Original Research Article

Finding on the Similarity between the Two Empirical Formulas: Temperature Dependence of Volumetric Expansion of Gas and Temperature Dependence of Resistivity of Conductor

ABSTRACT

This paper finds a similarity between the temperature dependence of volumetric expansion of gases and the temperature dependence of resistance of conductors. In the 1780s, Jacques Charles came to know that the volume of the gas was proportionally increasing as temperature increased. He also stated that the rate of volume expansion was not dependent on the kinds of gases. In the early 19th century, Georg Ohm discovered the electric resistance. It was known that the electric resistivity of conductors changed with temperature. At the room temperature, the measured temperature coefficients of resistance for silver, copper, aluminum and gold are 0.0038, 0.0039, 0.0039 and 0.0034 respectively. When the temperature coefficient 1/273 in Charles's law is expressed in a decimal, it is 0.0037. The similarity between the temperature dependence of volumetric expansion of gases and the temperature dependence of resistance of conductors is as follows: first, they have a linear relationship with respect to the temperature change; second, temperature coefficients in each formula are very close to each other; third, they are not dependent on the kinds of materials. This paper also shows that the formula of temperature dependence of resistivity can be converted into Charles's law formula mathematically. In this study, the temperature dependence of atomic vibration has been suggested as the cause of this similarity. In discussion, this paper suggests that the volumetric expansion of gas could be related with the atomic vibration.

Keywords: atomic vibration; temperature coefficient of resistance; Charles's law

1. INTRODUCTION

Charles's law, or the temperature dependence of volumetric expansion of gases (TDVEG), states that the volume of a gas is directly proportional to the temperature with constant pressure[1,2]. The obvious fact in the empirical gas law is that the increase of temperature gives a rise to the increase the speed of the molecules in a gas. Although it is not yet obvious how temperature increases the kinetic energy of gases, we know that temperature affects the amplitude of atomic vibration[3,4,5]. I here suggest another empirical formula that is similarly dependent on temperature and atomic vibration – namely, the formula of temperature dependence of resistivity of conductor (TDRC).

TDRC and TCVEG share a very similar temperature coefficient. The temperature coefficients of resistance of the good conductors such as silver, copper and aluminum are found to be close to 1/273 [6-10], which is the temperature coefficient of the volumetric expansion of gas in Charles's law formula. This similarity may imply that there is a common natural phenomenon between the two empirical formulas. In this study, the temperature dependence of atomic vibration has been suggested and examined as the cause of the similarity between the two.

This paper is organized as follows. First, the inherent features of the Charles's law have been analyzed, and the atomic vibration related assumption has been suggested. Second, general discussion of TDRC has been presented. Then, the temperature coefficients of resistance of the conductors with a low specific resistivity have been compared with the temperature coefficients of volumetric expansion of gas in Charles's law formula. It presents that the formula of the temperature dependence of resistivity of conductor can be converted into Charles's law formula mathematically. Later, how the gas molecules get the kinetic energy from temperature is discussed.

2. SIMILARITY BETWEEN THE FORMULAS OF THE TEMPERATURE DEPENDENCE OF VOLUMETRIC EXPANSION OF GAS (TDVEG) AND THE TEMPERATURE DEPENDENCE OF RESISTIVITY OF CONDUCTOR (TDRC)

2.1 Temperature Dependence of Volumetric Expansion of Gas (TDVEG)

To see Charles's law expressed by the equation (2.1), the most important feature is that the increase of temperature causes the volume expansion of gases. The temperature coefficients is constant whatever the gas is [1,2].

$$V_T = V_0[1 + \frac{1}{273}(T - T_0)] \qquad (2.1)$$

Here, V_T is the volume of gas at temperature T ℃, V_0 is the volume at 0 ℃, 1/273 is the temperature coefficient of volumetric expansion of gas. Charles's law is not a theoretically developed formula. It is not known so far how temperature gives a rise to volume expansion of gases. To see Avogadro's law, it states that there are the same numbers of molecules in the equal volume of gases under the same pressure and temperature whatever the gas is. Therefore, another important feature of the gas laws is that they are not dependent on the kinds of gases.

Let's look at the temperature dependent atomic vibration to understand these two important features of the gas laws. Every gas molecule is composed of atoms, and the amplitude of atomic vibration increases with temperature [3, 4, 5]. However, when temperature reaches 0 K, the volume of a gas disappears according to Charles's law. It means that the speed of molecules in a gas is zero with no atomic vibration. Therefore, considering Charles's law in relation to the atomic vibration, it can be assumed that the atomic vibration gives a rise to the kinetic energy of a gas. Under this assumption, Charles's law can be reinterpreted as follows: first, the increased temperature increases the amplitude of atomic vibration proportionally. Then, the increased amplitude of atomic vibration increases the speed of molecules in gases. Thus, temperature increase contributes the linear volume expansion of gases. Second, every gas molecule is composed of atoms and the atomic vibration is largely dependent on temperature. Thus, the temperature coefficient of gases, which is based on the atomic vibration, is constant whatever the gas is.

2.2 Temperature Dependence of Resistivity of Conductor (TDRC)

Electrical resistivity of a material shows how strongly the material opposes the flow of free electrons passing through it. If there is no free electron, the material will be a nonconductor. On the other hand, the conductors have a lot of free electrons. But they have different specific resistivity because of their different numbers of free electrons and different crystal lattice structures. Also, the electrical resistivity of conductors changes with temperature. It is well known that the increased amplitude of atomic vibration increases the collision of electrons with the atoms which makes up the crystal lattice[3]. In other words, the temperature dependence of resistivity of conductor is directly related to the atomic vibration [11]. In general, resistance is proportional to the temperature if the temperature does not vary too much. The well-known formula of the temperature dependence of resistivity of conductor is as follows [12]:

$$R_T = R_0[1 + \alpha(T - T_0)] \qquad (2.2)$$

Here, R_T is resistance at T ℃, R_0 is resistance at T_0 ℃, α is temperature coefficient of resistance at T_0 ℃. Let's look at the temperature coefficient of resistance for the good conductors having a low specific resistivity in Table 1 [6-10].

Table 1. The measured temperature coefficient of resistance for the good conductors.

Material	Specific resistivity($\Omega \cdot$ m)*	Temperature coefficient of resistance(1/℃)**, α
Ag	1.59×10^{-8}	0.0038
Cu	1.70×10^{-8}	0.0039~0.0040
Au	2.44×10^{-8}	0.0034~0.0037
Al	2.82×10^{-8}	0.0037~0.0043
W	5.6×10^{-8}	0.0045~0.0048
Zn	5.9×10^{-8}	0.0037~0.0038
Mo	6.0×10^{-8}	0.0044
Co	6.3×10^{-8}	0.0066
Ni	7.0×10^{-8}	0.006~0.0068
Fe	1.0×10^{-7}	0.005
Pt	1.1×10^{-7}	0.0039
V	1.9×10^{-7}	0.0039
Pb	2.2×10^{-7}	0.0039

*(* All values at 20 ℃, ** the exact value dependents on the purity of material as well as the temperature.)*

The temperature coefficient α is typically 0.003 to 0.006 for metals for room temperature.

2.3 The Similarities Between the Two Empirical Formulas

It is shown that the volume and the resistance are linearly dependent on the change of temperature in the two equations (2.1) and (2.2). It is also meaningful to make a comparison between the temperature coefficients in the two formulas.

Temperature coefficient of resistance (1/℃)

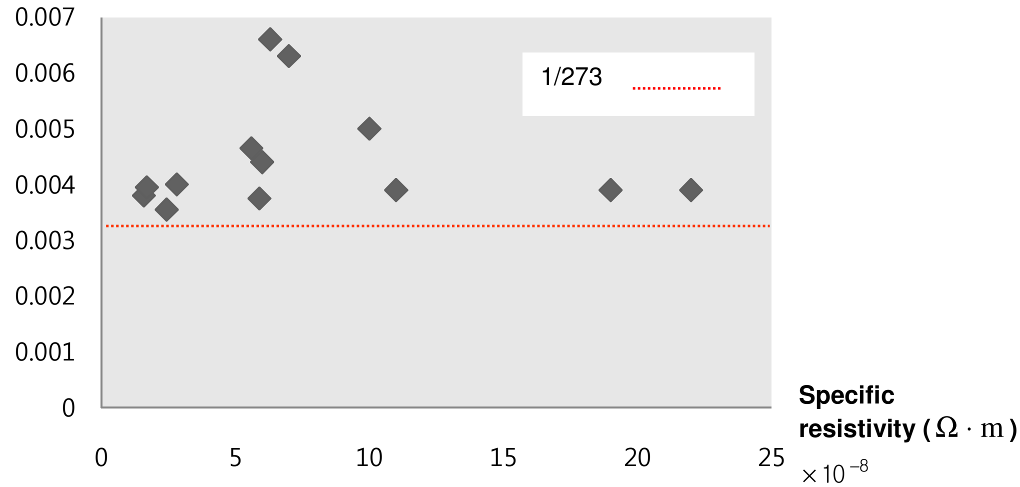

Fig. 1. The scatter diagram of the temperature coefficients of resistance of the good conductors in Table 1 with respect to the temperature coefficient in Charles's law.

In Table 1, the average temperature coefficient of resistance of the good conductors is 0.0045, which is about 21% larger than the value of 1/273. Most of the good conductors have temperature coefficients of resistance which are very close to the value of 1/273. In summary, there are some shared features in the two empirical formulas. First, they are linear dependent on the temperature changes. Second, the values of the temperature coefficients are very close to each other. Third, they are not dependent on the kinds of material. This similarity is remarkable given that the volume expansion of gases and the resistance of conductors are very different fields of physics.

To examine the shared features, atomic vibration can be considered to be a determinant factor, because all of the gases or the conductors are composed of atoms. In addition, the mean square amplitude of vibration of atoms is proportional to the temperature [3,4,5]. We know that temperature dependence of resistivity is caused by the atomic vibration [3,11]. In Charles's law, the volume of a gas becomes zero at the temperature of 0 K. At this temperature, atoms do not vibrate anymore. This is the evidence for that there must be a relationship between the atomic vibration and the volume expansion of a gas. Therefore, the similarities shown in the two empirical formulas could be explained by the temperature dependence of atomic vibration, as summarized in Table 2.

Table 2. The similarities between TDVEG and TDRC with cause

	TDVEG	**TDRC**
Formula	$V_T = V_0[1 + \frac{1}{273}(T - T_0)]$	$R_T = R_0[1 + \alpha(T - T_0)]$
Object	Resistance of conductor	Volume of gas
Temperature coefficient	1/273	α ($\cong$ 1/273)
Material dependency	Negligible	Week (for the good conductors)
Cause	Atomic vibration (inferred)	Atomic vibration

Evidence	When atomic vibration stops at 0K, the volume of gas becomes zero.

2.4 Conversion between the two empirical formulas

If the two formulas are equally temperature dependent and related with atomic vibration, are they convertible to each other? Note that resistance is linearly proportional to the amplitude of atomic vibration and the amplitude of vibration is proportional to the temperature. Then, the general relationship between resistance and temperature for the conductors can be expressed by:

$$R = k_1 T \tag{2.3}$$

Also, temperature has a relationship with the volume of gas by the ideal gas law. By applying the ideal gas law to the equation (2.3), it can be rewritten as:

$$R_T = k_1 \frac{PV_T}{k_2} \tag{2.4}$$

Finally, by using equation (2.4), equation (1) can be rewritten and summarized by:

$$k_1 \frac{PV_T}{k_2} = k_1 \frac{PV_0}{k_2}[1 + \alpha(T - T_0)]$$

$$V_T = V_0[1 + \alpha(T - T_0)] \tag{2.5}$$

Here, k_1, k_2 are proportional constants, P is pressure, V_T is the volume of gas at temperature T ℃, V_0 is the volume at 0 ℃, α is the temperature coefficient of volumetric expansion of gas in Charles's law. The converted equation (2.5) is mathematically isomorphic to Charles's law formula shown in equation (2.1).

3. DISCUSSIONS

By using the ideal gas law, the formula of the TDRC expressed by the equation (2.2) has been converted to Charles's law formula, TDVEG. In addition, it turns out that the temperature coefficients in the two formulas are very close to each other. In this study, the good conductors have been compared because their resistance changes are the most sensitive to atomic vibration. Not good conductor such as nickel-chrome or Manganin is not sensitive to the change of temperature and it has a very small value of temperature of coefficient of resistance.

This similarity implies that the two empirical formulas might be affected by a common feature of natural phenomena. If the common feature is the atomic vibration, the suggested assumption that the atomic vibration gives a rise to the kinetic energy of a gas is valid. How the atomic vibration is related with the volume expansion of a gas is not well investigated. Nonetheless, there is a research that the asymmetric repulsive forces on the nucleus by the atomic vibration give speed to the gas [13], although more evidences are needed to prove the suggested assumption.

4. CONCLUSIONS

In this study, the similarity between the two empirical formulas, which are the TDVEG and TDRC has been examined. First, the two formulas have a linear relationship with temperature change. Second, the temperature coefficients are very close to each other. Third, they both do not vary depending on the kind of materials. Lastly, it is shown that they are mathematically convertible. These findings are remarkable because the volume expansion of gases and the resistance of conductors are very different fields of physics. The temperature dependent atomic vibration has been suggested as a cause of this similarity. Although it is not well known in Charles's law how the volume expansion of a gas is related with atomic vibration, it is widely known that temperature dependence of resistance of the conductor is directly related with atomic vibration. Thus, the finding of such similarity shows that the kinetic energy of a gas in Charles's law could be obtained from the atomic vibration.

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