$ with respect to the laboratory reference frame $\mathrm{K}_t$,

$$S_p^{(t')} := \int_{t_1'}^{t_2'} dt' \mathrm{L}_p^{(t')}[r; dr/dt'] \tag{129}$$

on a temporal interval $[t_1', t_2'] \subset \mathbb{R}$ with respect to the moving reference frame $\mathrm{K}_{t'}$ and

$$S_p^{(\tau)} := \int_{\tau_1}^{\tau_2} d\tau \mathrm{L}_p^{(\tau)}[r; dr/d\tau] \tag{130}$$

on a temporal interval $[\tau_1, \tau_2] \subset \mathbb{R}$ with respect to the proper time reference frame $\mathrm{K}_\tau$, naturally related to the moving charged point particle $\xi$.

It was first observed by Poincaré and Minkowski [65] that the temporal differential $d\tau$ is not a closed differential one-form, which physically means that a particle can traverse many different paths in space $\mathbb{R}^3$ with respect to the reference frame $\mathrm{K}_t$ during any given proper time interval $d\tau$, *naturally* related to its motion. This fact was stressed [65] [111] [112] [113] [114] by Einstein, Minkowski and Poincaré, and later exhaustively analyzed by R. Feynman, who argued [1] that the dynamical equation of a moving point charged particle is physically sensible only with respect to its proper time reference frame. This is Feynman's proper time reference frame paradigm, which was recently further elaborated and applied both to the electromagnetic Maxwell equations in an [23] [24] [74] and to the Lorentz type equation for a moving charged point particle under external electromagnetic field in [47] [49] [50] [51]. As it was there argued from a physical point of view, the least action principle should be applied only to the expression (130) written with respect to the proper time reference frame $\mathrm{K}_\tau$, whose temporal parameter $\tau \in \mathbb{R}$ is independent of an observer and is a closed differential one-form. Consequently, this action functional is also mathematically sensible, which in part reflects the Poincaré's and Minkowski's observation that the infinitesimal quadratic interval

$$d\tau^2 = (dt')^2 - |dr - dr_f|^2, \tag{131}$$

relating the reference frames $\mathrm{K}_{t'}$ and $\mathrm{K}_\tau$, can be invariantly used for the four-dimensional relativistic geometry. The most natural way to contend with this problem is to first consider the quasi-relativistic dynamics of the charged point particle $\xi$ with respect to the moving reference frame $\mathrm{K}_{t'}$ subject to which the charged point particle $\xi_f$ is at rest. Therefore, it is possible to write down a suitable action functional (129), up to $O(1/c^4)$, as the light velocity $c \to \infty$, where the quasi-classical Lagrangian function $\mathrm{L}_p^{(t')}[r; dr/dt']$ can be naturally chosen as

$$\mathrm{L}_p^{(t')}[r; dr/dt'] := m'(r)\left|dr/dt' - dr_f/dt'\right|^2/2 - \xi\varphi'(r). \tag{132}$$

of the

where $m'(r) \in \mathbb{R}_+$ is the charged particle $\xi$ inertial mass parameter and $\varphi'(r)$ is the potential function generated by the charged particle $\xi_f$ at a point $r \in \mathbb{R}^3$ with respect to the reference frame $K_{t'}$. Since the standard temporal relationships between reference frames $K_t$ and $K_{t'}$ :

$$dt' = dt(1-|dr_f / dt'|^2)^{1/2}, \qquad (133)$$

as well as between the reference frames $K_{t'}$ and $K_\tau$ :

$$d\tau = dt'(1-|dr / dt' - dr_f / dt'|^2)^{1/2}, \qquad (134)$$

give rise, up to $O(1/c^2)$, as $c \to \infty$, to $dt'$ ; $dt$ and $d\tau$ ; $dt'$, respectively, it is easy to verify that the least action condition $\delta S_p^{(t')} = 0$ is equivalent to the dynamical equation

$$d\pi / dt = \nabla L_p^{(t')}[r; dr/dt] = (\frac{1}{2}|dr/dt - dr_f/dt|^2)\nabla m - \xi\nabla\varphi(r), \qquad (135)$$

where we have defined the generalized canonical momentum as

$$\pi := \partial L_p^{(t')}[r; dr/dt] / \partial(dr/dt) = m(dr/dt - dr_f/dt), \qquad (136)$$

with the dash signs dropped and denoted by " $\nabla$ " the usual gradient operator in $E^3$. Equating the canonical momentum expression (136) with respect to the reference frame $K_t$ to that of (125) with respect to the canonical reference frame $\tilde{K}_{t'}$, and identifying the reference frame $\tilde{K}_{t'}$ with $K_{t'}$, one obtains that

$$m(dr/dt - dr_f/dt) = m\,dr/dt - \xi A(r). \qquad (137)$$

This gives ~~giving~~ rise to the important inertial particle mass determining expression

$$m = -\xi\varphi(r), \qquad (138)$$

which right away follows from the relationship

$$\varphi(r) dr_f / dt = A(r). \qquad (139)$$

The latter is well known in the classical electromagnetic theory [2] [5] for potentials $(\varphi, A) \in T^*(M^4)$ satisfying the Lorentz condition

$$\partial\varphi(r)/\partial t + \langle \nabla, A(r) \rangle = 0, \qquad (140)$$

yet the expression (138) looks very nontrivial in relating the " *inertial*" mass of the charged point particle $\xi$ to the electric potential, being both generated by the ambient charged point particles $\xi_f$. As was argued in articles [49] [50], the above mass phenomenon is closely related and from a physical perspective shows its deep relationship to the classical electromagnetic mass problem.

Before further analysis of the ~~completely~~ relativistic motion of the charge $\xi$ ~~motion~~ under consideration, we substitute the mass expression (138) into the quasi-relativistic action functional (129) with the Lagrangian (132). As a result, we obtain two possible action functional expressions, taking into account two main temporal parameters choices:

$$S_p^{(t')} = -\int_{t_1'}^{t_2'} \xi\varphi'(r)(1 + \frac{1}{2}|dr/dt' - dr_f/dt'|^2)dt' \qquad (141)$$

on an interval $[t_1', t_2'] \subset R$, or

$$S_p^{(\tau)} = -\int_{\tau_1}^{\tau_2} \xi\varphi'(r)(1 + \frac{1}{2}|dr/d\tau - dr_f/d\tau|^2)d\tau \qquad (142)$$

on an $[\tau_1, \tau_2] \subset \mathrm{R}$ . The direct relativistic transformations of (142) entail that

$$\begin{aligned} S_p^{(\tau)} &= -\int_{\tau_1}^{\tau_2} \xi \varphi^{'}(r)(1+\frac{1}{2}\left|dr/d\tau - dr_f/d\tau\right|^2)d\tau ; \\ &; \ -\int_{\tau_1}^{\tau_2} \xi \varphi^{'}(r)(1+\left|dr/d\tau - dr_f/d\tau\right|^2)^{1/2} d\tau = \\ &= -\int_{\tau_1}^{\tau_2} \xi \varphi^{'}(r)(1-|dr/dt^{'} - dr_f/dt^{'}|)^{-1/2} d\tau = -\int_{t_1^{'}}^{t_2^{'}} \xi \varphi^{'}(r) dt^{'}, \end{aligned} \tag{143}$$

giving rise to the correct, from the physical point of view, relativistic action functional form (129), suitably transformed to the proper time reference frame representation (130) via the Feynman proper time paradigm. Thus, we have shown that the true action functional procedure consists in a physically motivated choice of either the action functional expression form (128) or (129). Then, it is transformed to the proper time action functional representation form (130) within the Feynman paradigm, and the least action principle is applied.

Concerning the above discussed problem of describing the motion of a charged point particle $\xi$ in the electromagnetic field generated by another moving charged point particle $\xi_f$, it must be mentioned that we have chosen the quasi-relativistic functional expression (132) in the form (129) with respect to the moving reference frame $\mathrm{K}_{t^{'}}$, because its form is physically reasonable and acceptable, since the charged point particle $\xi_f$ is then at rest, generating no magnetic field.

Based on the above relativistic action functional expression

$$S_p^{(\tau)} := -\int_{\tau_1}^{\tau_2} \xi \varphi^{'}(r)(1+\left|dr/d\tau - dr_f/d\tau\right|^2)^{1/2} d\tau \tag{144}$$

written with respect to the proper reference from $\mathrm{K}_\tau$, one finds the following evolution equation:

$$d\pi_p/d\tau = -\xi \nabla \varphi^{'}(r)(1+\left|dr/d\tau - dr_f/d\tau\right|^2)^{1/2}, \tag{145}$$

where the generalized momentum is given exactly by the relationship (136):

$$\pi_p = m(dr/dt - dr_f/dt). \tag{146}$$

Making use of the relativistic transformation (133) and the next one (134), the equation (145) is easily transformed to

$$\frac{d}{dt}(p + \xi A) = -\nabla \varphi(r)(1 - \left|u_f\right|^2), \tag{147}$$

where we took into account the related definitions: (138) for the charged particle $\xi$ mass, (139) for the magnetic vector potential and $\varphi(r) = \ \varphi^{'}(r)/(1-\left|u_f\right|^2)^{1/2}$ for the scalar electric potential with respect to the laboratory reference frame $\mathrm{K}_t$. Equation (147) can be further transformed, using elementary vector algebra, to the classical Lorentz type form:

$$dp/dt = \xi E + \xi u \times B - \xi \nabla < u - u_f, A >, \tag{148}$$

where

$$E := -\partial A/\partial t - \nabla \varphi \tag{149}$$

is the related electric field and

$$B := \nabla \times A \tag{150}$$

is the related magnetic field, exerted by the moving charged point particle $\xi_f$ on the charged point particle $\xi$ with respect to the laboratory reference frame $\mathrm{K}_t$. The Lorentz type force equation (148) was obtained in [49] [50] in terms of the moving reference frame $\mathrm{K}_t'$, and recently reanalyzed in [34] [50]. The obtained results follow in part [16] [17] from Ampère's classical works on constructing the magnetic force between two neutral conductors with stationary currents.

## 3. The self-interaction problem: historical preliminaries

The elementary point charged particle, like the electron, mass problem was inspiring many physicists [20] from the past as J. J. Thompson, G.G. Stokes, H.A. Lorentz, E. Mach, M. Abraham, P.A. M. Dirac, G.A. Schott, J. Schwinger and many others. Nonetheless, their studies have not given rise to a clear explanation of this phenomenon that stimulated new researchers to tackle it from different approaches based on new ideas stemming both from the classical Maxwell-Lorentz electromagnetic theory, as in [1] [21] [22] [24] [25] [26] [34] [74] [109], and modern quantum field theories of Yang-Mills and Higgs type, as in [40] [41] [43] [108] and others, whose recent and extensive review is given ~~done~~ in [44].

In the present work we mostly concentrate on a detailed quantum and classical analysis of the self-interacting shell model charged particle within the Fock many-temporal approach [115] [116] and the Feynman proper time paradigm [1] [22] [45] [46] subject to deriving the electromagnetic Maxwell equations and the related Lorentz like force expression within the vacuum field theory approach, devised in works [24] [49] [50] [51] [74] [117], and further, we elaborate the obtained results to treating the classical H. Lorentz and M. Abraham [12] [27] [28] [29] [30] [31] [32] [33] [35] [36] [37] [39] [52] [53] [54] [107] [118] electromagnetic mass origin problem. For the first time the proper time approach to classical electrodynamics and quantum mechanics was possibly suggested ~~still~~ in 1937 by V. Fock [119], in which, in particular, there was constructed an alternative proper time based Lagrangian description of a point charged particle under an external electromagnetic field. A more detailed motivation of using the proper time approach was later presented by R. Feynman in his Lectures [1]. Concerning the alternative and much later investigations of the *a priori* given quantum electromagnetic Maxwell equations in the Fock space one can mention the Gupta-Bleiler [120] [121] [122] and [61] [71] [88] approaches. The first one, as it is well known [71] [121], contradicts ~~a~~ one of the most important field theoretical principles - the positive definiteness of the quantum event probability and is strongly based on making nonphysical use of an indefinite metric on quantum states. The second one is completely non-relativistic and based on the canonical quantization scheme [71] in the case of the Coulomb gauge condition. Inspired by these and related classical results, we have stated that the self-interacting quantum mechanism of the charged particle with its self-generated electromagnetic field consists of two physically different phenomena, whose influence on the structure of the resulting Hamilton interaction operator appeared to be crucial and gave rise to a modified analysis of the related classical shell model charged particle within the Lagrangian formalism. As a result of our scrutinized study~~ing~~ of the classical electromagnetic mass problem there was demonstrated that it can be satisfactory solved within the classical H. Lorentz and M. Abraham reasonings augmented with the additional electron stability condition, which was not taken before into account yet appeared to be very important for balancing the related electromagnetic field and mechanical electron momenta. The latter, following the recent ~~enough~~ works [31] [35] [118] devoted to analyzing the electron charged shell model, was realized within the~~re~~ suggested *pressure-energy compensation principle,* suitably applied to the ambient electromagnetic energy fluctuations and the self-generated electrostatic Coulomb electron energy. In the case of a point charged particle the alternative relativistic invariant approach to studying the radiation reaction force was ~~before~~ suggested by Teitelbom [37], and ~~which~~ was based on a formal relativistic invariant splitting of the electromagnetic energy-momentum tensor, ~~and~~ deriv~~ing~~ed the related suitably renormalized charged particle equations of motion. He thereby

## 4. The charged particle self-interaction quantum origin

Consider a free relativistic quantum fermionic *a priori* massless particle field described [121] [123] by means of the secondly-quantized self-adjoint Dirac-Weil type Hamiltonian

$$H_f = \int_{\mathbb{R}^3} d^3x \psi^+ < c\alpha, \frac{\hbar}{i}\nabla > \psi, \tag{151}$$

where $\alpha \in \mathrm{E}^3 \otimes End\ \ M^4$ denotes the standard Dirac spin matrix vector representation in the Minkowski space $M^4$, $c \in \mathbb{R}_+$ is the light velocity, $<\cdot,\cdot>$ denotes the usual scalar product in the Euclidean space $\mathrm{E}^3$, $\psi : \mathbb{R}^3 \to (End\ \ \Phi)^4$ - a spinor of the quantum annihilation operators, acting in a suitable Fock space $\Phi$ endowed with the usual scalar product $(\cdot,\cdot)$ and $\psi^+ : \mathbb{R}^3 \to (End\ \ \Phi)^4$ - the respectively adjoint co-spinor of creation operators in the Fock space $\Phi$. The following anticommuting [121] [123] operator relationships

$$\begin{aligned} \psi_j(x)\psi_l^+(y) + \psi_l^+(y)\psi_j(x) &= \delta_{jl}\delta(x-y), \\ \psi_j(x)\psi_l(y) + \psi_l(y)\psi_j(x) &= 0, \\ \psi_j^+(x)\psi_l^+(y) + \psi_l^+(y)\psi_j^+(x) &= 0 \end{aligned} \tag{152}$$

hold for any $x, y \in \mathbb{R}^3$ and $j, l \in \overline{1,4}$, being compatible with the related Heisenberg operator dynamics, generated by the fermionic Hamiltonian operator (151):

$$\partial\psi / \partial\overline{t} := \frac{i}{\hbar}[H_f, \psi], \ \partial\psi^+ / \partial\overline{t} := \frac{i}{\hbar}[H_f, \psi^+] \tag{153}$$

with respect to its own laboratory reference frame $\mathrm{K}_{\overline{t}}$, parameterized by the evolution parameter $\overline{t} \in \mathbb{R}$.

It is clear that the Hamiltonian (151) possesses no information of such an important characteristic as the electric charge $\xi \in \mathbb{R}$, which generates the own electromagnetic field interacting both with it and with other ambient charged particles. As ~~it~~ is usually accepted, we will model a free electromagnetic field by its bosonic self-adjoint operator four-potential $(\varphi, A) : \mathbb{R}^3 \to Hom\ \ (\Phi, \Phi^4)$, whose evolution is generated by the self-adjoint Hamiltonian

$$H_b = 2\int_{\mathbb{R}^3} d^3k\,|k|^2\,[< A^+(k), A(k) > -\varphi(k)\varphi^+(k)], \tag{154}$$

acting in the ~~before~~ introduced common Fock space $\Phi$ and represented by means of the field operators expanded into the Fourier integrals

$$\begin{aligned} \varphi(x) \;\; : \;\; &= \frac{1}{(2\pi)^{3/2}}\int_{\mathbb{R}^3} d^3k\varphi(k)\exp(i < k, x >) + \frac{1}{(2\pi)^{3/2}}\int_{\mathbb{R}^3} d^3k\varphi^+(k)\exp(-i < k, x >), \\ A(x) \;\; : \;\; &= \frac{1}{(2\pi)^{3/2}}\int_{\mathbb{R}^3} d^3kA(k)\exp(i < k, x >) + \frac{1}{(2\pi)^{3/2}}\int_{\mathbb{R}^3} d^3kA^+(k)\exp(-i < k, x >). \end{aligned} \tag{155}$$

The coefficients of the expansions (155) satisfy the following [115] [116] [121] commutation operator relationships:

$$\begin{aligned} \varphi(k), \varphi^+(s)] \;\; &= \;\; -\frac{c\hbar}{2|k|}\delta(k-s), \\ \varphi(k), A_j(s)] \;\; &= \;\; 0, \\ [\varphi(k), \varphi(s)] \;\; &= \;\; 0 = [\varphi^+(k), \varphi^+(s)], \\ [A_j(k), A_l^+(s)] \;\; &= \;\; \frac{c\hbar}{2|k|}\delta_{jl}\delta(k-s), \\ [A_j(k), A_l(s)] \;\; &= \;\; 0 = [A_j^+(k), A_l^+(s)] \end{aligned} \tag{156}$$

for all $k, s \in \mathbb{E}^3$ and $j, l \in \overline{1,3}$, compatible with the related Heisenberg operator dynamics [121] generated by the electromagnetic field Hamiltonian (154):

$$\frac{\partial A}{\partial \tilde{t}} := \frac{i}{\hbar}[H_b, A], \frac{\partial \varphi}{\partial \tilde{t}} := \frac{i}{\hbar}[H_b, \varphi], \tag{157}$$

with respect to its own laboratory reference frame $\mathrm{K}_{\tilde{t}}$, parameterized by the temporal parameter $\tilde{t} \in \mathbb{R}$. In particular, based on the commutation relationships (156), one can check that the electric

$$E := -\nabla\varphi - \frac{1}{c}\frac{\partial A}{\partial \tilde{t}} \tag{158}$$

and magnetic

$$B := \nabla \times A \tag{159}$$

fields satisfy the operator Maxwell equations in vacuum, and the following weak Lorenz type constraints

$$\begin{aligned} C_0(k)\Phi \;\; : \;\; &= i[< k, A(k) > -|k|\varphi(k)]\Phi = 0, \\ C_0^+(k)\Phi \;\; : \;\; &= -i[< k, A^+(k) > -|k|\varphi^+(k)]\Phi = 0 \end{aligned} \tag{160}$$

hold in the Fock space $\Phi$ for all $k \in \mathbb{E}^3$. As the operators $C_0(k) : \Phi \to \Phi$ and $C_0^+(k) : \Phi \to \Phi$

are commuting both to each other for all $k \in \mathrm{E}^3$ and with the Hamiltonian (154), that is

$$C_0(k), C_0(l)] \;=\; 0 = [C_0(k), C_0^+(l)], \tag{161}$$
$$C_0(k), H_b] \;=\; 0 = [C_0^+(k), H_b]$$

for any $k, l \in \mathrm{E}^3$, the constraints (160) are compatible with the evolution operator equations (157). Moreover, concerning the Hamiltonian operator (154), whose equivalent operator expression is

$$H_b = \frac{1}{2}\int_{\mathbb{R}^3} (|E|^2 + |B|^2), \tag{162}$$

the following proposition holds.

**Proposition 9.** *The Hamiltonian operator (154) on the reduced by means of constraints (160) Fock subspace $\Phi$ is Hermitian and non-negative definite.*

*Proof.* ~~Really, if~~ In order to define the operator

$$B(k) := A(k) - \frac{k}{|k|^2} < k, A(k) >, \tag{163}$$

the Hamiltonian operator (154) can be rewritten equivalently as

$$H_b \;=\; 2\int_{\mathbb{R}^3} d^3k\,|k|^2\,\{< \frac{k}{|k|} \times B^+(k), \frac{k}{|k|} \times B(k) > + \tag{164}$$
$$+ \frac{i}{|k|}\varphi(k) C_0^+(k) + \frac{1}{|k|^2}[{}_0^+(k) - i|k|\varphi^+(k)] C_0(k)\}.$$

The latter, owing to the weak Lorenz type constraints (160), gives rise to the inequality

$$(f, H_b f) \;=\; 2\int_{\mathbb{R}^3} d^3k\,|k|^2|(< \frac{k}{|k|} \times B(k) f, \frac{k}{|k|} \times B(k) f >) = \tag{165}$$
$$= \;2\int_{\mathbb{R}^3} d^3k\,\|k \times B(k) f\|^2 \geq 0$$

for any vector $f \in \Phi$, proving the proposition.

**Remark 5.** *The Hamiltonian operator expression (154) easily follows* [116] [121] [123] *from the well known relativistic invariant classical Fock-Podolsky electromagnetic Lagrangian*

$$\mathrm{L}_b \;:\; = \frac{1}{2}\int_{\mathbb{R}^3} d^3x [< \nabla\varphi + \frac{1}{c}\frac{\partial A}{\partial \tilde{t}}, \nabla\varphi + \frac{1}{c}\frac{\partial A}{\partial \tilde{t}} > - \tag{166}$$
$$- \;<\; \nabla \times A, \nabla \times A > - (\frac{1}{c}\frac{\partial \varphi}{\partial \tilde{t}} + < \nabla, A >)^2]$$

*Based on the corresponding to (166) Euler-Lagrange equations one finds that*

$$\frac{1}{c^2}\frac{\partial^2 A}{\partial \tilde{t}^2}-\Delta A=0,\ \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial \tilde{t}^2}-\Delta\varphi=0, \tag{167}$$

*whose wave solutions allow to determine the electromagnetic fields (158) and (159) and to check that the related Maxwell field equations in vacuum are satisfied if the Lorenz condition*

$$C_0(\tilde{t},x):=\frac{1}{c}\frac{\partial\varphi}{\partial\tilde{t}}+<\nabla,A>=0 \tag{168}$$

*holds for all* $(\tilde{t},x)\in M^4$. *Moreover, from the Lagrangian expression (166) one easily obtains by means of the corresponding Legendre transformation* [64] [96] [121] *the Hamiltonian operator*

$$H_b=\frac{1}{2}\int_{\mathbb{R}^3}d^3x(|E|^2+|B|^2-C_0^2)+\int_{\mathbb{R}^3}d^3x(<\nabla,A>^2-<\nabla\varphi,\nabla\varphi>), \tag{169}$$

*being equivalent in the Fock space* $\Phi$, *modulo the solutions (155) of the wave equations (167), to the ~~written above~~ operator expression (154).*

Taking into account the operator equations (157), one easily obtains that

$$\begin{aligned} C_0(k) &= i[<k,A(k)>-|k|\varphi(k)]\neq 0, \\ C_0^+(k) &= -i[<k,A^+(k)>-|k|\varphi^+(k)]\neq 0, \end{aligned} \tag{170}$$

contradicting the imposed ~~above~~ Lorenz constraint *(168)*. As the latter should be vanishing in the Fock space, it was suggested in [115] to reduce the Fock space $\Phi$ to a subspace, on which ~~there~~ are satisfied only the weak Lorenz type operator constraints (160). Concerning these constraints, imposed on the Fock space $\Phi$, it is necessary to mention that a corresponding vacuum vector $|0>\ \in\Phi$ does not, evidently, annihilate the operators $\varphi(k):\Phi\to\Phi$ and $A^+(k):\Phi\to\Phi^3$, as they do not form computing pairs with operators $C_0^+(k)$ and $C_0(k)$, respectively.

## 5. The transformed Fock space, its Lorenz type reduction and the Quantum Maxwell equations

As we are interested in describing the self-interaction of the fermionic quantum particle field $\psi:\Phi\to\Phi^4$ with the related self-generated bosonic electromagnetic potentials field $(\varphi,A):\Phi\to\Phi^4$, we need, within the Fock many-temporal description approach [115] [116], first to consider the fermionic particle and bosonic electromagnetic fields with respect to the common reference frame $\mathrm{K}_t$ specified by the temporal parameter $t\in\mathbb{R}$. Secondly, we need to make use of the classical "*minimum interaction*" principle [47] [117], (whose sketched backgrounds are presented in Supplement, Section 9. and to apply to the Hamiltonian operator expression (151):

$$H_f^{(int)}=\int_{\mathbb{R}^3}d^3x\psi^+<c\alpha,\frac{\hbar}{i}\nabla>\psi+\int_{\mathbb{R}^3}d^3x(\xi\psi^+\psi\varphi-\xi\psi^+<c\alpha,A>\psi), \tag{171}$$

in which the fermionic $\psi:\Phi\to\Phi^4$ and bosonic $(\varphi,A):\Phi\to\Phi^4$ operators are commuting *a priori* to each other as quantum fields of different nature. Since the whole quantum field system consists of the fermionic particle and bosonic self-generated electromagnetic fields, its evolution is described by means of the joint Hamiltonian operator

$$H_{f-b} := H_f^{(int)} + H_b \tag{172}$$

via the Heisenberg equations

$$\begin{aligned} \frac{\partial \psi}{\partial t} &:= \frac{i}{\hbar}[H_{f-b}, \psi], \frac{\partial \psi^+}{\partial t} := \frac{i}{\hbar}[H_{f-b}, \psi^+], \\ \frac{\partial A}{\partial t} &:= \frac{i}{\hbar}[H_{f-b}, A], \frac{\partial \varphi}{\partial t} := \frac{i}{\hbar}[H_{f-b}, \varphi] \end{aligned} \tag{173}$$

with respect to the common temporal parameter $t \in \mathrm{R}$, as in this case there is assumed that the corresponding temporal parameters $\tilde{t} \in \mathrm{R}$ and $\bar{t} \in \mathrm{R}$ coincide, that is $\tilde{t} = \bar{t} = t \in \mathrm{R}$ and, by definition, the operator spinor $\psi(t,x) := \psi(\bar{t}, \tilde{t})|_{\tilde{t}=\bar{t}=t}$. Simultaneously, ~~there~~ should be ~~evidently,~~ satisfied ~~the before derived both~~ the electromagnetic field evolution equations (157), with respect to the own reference frame $\mathrm{K}_{\tilde{t}}$ and the modified fermionic charged particle field equations

$$\frac{\partial \psi}{\partial \bar{t}} := \frac{i}{\hbar}[H_f^{(int)}, \psi], \frac{\partial \psi^+}{\partial \bar{t}} := \frac{i}{\hbar}[H_f^{(int)}, \psi^+] \tag{174}$$

with respect to the own reference frame $\mathrm{K}_{\bar{t}}$.

Being mostly interested in the evolution of the quantum particle fermionic field $\psi : \Phi \to \Phi$, we can get rid of the bosonic Hamiltonian impact into (174) having applied to the Fock space $\Phi$ the unitary canonical transformation

$$\Phi \to \tilde{\Phi} := U(t)\Phi, \tag{175}$$

where we denoted by $U(t) : \Phi \to \Phi$ the unitary operator satisfying the determining equation

$$dU(t)/dt = \frac{i}{\hbar} H_b U(t) \tag{176}$$

subject to the bosonic Hamiltonian operator (154) and the temporal parameter $t \in \mathrm{R}$. As a consequence of the transformation (175) we obtain the effective fermionic particle field interaction Hamiltonian operator

$$\begin{aligned} \tilde{H}_f^{(int)} &:= U(t) H_f^{(int)} U^*(t) = \\ &= \int_{\mathrm{R}^3} d^3x \psi^+ < c\alpha, \frac{\hbar}{i}\nabla > \psi + \int_{\mathrm{R}^3} d^3x (\xi \psi^+ \psi \tilde{\varphi} - \xi \psi^+ < c\alpha, \tilde{A} > \psi), \end{aligned} \tag{177}$$

where, by definition,

$$\tilde{A} := U(t) A U^*(t), \tilde{\varphi} := U(t) \varphi U^*(t)], \tag{178}$$

subject to which the evolution in the transformed Fock space $\tilde{\Phi}$, induced by the Hamiltonian operator (154)

$$\tilde{H}_b := U(t) H_b U^*(t) = 2\int_{\mathrm{R}^3} d^3k \, |k|^2 \, [< \tilde{A}^+(k), \tilde{A}(l) > - \tilde{\varphi}(k) \tilde{\varphi}^+(k)], \tag{179}$$

became completely eliminated. Concerning the Hamiltonian operator (179) here we need to mention that the related commutation relationships for the operators $(\tilde{\varphi}(k), \tilde{A}(k)) : \tilde{\Phi} \to \tilde{\Phi}^4$