Critical comment on the paper "Some of the Complexities in the Special relativity: New paradoxes"

Abstract

Special Relativity, which the 110-anniversary was marked in the last year, is a very well elaborated theory. However, there might be some logically deduced discrepancies named paradoxes, which demand a painstaking examination. Nonetheless, every search for inherent contradictions is an uphill task. The authors of the considered paper proposed a situation with two coevals, believed to be contradictory, but the relevant mathematical analysis proves that it is none other than a pseudo-paradox, as well as several others.

Keywords: Special relativity theory, paradoxes, relativity of simultaneity.

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1. Introduction

The ‘twin paradox’ – a scientific thought experiment – is the most well-known generally because of the discussion about a journey to a neighbouring star and the return back in the course of a human lifetime. Briefly summarised it is as follows: there are two persons of the same age (identical twins). One of them remains stationary, whereas the other travels with acceleration-deceleration “to and fro”. When the traveller returns he occurs less aged than his home-sitting brother. A solution may be found, for example, in [1]. According to the rules of Special Relativity Theory (SRT) the divergence the ages is said to occur due to the unavoidable dynamic asymmetry: the travelling twin must undergo a plus-minus acceleration in contrast with the inertial counterpart. According to French: “*There is no paradox, and the asymmetrical ageing is real.*” [1, p. 156]. That is why up to date authors came to use either inverted commas like [2], or prefix *pseudo* like [3].

In [4] a new thought experiment, including three different observers (triplets) is presented. As distinct from the well-known “twin paradox”, two persons are moving in opposite directions in a manner symmetrical to the basic Inertial Reference Frame (IRF) staying at rest. This consideration leads to a contradiction in the notion of “relativity of simultaneity”, which restricts the area of the lawful implementation ability for Lorentz transformations. The authors of the paper ‘Some of the Complexities in the Special relativity: New paradoxes’ [5] borrowed the idea and it seemed to them that new paradoxes are found. Nevertheless, just symmetry in travels and homogeneity-isotropyness in space are not yet sufficient for a paradoxal situation to be begotten.

2. Pseudo-paradox of coevals

In the paper [5, pp. 3-4] one can read: “*Let two babies were born on each spacecraft just after accelerations became equal to zero. … The babies differ in that they moved relative to each other at a certain constant speed within their entire lives. They travelled equal distances* $|OA_1| = |OB_1|$ *up to the meeting at the beacon. … From the SRT viewpoint, the first child can reason in the following manne*r. *The second child moved relative me with a big velocity all my life (17 years). Therefore his age must be less than mine own. Besides, if he counts out the age of the second child starting from the moment of the receipt of the confirmation from* B_1 *, then he will believe that he will see an infant with his feeding bottle at* *the meeting. But the second child can reason about the first child in the same manner.*" Perhaps, young teenagers may reason in the above manner, owing to lack of the specific education, but the relativistic formalism rules out such a sort of arguing, as it is shown below.

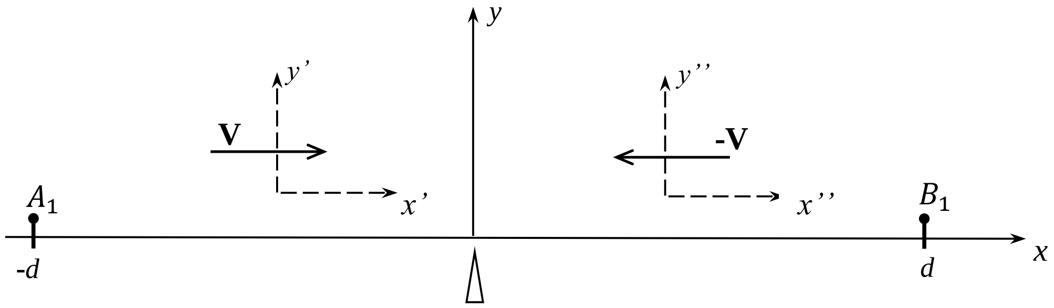

Figure 1. ***Symmetrically moving IRFs***

We have two inertial reference frames: IRF' – the child's A own, and IRF'' – the child's B own, moving relative to un-primed IRF – beacon's own (a body is at rest in its own IRF) with speeds V and $-V$, correspondingly, along the x-axis (Figure 1). The starting positions A_1 and B_1 are at the abscissas $-d$ and d of the beacon coordinate system. Un-primed IRF with space-time coordinates (x, y, z, t) and primed IRF' with space-time coordinates (x', y', z', t') are reciprocally connected. In the case of identities $y' \equiv y$, $z' \equiv z$, relation between primed and un-primed coordinates is provided by the simple version of the Lorentz transformation:

$$x' = \frac{x - Vt}{\sqrt{1 - V^2/c^2}} = \gamma(x - Vt),\ t' = \frac{t - \frac{V}{c^2}x}{\sqrt{1 - V^2/c^2}} = \gamma\left(t - \frac{V}{c^2}x\right), \qquad (1)$$

where c is speed of light and the relativistic factor $\gamma = \left(1 - V^2/c^2\right)^{-1/2} > 1$.

Denote $(-d, t_A)$ and (d, t_B) the starting events of children A and B. In the un-primed IRF both starts are simultaneous, i.e. $t_A = t_B = t_0$. In the primed IRF' by virtue of formulae (1) we have

$$x'_A = \gamma(-d - Vt_0),\ t'_A = \gamma\left(t_0 + \frac{V}{c^2}d\right),\ x'_B = \gamma(d - Vt_0),\ t'_B = \gamma\left(t_0 - \frac{V}{c^2}d\right). \qquad (2)$$

The meeting of children occurs at the beacon with abscissa $x_0 = 0$ at the moment t_m. Since the meeting moment on the watch of beacon is $t_m = t_0 + d/V$, the meeting event (x_0, t_m) from the first child point of view has the components

$$x'_0 = \gamma(0 - Vt_m) = -\gamma V t_m = -\gamma(Vt_0 + d)\,,\; t'_m = \gamma(t_m - 0) = \gamma t_m = \gamma(t_0 + d/V). \quad (3)$$

Thus, we can calculate the age of child A at the meeting moment, using formulae (2) and (3):

$$T'_A = t'_m - t'_A = \gamma\left(t_0 + \frac{d}{V} - t_0 - \frac{V}{c^2}d\right) = \gamma\frac{d}{V}\left(1 - \frac{V^2}{c^2}\right) = \frac{d}{\gamma V}\,. \quad (4)$$

From the first child point of view, the starting event of the second child is $(x'_B\,, t'_B)$, and the meeting event is $(x'_m\,, t'_m)$, therefore the time elapsed from the second child birth to the meeting is

$$T'_B = t'_m - t'_B = \gamma\left(t_0 + \frac{d}{V} - t_0 + \frac{V}{c^2}d\right) = \gamma\frac{d}{V}\left(1 + \frac{V^2}{c^2}\right). \quad (5)$$

However, this value isn't the second child age because he is moving relative to the first one. According to the relativistic rule for velocity addition, the speed (absolute value) of child B relative to the child A is

$$v = \frac{2V}{1 + V^2/c^2}\,.$$

The corresponding relativistic factor

$$\Gamma = \left(1 - {v^2}/{c^2}\right)^{-1/2} = \left[1 - \frac{4V^2/c^2}{(1 + V^2/c^2)^2}\right]^{-1/2} = \left[\frac{1 + \frac{2V^2}{c^2} + V^4/c^4 - 4V^2/c^2}{(1 + V^2/c^2)^2}\right]^{-1/2}$$

$$= \left[\frac{(1 - V^2/c^2)^2}{(1 + V^2/c^2)^2}\right]^{-1/2} = \frac{1 + V^2/c^2}{1 - V^2/c^2}\,. \quad (6)$$

In order to find the second child age, it is necessary to divide the value T'_B by factor Γ. Using formulae (5) and (6) we obtain

$$T''_B = \frac{T'_B}{\Gamma} = \gamma\frac{d}{V}\left(1 + \frac{V^2}{c^2}\right)\Big/\frac{1 + V^2/c^2}{1 - V^2/c^2} = \gamma\frac{d}{V}\left(1 - \frac{V^2}{c^2}\right) = \frac{\gamma d}{V\gamma^2} = \frac{d}{\gamma V}\,. \quad (7)$$

From comparing (4) and (7) it is obvious that age of both coevals is the same ($T''_B = T'_A$), when they meet one another at the beacon.

"*But the second child can reason about the first child in the same manner*" [5, p. 4]. Let us consider in brief the second child reasoning. In the double-primed IRF'' by virtue of formulae (1) we have

$$x''_A = \gamma(-d + Vt_0),\, t''_A = \gamma\left(t_0 - \tfrac{V}{c^2}d\right),\; x''_B = \gamma(d + Vt_0),\, t''_B = \gamma\left(t_0 + \tfrac{V}{c^2}d\right).$$

The meeting event (x_0, t_m) from the second child point of view has the components

$$x''_0 = \gamma(0 + Vt_m) = \gamma V t_m = \gamma(Vt_0 + d)\,,\; t''_m = \gamma(t_m +\ 0) = \gamma t_m = \gamma(t_0 + d/V).$$

Thus, we can calculate the age of child B at the meeting moment:

$$T''_B =\ t''_m - t''_B = \gamma\left(t_0 + \frac{d}{V} - t_0 -\ \frac{V}{c^2}d\right) = \gamma\frac{d}{V}\left(1 -\ \frac{V^2}{c^2}\right) = \frac{d}{\gamma V}\,.$$

The time elapsed from the first child birth to the meeting is

$$T''_A =\ t''_m - t''_A = \gamma\left(t_0 + \frac{d}{V} - t_0 +\ \frac{V}{c^2}d\right) = \gamma\frac{d}{V}\left(1 +\ \frac{V^2}{c^2}\right).$$

Since the factor Γ here is the same, the first child age is

$$T'_A = \frac{T''_A}{\Gamma} = \frac{d}{\gamma V}\,.$$

But again the above deduced equality $T'_A = T''_B$ persists. In this way we are persuaded that paradox of coevals does not exist in the realm of SRT, despite completely symmetric travels in the isotropic and homogeneous space. ***Quod erat demonstrandum.***

3. Some more putative paradoxes

"*Then it seems rather strange that the difference between lengths of bodies vanishes with the return to the initial place in the SRT (for example, in the paradox of twins), but the disparity remains in the time elapsed*" [5, p. 5]. This is not a coherent comparison. A time segment between two events also is invariant in the given IRF in spite of how many sundry $IRF',\ IRF'',\ IRF''',$ ... we are using in our manoeuvres with the Lorentz transformation.

"*The movement of a spaceship compresses the entire universe. It is without any physical mechanism*" [5, p. 5]. NO! The entire universe is already compressed to different degrees depending on corresponding different IRF. That is just an observer who must apply any physical mechanism in order to swop his own inertial frame of reference for other one.

The pseudo-paradox of two pedestrians. "*Two pedestrians from the ends of the segment begin to go with some equal speed towards a single preselected side, along the strait line containing the given segment, and they will walk for several seconds. The moving segment* (between pedestrians!) *should be shortened relative to the motionless segment* (between starting positions!) *by some hundreds of kilometres. However, none of the pedestrians will fly away for hundreds of kilometres during these seconds. Since the Lorentz transformation laws are continuous, the moving segment cannot likewise be torn off at the middle. In such a case, where has this segment been shortened?*" [5, p. 6]. This last question is senseless. The phrase "*pedestrians begin to go*" implies a transition process, whose result hardly depends on the type of acceleration. The final distance between pedestrians may get longer or shorter in their own IRF even if chosen law of acceleration is the same for both of them.

About "*two identical spaceships flying along two identical circular orbits in the opposite directions*" it should be said the following. The well-known formula

$$\Delta\tau = \int_0^T \sqrt{1 - \left(\frac{V(t)}{c}\right)^2}\ dt$$

determines a one-to-one correspondence between time t in an inertial system and time τ in a non-inertial system. If T is the half-period of orbiting, we have the following equality:

$$T' = \int_0^T \sqrt{1 - \left(\frac{V'(t)}{c}\right)^2}\ dt = \int_0^T \sqrt{1 - \left(\frac{V''(t)}{c}\right)^2}\ dt = T''.$$

Time between two consecutive meetings elapsed on the first spaceship is equal to time elapsed on the second one, since both functions $V'(t)$ and $V''(t)$ are identical.

The pseudo-paradox of a cut-in-half ruler. "*Being cut on two equal parts (1 and 2) the ruler C will move during the experiment. But the same ruler D will move as the whole during the experiment. At first, the movement of the first half of ruler $C - 1$ will be considered separately. The ruler $C - 1$ starts to move with uniform acceleration, reachs some large speed V, flies with such a constant speed and crosses the finish line F by its right end. Suppose now that the second half $C - 2$ of the ruler started to move simultaneously with the first half $C - 1$ under the same law (as the first half $C - 1$). Then, its right end will cross the line first half O' at the time of crossing of the finish by the first half of ruler $C - 1$. … We have a logic contradiction*" [5, p. 7].

NO! We have neither logical, nor physical contradictions, but we have the same error as in the previous "paradox". "*First, whence can the ruler C know about its cutting?*" The ruler cannot, but the driver, providing a fixed law of acceleration, do can. "*Secondly, the cut of the zero value cannot turn to a nonzero spatial gap according SRT.*" On the contrary, there exists such a law of acceleration (common for both parts of the ruler), that nonzero spatial gap occurs. Please, take a look on the so called **Bell's effect** in SRT.

4. Conclusion

As regards 'the relativistic paradox of the turn of sliding rods', it remains unclear for me: why the authors envisage two velocities **V** and **v** separately? I believe the calculation with due regards for the relativistic velocity addition would remove the problem from agenda despite the notorious non-commutativity.

The 'devil's advocate' kindly recommends authors to bring the list of references in accordance with reality. Firstly, substitute their erroneous reference [25] for the correct one present here as [4]. Secondly, in the passage where the well-known problem 'parallel flying charges' is discussing, the reference to the famous textbook 'Feynman's lectures on physics' is obligatory.

There are some minor mistakes and misprints in the text. It is possible readily correct them in the on-line version of the paper. Unfortunately, the effect of publication as a whole is the exact opposite of what the authors expected to reach.

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