Original Research Article

Chemical Reaction and Radiative MHD Heat and Mass Transfer Flow with Temperature Dependent Viscosity past an Isothermal Oscillating Cylinder

ABSTRACT

The numerical analysis is performed to examine the effects of magnetic, radiation and chemical reaction parameters on the unsteady heat and mass transfer flow past a temperature dependent viscosity of an isothermal oscillating cylinder. The dimensionless momentum, energy and concentration equations are solved numerically with stability and convergence analysis of the solution parameters by using explicit finite difference method. The effects on the velocity, temperature, concentration field, skin-friction, Nusselt number, streamlines and isotherms of various parameters entering into the problem separately are discussed with the help of graphs.

Keywords: Chemical reaction, magnetic, Radiation, Oscillating Cylinder, explicit finite difference.

1. INTRODUCTION

Free convection flow is often encountered in cooling of nuclear reactors or in the study of structure of stars and planets. Along with the free convection flow the phenomenon of mass transfer is also very common in the theories of stellar structure. The study of convective flow with mass transfer along a vertical porous plate is receiving considerable attention of many researchers because of its varied applications in the field of cosmical and geophysical sciences. Permeable porous plates are used in the filtration processes and also for a heated body to keep its temperature constant and to make the heat insulation of the surface more effective. The study of stellar structure on the solar surface is connected with mass transfer phenomena. Its origin is attributed to difference in temperature caused by the non-homogeneous production of heat, which in many cases can rest not only in the formation of convective currents but also in violent explosions. Mass transfer certainly occurs within the mantle and cores of planets of the size of or larger than the earth. It is therefore interesting to investigate this phenomenon and to study in particular, the case of mass transfer on the free convection flow. Magneto hydrodynamic (MHD) is the branch of continuum mechanics which deals with the flow of electrically conducting fluids in electric and magnetic fields. Several natural phenomena and engineering problems are important being subjected to a magneto hydrodynamic analysis. Magneto hydrodynamic has drawn the attention of a broad number of scholars due to its variant applications. The study of flow problems, which involve the interaction of several phenomena, has a wide range of applications in the field of science and technology. One such study is related to the effects of free convection MHD flow, which plays an important role in agriculture, engineering, and petroleum industries. The problem of free convection under the influence of a magnetic field has attracted the interest of many researchers in view of its application in geophysics and in astrophysics. Abd EL–Naby *et al.* (2004) presented finite difference solution of radiation effects on MHD unsteady free-convection flow on vertical porous plate. Dufour and soret effects on mixed convection flow past a vertical porous flat plate with variable suction have been studied by Alam *et al.* (2006). C. O. Popiel (2008) presented free convection heat transfer from vertical slender cylinder. Numerical study of free convection magneto hydrodynamic heat and mass transfer from a stretching surface to a saturated porous medium with soret and dufour effects is presented by Beg Anwa *et al.* (2009). Mass transfer effects on MHD flow and heat transfer past a vertical porous plate through porous medium under oscillatory suction and heat source studied by S.S.Das *et al.* (2009). H. P. Rani *et al.* (2010) studied about a numerical study on unsteady natural convection of air with variable viscosity over an isothermal vertical cylinder. Chemically reactive species and radiation effects on MHD convective flow past a moving vertical cylinder have been studied by Gnaneswara Reddy Machireddy (2013). Md. A Hossain *et al.* (2015) studied about a numerical study on unsteady natural convection flow with temperature dependent viscosity past an isothermal vertical cylinder. Free convection and mass transfer flow through a porous medium with variable temperature have been presented by R. K. Mondal *et al.* (2015). V. Rajesh *et al.* (2016) studied finite difference analysis of unsteady MHD free convective flow over moving semi-infinite vertical cylinder with chemical reaction and temperature oscillation effects.

The main objective of this research is to investigate the effect of radiation, chemical reaction, heat and mass transfer effect on unsteady MHD free convection flow with temperature dependent viscosity past an isothermal oscillating cylinder. Then these governing equations will be transformed into dimensionless momentum, energy and concentration equations and then the equations will be solved numerically by using explicit finite difference technique with the help of a computer programming language COMPAQ VISUAL FORTRAN 6.6a.

2. MATMEMATICAL MODEL OF THE FLOW

A two-dimensional unsteady free convection flow of a viscous incompressible electrically conducting and radiating optically thick fluid past an impulsively started semi-infinite oscillating cylinder of radius r_0 is considered. Here the x-axis is taken along the axis of cylinder in the vertical direction and the radial coordinate r is taken normal to the cylinder. Initially the cylinder and the fluid are at the same temperature T'_∞ and concentration C'_∞. At time $t' > 0$, the cylinder starts moving in the vertical direction with a uniform velocity u_0.

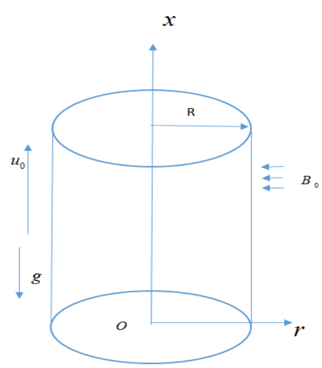

Fig-1**:** Flow model and Physical Co-ordinate.

The temperature of the surface of the cylinder is increased to T'_w and concentration C'_w and are maintained constantly thereafter. A uniform magnetic field is applied in the direction perpendicular to the cylinder. The field is assumed to be slightly conducting, and hence the magnetic Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the applied magnetic field. It is further assumed that there is no applied voltage, so that electric field is absent. It is also assumed that the irradiative heat flux in the x -direction is negligible as compared to that in the radial direction and the viscous dissipation is also assumed to be negligible in the energy equation due to slow motion of the cylinder. It is also assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. But here we assume the level of species concentration to be very low and hence heat generated during chemical reaction can be neglected. In this reaction, the reactive component given off by the surface occurs only in very dilute form. Hence, any convective mass transport to or from the surface due to a net viscous dissipation effects in the energy equation are assumed to be negligible. It is also assumed that all the fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term. The foreign mass present in the flow is assumed to be at low level, and Soret and Dufour effects are negligible. Then, the flow under consideration is governed by the following system of equations:

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0 \tag{1}$$

$$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+\frac{\partial u}{\partial r}=g\beta\left(T'-T'_\infty\right)+g\beta^*\left(C'-C'_\infty\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(\upsilon r\frac{\partial u}{\partial r}\right)-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+\frac{\partial T'}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T'}{\partial r}\right)-\frac{1}{\rho c_p}\frac{1}{r}\frac{\partial}{\partial r}\left(rq_r\right) \tag{3}$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+\frac{\partial C'}{\partial r}=\frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C'}{\partial r}\right)-K_1C' \tag{4}$$

With boundary conditions,

$$\left.\begin{array}{lll} t'\le 0: u=0,\ \ v=0,\ \ T'=T'_\infty,\ \ C'=C'_\infty & \text{for all } x\ge 0 \text{ and } r\ge 0 \\ t'>0: u=u_0,\ \ v=0,\ \ T'=T'_w,\ \ C'=C'_w & \text{at } \ r=r_0 \\ \quad u=0,\ \ v=0,\ \ T'=T'_\infty,\ \ C'=C'_\infty & \text{at } \ x=0 \text{ and } r\ge r_0 \\ \quad u\to 0,\ \ T'\to T'_\infty,\ \ C'\to C'_\infty & \text{as } \ r\to\infty \end{array}\right\} \tag{5}$$

It is required to make the equations (1) to (4) with boundary conditions (5) dimensionless. For this intention we introduce the following dimensionless quantities

$$\begin{array}{l} U=\dfrac{u}{u_0},\ R=\dfrac{r}{r_0},\ X=\dfrac{x\upsilon}{u_0r_0^2},\ V=\dfrac{vr_0}{\upsilon},\ t=\dfrac{t'\upsilon}{r_0^2},T=\dfrac{T'-T'_\infty}{T'_w-T'_\infty} \\ Gr=\dfrac{g\beta r_0^2(T'_w-T'_\infty)}{\upsilon u_0},\ Gc=\dfrac{g\beta^* r_0^2(C'_w-C'_\infty)}{\upsilon u_0},C=\dfrac{C'-C'_\infty}{C'_w-C'_\infty} \\ Pr=\dfrac{\upsilon}{\alpha},\ \ N=\dfrac{KK_e}{4\sigma_s T'^3_\infty},\ \ Sc=\dfrac{\upsilon}{D},\ \ K=K_r\dfrac{r_0^2}{\upsilon},M=\sigma B_0^2\dfrac{r_0^2}{\rho\upsilon} \end{array} \tag{6}$$

If γ denotes the non-dimensional viscosity variation parameter then $\gamma = \lambda\ (T^*_w - T^*_\infty)$. By introducing the non-dimensional variables of (6) into the equations (1) to (4) along with (5), we get the following no dimensional equations (7) to (10) with boundary conditions (11)

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial R}+\frac{V}{R}=0 \tag{7}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial R}=GrT+GcC+\left(1+\gamma T\right)\left(\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}\right)+\gamma\frac{\partial U}{\partial R}.\frac{\partial T}{\partial R}-MU \tag{8}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial R}=\frac{1}{Pr}\left(1+\frac{4}{3N}\right)\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) \tag{9}$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial R}=\frac{1}{Sc}\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial C}{\partial R}\right)-KC \tag{10}$$

The corresponding boundary conditions in terms of non-dimensional variables are

$$\left.\begin{array}{ll} t\le 0: U=0,\ \ V=0,\ \ T=0,\ \ C=0 & \text{for all } X\ge 0 \text{ and } R\ge 0 \\ t>0: U=1+cos(wt), V=0,\ T=1,\ C=1 & \text{at } \ R=1 \\ \quad U=0,\ \ T=0,\ \ C=0 & \text{at } \ X=0 \ \text{ and } R\ge 1 \\ \quad U\to 0,\ T\to 0,\ C\to 0 & \text{as } \ R\to\infty \end{array}\right\} \tag{11}$$

We have calculated average skin friction coefficient as

$$\overline{C_f}=-\int_0^1\left(\frac{\partial U}{\partial R}\right)_{R=1}dX \tag{12}$$

The average heat transfer rate (Nusselt number) is expressed as

$$\overline{Nu}=-\int_0^1\left(\frac{\partial T}{\partial R}\right)_{R=1}dX \tag{13}$$

3. NUMERICAL ANALYSIS OF THE PROBLEM

In order solve the nonlinear partial differential equations (7)-(10) along with boundary condition (11) an explicit finite difference method has been employed. The finite difference equations for the equations (7)-(10) can be written as the equations (14) to (17) respectively

$$\frac{U(i,j)-U(i-1,j)}{\Delta X}+\frac{V(i,j)-V(i-1,j)}{\Delta R}+\frac{V(i,j)}{1+(j-1)\Delta R}=0 \tag{14}$$

$$\begin{aligned}&\frac{U'(i,j)-U(i,j)}{\Delta\tau}+U(i,j)\frac{U(i,j)-U(i-1,j)}{\Delta X}+V(i,j)\frac{U(i,j+1)-U(i,j)}{\Delta R}=\\&GrT(i,j)+GcC(i,j)-MU(i,j)-\left[1+\gamma T(i,j)\right]\\&\left[\frac{U(i,j+1)-2U(i,j)+U(i,j-1)}{(\Delta R)^2}+\frac{1}{\left[1+(j-1)\Delta R\right]}\frac{U(i,j+1)-U(i,j)}{\Delta R}\right]\\&+\gamma\frac{T(i,j+1)-T(i,j)}{\Delta R}\frac{U(i,j+1)-U(i,j)}{\Delta R}\end{aligned} \tag{15}$$

$$\begin{aligned}&\frac{T'(i,j)-T(i,j)}{\Delta\tau}+U(i,j)\frac{T(i,j)-T(i-1,j)}{\Delta X}+V(i,j)\frac{T(i,j)-T(i-1,j)}{\Delta R}\\&=\frac{1}{\Pr}\left[1+\frac{4}{3N}\right]\frac{1}{\left[1+(j-1)\Delta R\right]}\frac{T(i,j+1)-T(i,j)}{\Delta R}+\frac{1}{Pr}\left[1+\frac{4}{3N}\right]\left[\frac{T(i,j+1)-2T(i,j)+T(i,j-1)}{(\Delta R)^2}\right]\end{aligned} \tag{16}$$

$$\begin{aligned}&\frac{C'(i,j)-C(i,j)}{\Delta\tau}+U(i,j)\frac{C(i,j)-C(i-1,j)}{\Delta X}+V(i,j)\frac{C(i,j)-C(i-1,j)}{\Delta R}\\&=\frac{1}{Sc}\left[\frac{1}{\left[1+(j-1)\Delta R\right]}\frac{C(i,j+1)-C(i,j)}{\Delta R}+\frac{C(i,j+1)-2C(i,j)+C(i,j-1)}{(\Delta R)^2}\right]-KC(i,j)\end{aligned} \tag{17}$$

To obtain the finite difference equations the region of the flow is divided into the grids or meshes of lines parallel to *X* and *R* is taken normal to the axis of the cylinder. Here we consider that the height of the cylinder is $X_{max}=20.0$ i.e. *X* varies from 0 to 20 and regard $R_{max}=50.0$ as corresponding to $R\to\infty$ i.e. *R* varies from 0 to 50. In the above equations (11) to (14) the subscripts *i* and *j* designate the grid points along the *X* and *R* coordinates, respectively, where $X=i\Delta X$ and $R=1+(j-1)\Delta R$. There are $M=400$ and $N=300$ grid spacing in the *X* and *R* directions respectively. By experimenting with different set of mesh sizes, we have been fixed at the level $\Delta X=0.067$, ΔR=0.25 and the time step $\Delta\tau=0.001$. In this case, spatial mesh sizes are reduced by 50% in one direction, and then in both directions, and the results are compared. It is regarded that, when the mesh size is reduced by 50% in both the direction, the results differ in the fourth decimal places. The computer takes more time to compute, if the size of the time-step is small. Hence, the previous mentioned sizes have been taken as suitable mesh sizes for calculation.

From the boundary conditions given in equation (11), the values of velocity *U, V* and temperature *T* are known at time $\tau=0$; then the values of *U, V* and *T* at the next time step can be evaluated. Generally, when the above variables are known at $\tau=n\Delta\tau$, the values of variables at $\tau=(n+1)\Delta\tau$ are calculated as follows. The finite difference equations (14) and (17) at every internal nodal point on a particular *i*-level constitute a rectangular system of equations. The temperature *T* is calculated from equation (16) at first at every *j* nodal point on a particular *i* -level at the (*n+1*) time step. By making the use of these known values of *T*, in a similar way the velocity *U* at the *(n+1)* time step is calculated from equation (13). Thus the values of *T* and *U* are known at a particular i -level. Then the velocity *V* is calculated from equation (12) explicitly. This process is repeated for the consecutive *i* -levels. Thus the values of and *T* are known at all grid points in the rectangular region at the $(n+1)^{th}$ time step. This iterative procedure is repeated for many time steps until the steady state solution is reached.

4. RESULT AND DISCUSSION

To assess the physical situation of the problem of the study, the velocity field, temperature field and concentration field are express by assigning numerical values to different parameters encountered into the corresponding equations. To be realistic, the value of Schmidt number (*Sc*) are chosen for Hydrogen gas diffusing in electrically-conducting Air (*Sc*=0.20), Helium (*Sc*=0.30), Steam (*Sc*=0.60), Oxygen (*Sc*=0.66), NH_3 (*Sc*=0.78) and CO_2 at 25^0C (*Sc*=0.94). The value of Prandtl number (*Pr*) number are chosen for air (*Pr*=0.71), water (*Pr*=7.0) and water at 4^0C (*Pr*=11.62). The Fig-2 depicts that when *Pr* and *Sc* changes then the velocity curves show different shapes for fixed values of *Gr, Gc, N, k, γ* and *M*. It is also noticed that the decreasing value of *Pr* and *Sc* results to an increasing of velocity main flow. The Prandtl number physically relates the relative thickness of the hydrodynamic boundary layer and thermal boundary layer. The Fig-3 displays that when the *K* changes then the velocity curves evince different shapes for fixed values of rest parameters. The increase values of magnetic parameter create a drug force known as Lorent force that opposes the fluid motion. The Fig-4 indicates that when *γ* changes then the velocity, curves show different shapes for fixed values of other parameters. The velocity curve is in downward direction at the increasing values of *γ*. The thermal Grashof number signifies the ratio of the species buoyancy force to the hydrodynamic viscous force and the mass Grashof number signifies the relative effect of the buoyancy force to the viscous hydrodynamic force. When *Gr, Gc, M* changes then the velocity curves exhibit different shapes *is* uncovered by the Fig-5. The curves are in upward direction for the increasing value of *Gr* and *Gc*. The temperature profiles curves exhibit different shapes when *Sc* and *Pr* changes with fixed values of *Gr, Gc, N, k,γ* and *M* is shown by the Fig-6. The temperature profiles curve is in downward direction at the increasing values of *Sc* and *Pr.* Schmidt number decrease the molecular diffusivity. When the *Sc, Pr* and *K* changes then the concentration curves let on different shapes for fixed values rest parameters as shown in Fig-8 and Fig-9. By analyzing Fig-8 it is apparent that the curves are upward direction with the combination of decreasing values of *Sc* and *Pr*. Nusselt number (*Nu*) is increases with the decreases of *γ* which is uncovered by the Fig-9. Skin-friction increases with an increase of *γ* which is shown by the Fig-10. With the increases of viscosity variation parameter (*γ*) increases the values of stream which as shown in Fig-11 to Fig-13. The isotherm lines increases for the increasing values of viscosity variation parameter (*γ*) which is noted by the Fig-14 to Fig-16.

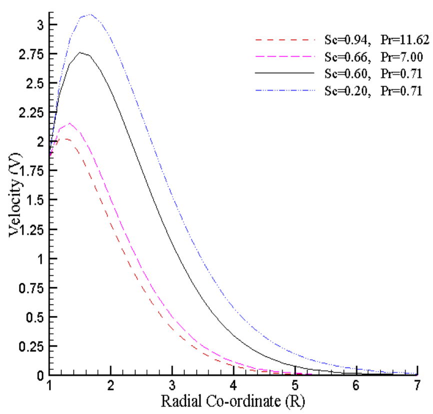

Fig-2:Velocity profiles for different values of *Sc* and *Pr* against *R.*

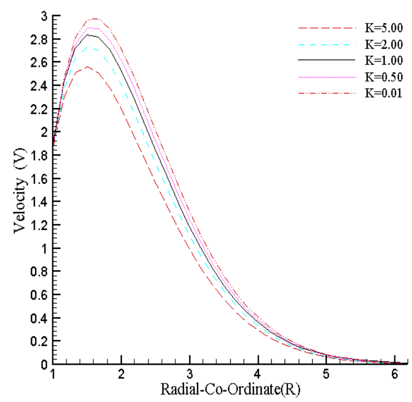

Fig-3:Velocity profiles for different values of *K* against *R.*

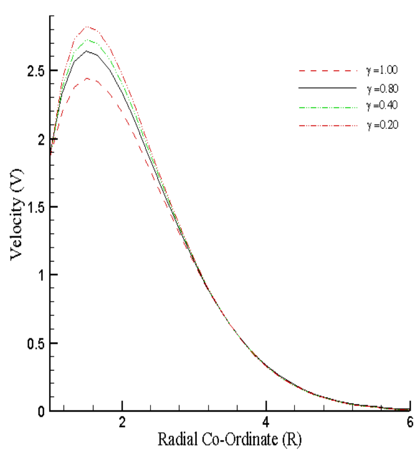

Fig-4: Velocity profiles for different values of *γ* against *R.*

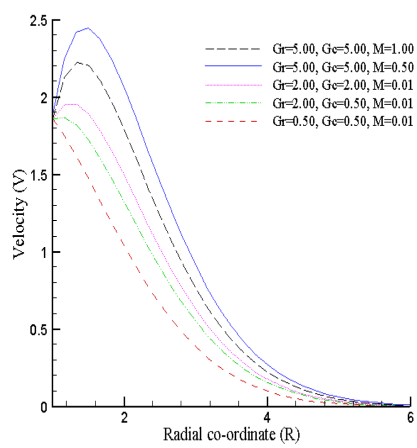

Fig-5: *Velocity profiles for different values of Gr, Gc and M against R.*

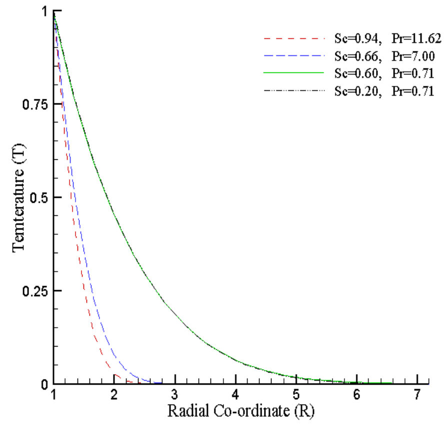

Fig-6: Temperature profiles for different values of *Sc* and *Pr* against *R*.

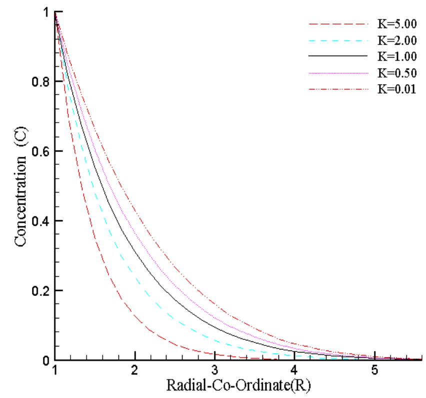

Fig-7: Concentration profiles for different values *K* against *R*.

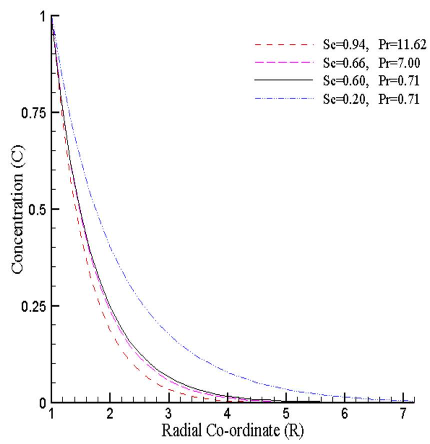

Fig-8: Concentration profiles for different values of *Sc* and *Pr* against *R*.

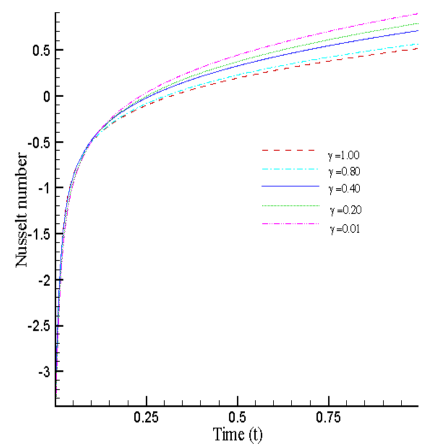

Fig-9: Nusselt number for different values of γ against *R*.

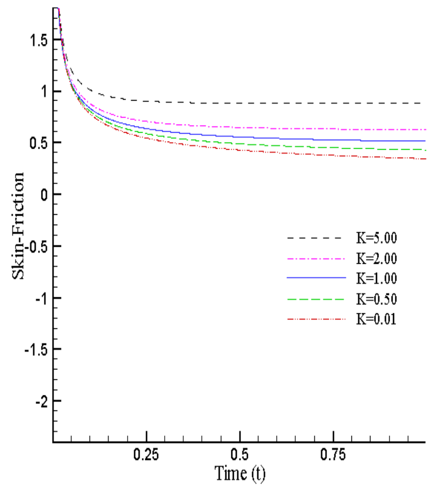

Fig-10: Skin-Friction for different values of *K* against *R*.

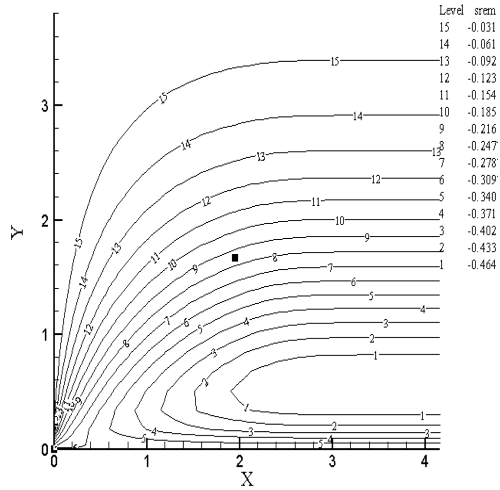

Fig-11: The streamlines with respect to *γ=-0.20* at *Pr=0.71*

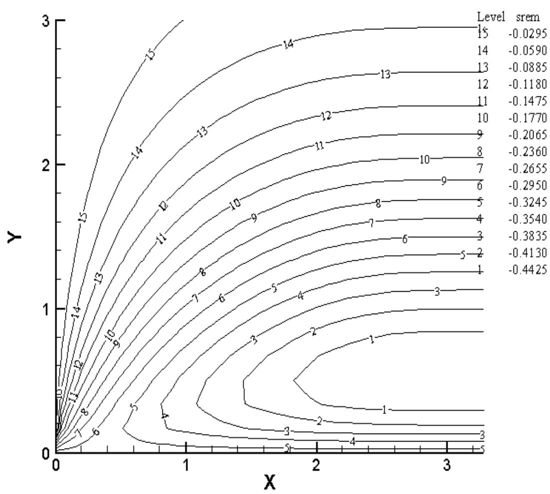

Fig-12: The streamlines with respect to γ=0.01at Pr=0.71

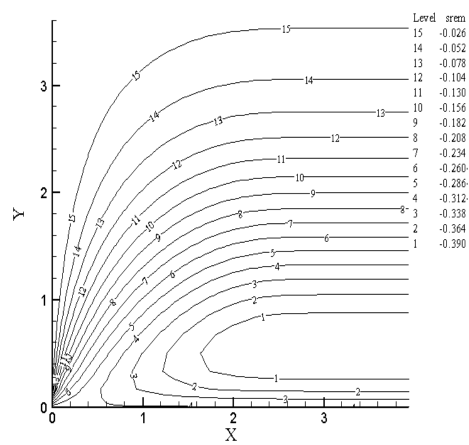

Fig-13: The streamlines with respect to *γ=0.80* at *Pr=0.71*

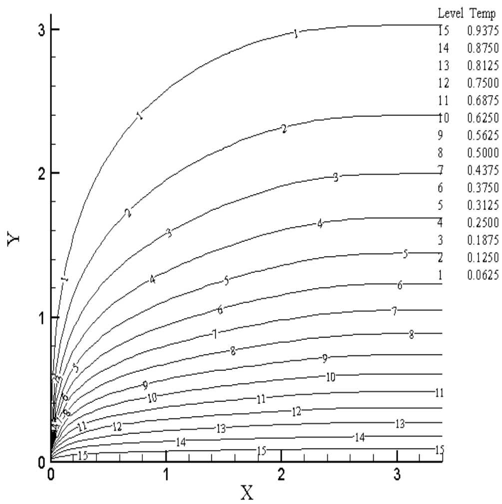

Fig-14: The isotherm lines with respect to *γ=-0.20* at *Pr=0.71*

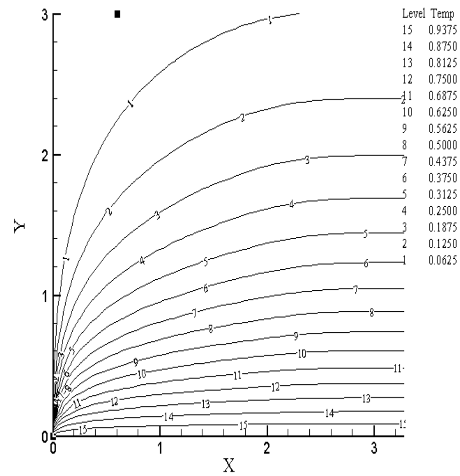

Fig-15: The isotherm lines with respect to *γ=0.01* at *Pr=0.71*

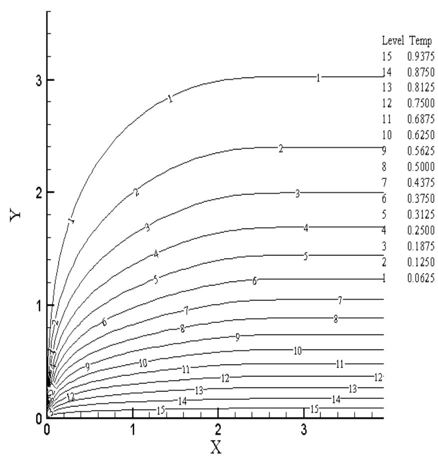

Fig-16: The isotherm lines with respect to *γ=0.80* at *Pr=0.71*

5. CONCLUTION

In the present research work, boundary layer equations become non-dimensional by using non-dimensional quantities. The non-dimensional boundary layer equations are nonlinear partial differential equations. These equations are solved by explicit finite difference method. Results are given graphically to display the variation of velocity, temperature, concentration, Nusselt number, Skin-friction, stream and Isotherm lines. The following conclusions are set out through the overall observations.

1) The velocity decreases with an increase of Scmidth number (*Sc*) and Prandtl number (*Pr).*
2) With the decreasing of chemical reaction parameter (*K*), viscosity variation parameter (*γ*), result to increasing the velocity profiles.
3) For the decreasing values of Scmidth number (*Sc*) and Prandtl number (*Pr*) the temperature increases.

4) The concentration increases with the decreasing values of Scmidth number (*Sc*), Prandtl number (*Pr*) and chemical reaction parameter (*K*).

5) Skin friction increases with an increase of chemical reaction parameter (*K*).

6) The Nusselt number (*Nu*) is increases with the decreases of γ.

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