#### Original Research Article 1 2 Natural Convective Mass Transfer MHD Flow of Chemically 3 **Reactive Micropolar Fluid past** 4 a Vertical Porous Plate 5 6 7 ABSTRACT Magnetic field effects on a free convective mass transfer flow of chemically reactive micropolar fluid over a vertical porous plate are investigated in this work. A mathematical related to the problem is developed from the basis of model studvina magnetohydrodynamics(MHD). A usual mathematical transformation is applied on the model to obtain a system of non-dimensional equations. Analytical solution of the problem is calculated by the use of perturbation technique. The computed numerical values of fluid velocity, angular velocity and species concentration are plotted in different figures. To observe the effects of various parameters on the above mentioned physical quantities, the results are discussed in detailed with the help of graphs. Finally, a conclusion is listed here. 8 9 Keywords: MHD, Micropolar fluid, Porous plate, Chemical reaction, Perturbation technique. 10 **1. INTRODUCTION** 11 12 The behaviors of fluid that contain suspended, metal or dust particles in many practical situations 13 are first observed by the micropolar fluid theory (Eringen, 1966) with internal structures in which 14 coupling between the spin of each particle and the macroscopic velocity field is taken into account. 15 Physically, the micropolar fluids contain dilute suspension of small, rigid, cylindrical 16 macromolecules with individual motion and are influenced by spin inertia. Since the theory is used 17 18 to investigate the flow character of polymeric fluids, colloidal suspension (Hadimoto and Tokioka, 19 1969; Kucaba-Pietal, 2004; Khedr, 2009), human and animal blood(Arimanet al., 1974; Muthu, 2008), 20 liquid crystal (Lockwood et al., 1987) and exotic lubricants so many scientists have been received a great interest to observe the micropolar fluid dynamics at present time. 21 22 The authors (Ferraro and Plumpton, 1996; Cramer and Pai, 1973; Raptis, 2011; Samiulhaqet al., 23 2012 and Seth et al., 2015) are notable for major contribution about MHD free convection flows and 24 their significant application in the field of stellar and planetary magnetospheres, aeronautics, chemical 25 engineering, electronics, and so on. In addition, many transport processes exist in industries 26 andtechnology where the transfer of heat and mass occurs simultaneously as a result of thermal diffusion and diffusion of chemical species. An extensive contribution on heat and mass transfer flow 27 28 has been made (Gebhart, 1971) to highlight the insight on the phenomena. Thereafter, several 29 authors, (Chamkha, 2000; Chaudhary, 2007; Hague and Alam, 2009; Samad and Mohebujjaman, 2009; Eldabe, 2011 and Seth, 2015) have paid attention to the study of MHD free convection and 30 31 mass transfer flows. 32 The growing needs for chemical reactions in chemical and hydrometallurgical industries require the study of heat and mass transfer with chemical reaction. The effect of the first order homogeneous 33 chemical reaction of an unsteady flow past a vertical plate with the constant heat and mass transfer 34 35 has been investigated (Das et al., 1994). The chemical reaction effects on an unsteady MHD free 36 convection fluid flow past a semi-infinite vertical plate embedded in a porous plate with heat 37 absorption have been studied by (Ibrahim et al., 2008; Anand Rao et al., 2012; Das, 2012; and Raju 38 et al., 2013). 39 The author (Peddision and McNitt, 1970; Bakr, 2011) has recognized the boundary layer situation 40 for steady micropolar fluid flow past a semi-infinite flat plate due to its important role in a number of 41 technical processes. The micro inertia effects on the flow of a micropolar fluid past a semi-infinite plate are investigated (Ahmadi, 1976; Kucaba-Pietal, 2004; Khedr et al., 2009). The free convective 42 43 micropolar fluid flow induced by the simultaneous action of buoyancy forces is of great interest in

nature and in many industrial applications as drying processes, solidification of binary alloy as well asin astrophysics, geophysics and oceanography.

The processes of mass transfer play an important role in the production of materials in order to obtain 46 the desired properties of a substance. Separation processes in chemical engineering such as the 47 48 drying of solid materials, distillation, extraction and absorption are all affected by the process of mass 49 transfer. Chemical reactions including combustion processes are often decisively determined by the 50 mass transfer. The authors (Callahan and Marner1976; Bakr 2011) studied a free convective steady 51 flow with mass transfer past a semi-infinite plate. An investigation on free convective steady flow with 52 mass transfer past asemi-infinite vertical porous plate with constant suction has been completed 53 (Soundalgekar and Wavre 1977; Ahmed and Das 2013). Transient free convection flow on a semi-54 infinite vertical plate with mass transfer has been observed (Soundalgekar and Ganesan1980). Hence 55 our main goal is to investigate a free convective mass transfer steady flow of a chemically reactive 56 micropolar fluid past a semi-infinite porous plate.

### 57 2. ANALYSIS AND SOLUTION

#### 58 2.1 MATHEMATICAL FLOW

A natural convective mass transfer steady flow of a chemically reactive micropolar fluid along a semiinfinite vertical porous plate is considered in the presence of a uniform magnetic field. The flow is assumed to be in the *x*-direction which is taken along the plate in the upward direction and *y*-axis is normal to it. Initially, we consider that the plate as well as the micropolar fluid particles is at rest at the same species concentration level  $C(=C_{\infty})$  at all points, where  $C_{\infty}$  is species concentration of uniform flow. It is also assumed that a magnetic field *B* of uniform strength is applied normal to the flow region. The physical configuration and co-ordinate system of the problem is presented in the

following Fig.1.



68 69 70

71

Fig.1. Physical configuration of the flow

72 Within the framework of the above stated assumption, the equations relevant to the problem are 73 governed by the following system of coupled non-linear partial differential equations under the 74 boundary-layer approximations,

75

76 Continuity Equation  $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ 

Momentum Equation

$$78 \qquad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta^* \left(C - C_{\infty}\right) + \left(v + \frac{\chi}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\chi}{\rho}\frac{\partial\Gamma}{\partial y} - \frac{\sigma' uB_0^2}{\rho} - \frac{v}{K'}u$$

Angular Momentum Equation

$$\begin{array}{l} 80 \\ 81 \end{array} \quad u \frac{\partial \Gamma}{\partial x} + v \frac{\partial \Gamma}{\partial y} = \frac{\gamma}{\rho j} \left( \frac{\partial^2 \Gamma}{\partial y^2} \right) - \frac{\chi}{\rho j} \left( 2\Gamma + \frac{\partial u}{\partial y} \right) \end{array}$$

**Concentration Equation** 

82 
$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - K_c (C - C_\infty)$$

83 with boundary condition,

84 
$$u = 0, \Gamma = -s \frac{\partial u}{\partial y}, C = C_w \text{ at } y = 0$$

85  $u = 0, \Gamma = 0, C = C_{\infty} \text{ at } y \rightarrow \infty$ 

where u is the velocity component,  $\Gamma$  is the velocity acting in z - direction (the rotation of  $\Gamma$  is in the x-y plane),  $B_0$  is the magnetic field component, g is local acceleration due to gravity,  $\chi$  is the

vortex viscosity,  $\gamma$  is the spin gradient viscosity,  $\beta^*$  is concentration expansion coefficient.

#### 90 2.2 MATHEMATICAL FORMULATION

91 Since our goal is to attain analytical solutions of the problem so we introduce the following 92 dimensionless variables,

93 
$$\eta = y \sqrt{\frac{U_0}{2vx}}, \psi = \sqrt{2vU_0x} f(\eta), u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}, \phi(\eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}} \text{ and } \Gamma = \sqrt{\frac{U_0^3}{2vx}} g(\eta)$$

94 The dimensionless equations are,

95 
$$(1+\Delta) f'''(\eta) + \Delta g'(\eta) + f(\eta) f''(\eta) + G_M \varphi(\eta) - (K+M) f'(\eta) = 0$$
 (4)

96 
$$\wedge g''(\eta) + f'(\eta)g(\eta) + f(\eta)g'(\eta) - 2\lambda g(\eta) - \lambda f''(\eta) = 0$$
(5)

97 
$$\varphi''(\eta) + S_c f(\eta) \varphi'(\eta) - S_c C_r \varphi(\eta) = 0$$
 (6)

98 with boundary conditions

99 
$$f(\eta) = f_w, f'(\eta) = 0, g(\eta) = -sf''(\eta), \varphi(\eta) = 1 \eta = 0$$

100 
$$f'(\eta) = 0, g(\eta) = 0, \varphi(\eta) = 0, \eta \to \infty$$

101 Where, micro-rotational number 
$$\Delta = \frac{\chi}{\rho v}$$
, modified Grashof number,

102 
$$G_m = \frac{g\beta^*(C_w - C_\infty)2x}{U_0^2}$$
 permeability of porous plate,  $K = \frac{2\nu x}{K'U_0}$ , magnetic force number,

103 
$$M = \frac{\sigma B_0^2 2x}{U_0 \rho}$$
, Spin Gradient number,  $\wedge = \frac{\gamma}{v \rho j}$ , Vortex viscosity,  $\lambda = \frac{2x \chi}{\rho j U_0}$ , Schmidt  
104 number,  $S_c = \frac{v}{D_m}$ , chemical reaction parameter,  $C_r = K_c \frac{2x}{U_0}$ .

105

### 106 2.3 MATHEMATICAL ANALYSIS

107 Since the solution is sought for the large suction further transformation can be made as,

108 
$$\xi = \eta f_w$$
 (1)  
109  $f(\eta) = f_w F(\xi)$  (2)  
110  $\phi(\eta) = f_w^2 G(\xi)$  (3)

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$$\begin{array}{ll} 144 & F_2 = \frac{1}{A_2 \varepsilon^2} - \frac{\xi}{\varepsilon^2} - \frac{1}{A_2 \varepsilon^2} e^{-A_2 \xi} + A_3 e^{-S_z \xi}, \ H_2 = 0 \ \text{and} \ G_2 = -C_z e^{-S_z \xi} \\ 145 & \text{From Equation (4.4.6) we get third order solutions,} \\ 146 & F_3 = -\frac{1}{A_2 \varepsilon^2} + \frac{\xi}{\varepsilon^3} + \frac{1}{A_2 \varepsilon^2} e^{-A_2 \xi} + A_{15} e^{-A_2 \xi} + A_{16} e^{-S_z \xi} + A_{17} e^{-S_z \xi} + A_{18} e^{-A_2 \xi} - A_{19} e^{-S_z \xi} \\ 147 & H_3 = A_8 e^{-A_2 \xi} + A_9 e^{-S_z \xi} \\ 148 & G_3 = A_4 e^{-S_z \xi} - A_5 e^{-S_z \xi} + A_0 \xi e^{-S_z \xi} - A_7 e^{-(S_z + A_1)\xi} + A_{20} e^{-2S_z \xi} \\ 149 & \text{From the Equation (9), (10) and (11) we have series for the solution. Putting the equations of first, second and third order solution in the Equations (9), (10) and (11), we get the values of F, H and G. \\ 151 & \text{Substituting the values of F, H and G to the equation.} \\ 152 & \text{The fluid velocity equation,} \\ 153 & f = f_w + \left(\varepsilon^2 f_w A_3 + \varepsilon^3 f_w A_{16} + \varepsilon^3 f_w A_{17} e^{-S_z \xi} - \varepsilon^3 f_w A_{19}\right) e^{-S_z \xi} + \left(\varepsilon^3 f_w A_{15} + \varepsilon^3 f_w A_{18}\right) e^{-A_2 \xi} \\ 154 & \Rightarrow f = f_w + \left(\varepsilon^2 f_w A_3 - S_c \varepsilon^3 f_w A_{16} - S_c \varepsilon^3 f_w A_{17} e^{-S_z \xi} + S_c \varepsilon^3 f_w A_{19}\right) e^{-S_z \xi} \\ 155 & \Rightarrow f' = \left(-S_c \varepsilon^2 f_w A_3 - S_c \varepsilon^3 f_w A_{16} - S_c \varepsilon^3 f_w A_{17} e^{-S_z \xi} + S_c \varepsilon^3 f_w A_{19}\right) e^{-S_z \xi} \\ 156 & -\left(A_2 \varepsilon^3 f_w A_{15} + A_2 \varepsilon^3 f_w A_{18}\right) e^{-A_2 \xi} \\ 157 & \text{The fluid angular velocity equation,} \\ 158 & g(\eta) = \varepsilon^3 f_w^3 A_8 e^{-A_2 \xi} + \varepsilon^3 f_w^3 A_9 e^{-S_z \xi} \\ 159 & \Rightarrow g(\eta) = \varepsilon^3 f_w^3 A_8 e^{-A_2 \xi} + \varepsilon^3 f_w^3 A_9 e^{-S_z \xi} \\ 159 & \Rightarrow g(\eta) = \varepsilon^3 f_w^3 A_8 e^{-A_2 \xi} + \varepsilon^3 f_w^3 A_9 e^{-S_z \xi} \\ 160 & \text{The fluid concentration equation,} \\ 161 & \phi(\eta) = f_w^2 G(\xi) \\ 162 & \Rightarrow \varphi(\eta) = f_w^2 \varepsilon e^{-S_z \xi} - \varepsilon^2 f_w^2 C_r e^{-S_z \xi} + \varepsilon^3 f_w^2 A_4 e^{-S_z \xi} - \varepsilon^3 f_w^2 A_5 e^{-S_z \xi} + \varepsilon^3 f_w^2 A_6 \xi e^{-S_z \xi} \\ 163 & -\varepsilon^3 f_w^2 A_7 e^{-(S_z + A_z) \xi} \\ 164 \\ 3. \text{ RESULT AND DISCUSSION} \\ 166 \end{array}$$

first.

167 To discuss the results of the problem, the analytical solutions are obtained by using the perturbation 168 technique. In order to analyze the physical situation of the model, we have computed the numerical 169 values of the flow variables for different values of Modified grashof number  $G_m$ , Suction parameter  $f_w$ , 170 Magnetic force number M, permeability of porous plate K, Micro-rotational number A, Vortex viscosity 171  $\lambda$ , Spin gradient viscosity number ( $\wedge$ ), Schmidt number  $S_c$  and Chemical reaction parameter  $C_r$ . The 172 fluid velocity, angular velocity and concentration versus the non-dimensional coordinate variable  $\eta$ 173 are displayed in Figures. 174 The increase values of magnetic parameter create a drug force known as Lorent force. The velocity 175 profiles are illustrated in Fig. 1. As it is observed, the velocity profiles curve climb up at the increase of

176 magnetic force number. Afterwards, the suction parameter stabilize the boundary layer growth. So the 177 velocity profiles curve decline with go up suction parameters. And same behavior is shown for 178 Schmidt number, modified Grashof number. The velocity profiles curve go down for permeability of 179 prous plate. Then with increase of vortex viscosity, the velocity profiles plunge. The angular velocity 180 profiles are displayed in Fig. 2. Firstly, angular velocity profiles curve decline with rise of suction 181 parameter. Afterthat, it increases at the modified Grashof number. But in Fig. 2(c) angular velocity 182 profiles show flactution for Schmidt number. Then at the upsurge of micro-rotational number the angle 183 velocity increase at the end of the list of figure, angular velocity decline due to soar of spin gradient 184 viscosity number and vortex viscosity.

185 Schmidt number decrease the molecular diffusivity. Fig. 3 describes the concentration profiles. As it is 186 noticed, concentration profiles curve with increase of suction parameter, Schmidt number, chemical 187 number and micro-rotational number.



(c) (f) Fig. 1. Velocity profiles for different values of (a) modified Grashof number (b) suction parameter (c) Schmidt number (d) magnetic force number (e) permeability of porous plate (f) spin gradient viscosity number.



(c) (f) Fig. 2. Angular velocity profiles for different values of (a) modified Grashof number (b) Schmidt number (c) suction parameter (d) micro-rotational number (e) spin gradient viscosity number (f) vortex viscosity.

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Fig. 3. Concentration profiles for different values of (a) suction parameter (d) Schmidt number (c) micro-rotational number (d) chemical reaction parameter.

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### 189 4. CONCLUTION

Some of the important findings of the present work obtained from the graphical representation of theresults are listed below:

- The fluid velocity and angular velocity profiles decreases with the increase of Modified
   grashof number.
- The velocity and angular velocity profiles decreases with the increase of Suction parameter
   and also the concentration profile decreases with the increase of Suction parameter.
- The velocity profile decreases and angular velocity profiles decreases with the increase of
   Schmidt number and also the concentration profile decreases with the increase of
   number.
- 199 4. The velocity profile go up with Magnetic force number.
- 5. The velocity profiles decreases with the increase of Permeability of porous plate.
- 201 6. The concentration profile decreases with the increase of Chemical reaction parameter.

202 7. The angular velocity profile increases with the increase of Micro-rotational number and the 203 concentration profile decreases with the increase of Micro-rotational number. 204 8. The angular velocity profile decreases with the increase of Spin gradient viscosity number. 205 The angular velocity profile decreases with the increase of Vortex viscosity. 206 REFERENCES 207 208 209 1. Ferraro VCA and Plumpton C. An Introduction to Magneto Fluid Mechanics. Oxford: 210 Clarendon Press, 1966. 211 2. Kucaba-Pietal A. Microchannels flow modelling with the micropolar fluid theory. Bulletin of the 212 Polish Academy of Science. 2004;52(3):209-214. 213 Khedr MEM, Chamkha AJ, Bayomi M. MHD Flow of a Micropolar Fluid past a Stretched 3. 214 Permeable Surface with Heat Generation or Absorption. Nonlinear Analysis: Modelling and 215 Control. 2009;14(1):27-40. 216 Muthu P, Rathish Kumar V and Peeyush Chandra. A study of micropolar fluid in an annular 4. 217 tube with application to blood flow. Journal of Mechanics in Medicine and Biology. 218 2008:8(4):561-576. 219 5. Lockwood F. Benchaitra M and Friberg S. ASLE Tribology Transactions. Astrophys Space 220 1987;30:539. 221 Cramer KP and Pai SI. Magneto Fluid Dynamics for Engineers and Applied Physics. New 6. 222 York: McGraw-HillBook Co., 1973. 223 Raptis A. Free convective oscillatory flow and mass transfer past a porous plate in the 7. presence of radiation for an optically thin fluid. Thermal Science. 2011;15:849-857. 224 225 8. Samiulhag I, Khan F, Ali and Shafie S. MHD free convection flow in a porous medium with 226 thermal diffusion and ramped wall temperature. Journal of the Physical Society of Japan. 227 2012;81:044401. 228 Seth GS. Sharma R and Sarkar S. Natural convection heat and mass transfer flow with Hall 9. 229 current, rotation, radiation and heat absorption past an accelerated moving vertical plate with 230 ramped temperature. Journal of Applied Fluid Mechanics. 2015;8:7-20. 231 10. Chamkha AJ and Khaled ARA. Hydromagnetic combined heat and mass transfer by natural 232 convection from a permeable surface embedded in a fluid saturated porous medium. 233 International Journal Numerical Methods Heat Fluid Flow. 2000;10:455-477. 234 Gebhart, B. Heat Transfer. New York: McGraw-Hill Book Co., 1971. 11. 235 12. Chaudhary RC and Jain A. Combined heat and mass transfer effects on MHD free convection 236 flow past an oscillating plate embedded in porous medium. Romanian Journal of Physics. 237 2007;52:505-524. 238 13. Haque MM, Alam MM. Transient heat and mass transfer by mixed convection flow from a 239 vertical porous plate with induced magnetic field, constant heat and mass fluxes. Mechanics 240 and Thermics - AMSE. 2009;78(4):54-74. 241 14. Samad MA, Mohebujiaman M. MHD heat and mass transfer free convection flow along a 242 vertical stretching sheet in presence of magnetic field with heat generation. Research Journal 243 of Applied Sciences, Engineering and Technology. 2009;1(3):98-106. Eldabe NTM, Elbashbeshy EMA, Hasanin WSA and Elsaid EA. Unsteady motion of MHD 244 15. 245 viscous incompressible fluid with heat and mass transfer through porous medium near a 246 moving vertical plate. International Journal of Energy Technology and policy. 2011;3:1–11. 247 Das UN, Dekha R and Soungalgekar VM. Effects on mass transfer on flow past an 16. 248 impulsively started infinite vertical plate with constant heat flux and chemical reaction. 249 Forschungim Ingenieurwesen. 1994:60:284-287. 250 Ibrahim FS, Elaiw AM and Bakr AA. Effect of the chemical reaction and radiation absorption 17. 251 on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving 252 plate with heat source and suction. Communication Nonlinear Science and Numerical 253 Simulation. 2008;13:1056-1066. 254 18. Annad Rao J, Sivaiah, S. and Srinivasa Raju, R. "Chemical Reaction Effects on an Unsteady 255 MHD Free Convection Fluid Flow past a Semi-infinite Vertical plate Embedded in a Porous 256 Medium with Heat Absorption." Journal of Applied Fluid Mechanics. 2012;3:63-70.

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- 274 **APPENDIX**

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275 
$$A_1 = \frac{1}{\Lambda}, \quad A_2 = \frac{1}{1+\Delta}, \quad A_3 = \frac{-A_2 G_M}{\left(-S_c^3 + A_2 S_c^2\right)}, \quad A_5 = \frac{S_c}{A_2 \varepsilon^2}, \quad A_6 = \frac{S_c}{\varepsilon^2}$$

276 
$$A_7 = \frac{S_c^2}{A_2 \varepsilon^2 (A_2^2 + S_c A_2)}, \quad A_{20} = \frac{A_3}{2}, \quad A_8 = \frac{-\frac{A_2 \lambda}{\varepsilon^2}}{(A_2^2 - A_1 A_2)}, \quad A_9 = \frac{A_3 \lambda S_c^2}{S_c^2 - A_1 S_c}, \quad A_{10} = \Delta A_2^2 A_8$$

277 
$$A_{11} = \Delta A_2 A_9 S_c, \quad A_{12} = G_m C_r A_2, \quad A_{13} = \frac{A_2 (K + M)}{\varepsilon^2}, \quad A_{14} = A_2 S_c A_3 (K + M), \quad A_{15} = \frac{A_{10}}{A_2^2}$$

278 
$$A_{16} = \frac{A_{11}}{\left(-S_c^3 + S_c^2 A_2\right)}, \quad A_{17} = \frac{A_{12}}{\left(-S_c^3 + S_c^2 A_2\right)}, \quad A_{18} = \frac{A_{13}}{A_2^2}, \quad A_{19} = \frac{A_{14}}{\left(-S_c^3 + S_c^2 A_2\right)}$$