Kaluza-Klein Bouncing Cosmological Model in General Relativity

ABSTRACT (ARIAL, BOLD, 11 FONT, LEFT ALIGNED, CAPS)

Kaluza-Klein cosmological model has been obtained in the general theory of relativity. The source for energy-momentum tensor is a perfect fluid. The field equations have been solved by using a special form of the average scale factor $R(t)=\left((t-t_0)^2+\frac{t_0}{1-\beta}\right)^{\frac{1}{1-\beta}}$ proposed by Scheerer R.J. [Phys. Rev. Letts. 93: 011301, 2004]. The physical properties and the bouncing behavior of the model are also discussed.

Keywords: Kaluza-Klein space time, Bouncing Universe.

1. INTRODUCTION

According to recent cosmological observations in terms of Supernovae Ia [1-2], large scale structure [3-4] with the baryon acoustic oscillations [5], cosmic microwave background radiations [6-11], and weak lensing [12], the current expansion of the universe is accelerating. From the observations, it is suggested that the universe is homogeneous. At the present time, the cosmic acceleration is explained by two ways: One is the introduction of the so called dark energy with negative pressure in general relativity [13-19]. The other is the modification of gravity on the large distances. As a simple way of modification of gravity, f(R) gravity, f(t) gravity, f(R,T) gravity etc. [20-28].

The solution of the singularity problem of the standard Big Bang cosmology is known as bouncing universe. A bouncing universe with an initial contraction to a non-vanishing minimal radius and then subsequent an expanding phase provides a possible solution to the singularity problem of the standard Big Bang cosmology. Moreover, for the universe entering into the hot Big Bang era after the bouncing, the EoS of the matter content ω in the universe must transit from $\omega<-1$ to $\omega>-1$. In the contracting phase, the scale factor $R(t)$ is decreasing, this means $\dot{R}(t)<0$, and in the expanding phase, scale factor $\dot{R}(t)>0$. Finally at the bouncing point, $\dot{R}(t)=0$ and near this point $\ddot{R}(t)>0$, for a period of time. In other view, in the bouncing cosmology, the Hubble parameter H passes across zero $(H=0)$ from $H<0$ to $H>0$. Cai *et al.* have investigated bouncing universe with quintom matter. He showed that a bouncing universe has an initial narrow state by a minimal radius and then develops to an expanding phase [29]. This means for the universe arriving to the Big-bang era after the bouncing, the equation of state parameter should crossing from $\omega<-1$ to $\omega>-1$.

Sadatian [30] have studied rip singularity scenario and bouncing universe in a Chaplygin gas dark energy model. Recently, Bamba *et al.* [31] have investigated bounce cosmology from $f(R)$ gravity and $f(R)$ bi-gravity. Astashenok [32] has studied effective energy models and dark energy models with bounce in frames of $f(T)$ gravity. Solomans *et al.* [33] have investigated bounce behavior in Kantowski-Sach and Bianchi cosmology. Silva *et al.* [34] have investigated bouncing solutions in Rastall's theory with a barotropic fluid. Brevik and Timoshkin [35] have obtained inhomogeneous dark fluid and dark matter leading to a bounce cosmology. Singh *et al.* [36] have studied k-essence cosmologies in Kantowski-Sachs and Bianchi space times.

The Kaluza-Klein theory [37-38] was introduced to unify Maxwell's theory of electromagnetism and Einstein's gravity theory by adding the fifth dimension. Due to its potential function to unify the fundamental interaction, Kaluza-Klein theory has been regarded as a candidate of fundamental theory.
Ponce [39], Chi [40], Fukui [41], Liu and Wesson [42], Coley [43] have studied Kaluza-Klein cosmological models with different contexts. Adhav et al. [44] have obtained Kaluza-Klein inflationary universe in general relativity. Reddy *et al.* [45] have discussed a five dimensional Kaluza- Klein cosmological model in the presence of perfect fluid in $f(R,T)$ gravity. Ranjeet *et al.* [46] have studied variable modified Chaplygin gas in anisotropic universe with Kaluza-Klein metric. Katore *et al.* [47] have obtained Kaluza-Klein cosmological model for perfect fluid and dark energy. Ram and Priyanka [48] have presented some Kaluza-Klein cosmological models in $f(R,T)$ gravity theory. Sahoo *et al.* [49] have investigated Kaluza-Klein cosmological model in $f(R,T)$ gravity with $\lambda(T)$. Recenty, Reddy *et al.* [50] have studied Kaluza-Klein minimally interacting holographic dark energy model in a scalar tensor theory of gravitation. Ghate and Mhaske [51] have investigated Kaluza-Klein barotropic cosmological model with varying gravitational constant G in creation field theory of gravitation.
In this paper, Kaluza-Klein bouncing cosmological model has been obtained in the general theory of relativity. This work is organized as follows: In section 2, the metric and field equations have been presented. The field equations have been solved in section 3 by using the physical condition that the expansion scalar θ is proportional to shear scalar σ and the special form of average scale factor $R(t)=\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^{\frac{1}{1-\beta}}$. The physical and geometrical behavior of the model have been discussed in section 4 . In the last section 5, concluding remarks have been expressed.

2. METRIC AND FIELD EQUATIONS

Five dimensional Kaluza-Klein metric is considered in the form

$$ds^2 = dt^2 - A^2(dx^2+dy^2+dz^2) - B^2 d\psi^2 \quad (1)$$

where A and B are functions of cosmic time t and the fifth coordinate ψ is taken to be space-like.
The energy-momentum tensor of the source is given by

$$T_i^j = (\rho+p)u_i u^j - p\delta_i^j, \quad (2)$$

where u^i is the flow vector satisfying $g_{ij}u^i u^j = 1$. Here ρ is the total energy density of perfect fluid and p is the corresponding pressure. p and ρ are related by and equation of state

$$p = \omega\rho, \quad 0 \le \omega \le 1. \quad (3)$$

In co-moving system of coordinates, from equation (2), one can find

$$T_0^0 = \rho \text{ and } T_1^1 = T_2^2 = T_3^3 = T_4^4 = -p. \quad (4)$$

The Einstein's field equations are given by

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j. \quad (5)$$

Using equation (2), for the metric (1), the field equations (3) are given by

$$3\frac{\dot{A}^2}{A^2}+3\frac{\dot{A}\dot{B}}{AB}=\rho\ , \quad (6)$$

$$2\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}^2}{A^2}+2\frac{\dot{A}\dot{B}}{AB}=-\omega\rho \quad (7)$$

$$3\frac{\ddot{A}}{A}+3\frac{\dot{A}^2}{A^2}=-\omega\rho\ , \quad (8)$$

where an overhead dot represents differentiation with respect to t.

The average scalar factor R and volume scalar V are given by

$$R^4=V=A^3B\ . \quad (9)$$

The generalized mean Hubble parameter H is defined by

$$H=\frac{\dot{R}}{R}=\frac{1}{4}\left(H_x+H_y+H_z+H_\phi\right), \quad (10)$$

where the directional Hubble parameters H_x, H_y, H_z and H_ϕ are given by

$$H_x=H_y=H_Z=\frac{\dot{A}}{A},H_\phi=\frac{\dot{B}}{B}\ . \quad (11)$$

The expansion scalar θ and shear scalar σ are given by

$$\theta=4H=\left(3\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right) \quad (12)$$

$$\sigma^2=\frac{1}{2}\left[\sum_{i=1}^{4}H_i^{\,2}-4H^2\right]. \quad (13)$$

The deceleration parameter q is defined by

$$q=-1+\frac{d}{dt}(H)\ . \quad (14)$$

The sign of q indicates whether the model inflates or not. A positive sign of q corresponds to the standard decelerating model whereas the negative sign of q indicates inflation. The recent observations of SN Ia (Reiss *et al.* [2]-[52], Perlmutter *et al.* ([1], [53-54]) reveal that the present universe is accelerating and the value of DP lies somewhere in the range $-1<q<0$.

3. SOLUTION OF FIELD EQUATIONS

The field equations (6) to (8) are a system of three highly non-linear differential equations in four unknowns A,B,ρ and ω. The system is thus initially undetermined. We need one extra conditions for solving the field equations completely.

The expansion scalar θ is proportional to the shear scalar σ which leads to

$$B=A^m\ , \quad (15)$$

where m is a proportionally constant.

The motive behind assuming condition is explained with reference to Thorne [55] the observations of the velocity red-shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropy today within $\approx$ 30 percent (Kantowski and Sachs [56]; Kristian and Sachs [57]).To put more precisely, red-shift studies place the limit $\frac{\sigma}{H}\leq 0.3$ on the ratio of shear σ to Hubble constant H in the neighborhood of our galaxy today Collins *et al.* [58] have pointed out that congruence to the homogenous expansion satisfies that the condition $\frac{\sigma}{H} \le 0.3$ is constant.

Now we take following ansatz for the scale factor, where increase in terms of time evolution

$$R(t) = \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{1}{1-\beta}} \tag{16}$$

where t_0 is initial time and $\beta < 1$ is constant.

The motivation to choose such scale factor is behind the fact that the universe is accelerated expansion at present and decelerated expansion in the past. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. Thus, in general, the DP is not a constant but time variable. By the above choice of scale factor yields a time dependent DP,

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -1 + \frac{d}{dt}(H) . \tag{17}$$

Solving equations $A = B^m$ and $R(t) = (A^3 B)^{\frac{1}{4}}$, we get

$$B = \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{4m}{(1-\beta)(m+3)}} . \tag{18}$$

With the help of equation (18), equation (15) takes the form

$$A = \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{4}{(1-\beta)(m+3)}} . \tag{19}$$

Using above two equations (18) and (19), the metric (1) takes the form

$$ds^2 = dt^2 - \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{8}{(1-\beta)(m+3)}} \left(dx^2 + dy^2 + dz^2\right) - \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{8m}{(1-\beta)(m+3)}} d\psi^2 . \tag{20}$$

Equation (20) represents Kaluza-Klein cosmological model with time dependent scale factor.

4. PHYSICAL PROPERTIES OF THE MODEL

The physical quantities such as spatial volume V, Hubble parameter H, expansion scalar θ, mean anisotropy A_m, shear scalar σ^2, energy density ρ, equation of state parameter ω are obtained as follows:

The average scale factor is

$$R(t) = \left[(t - t_0)^2 + \frac{t_0}{1-\beta} \right]^{\frac{1}{1-\beta}} .$$

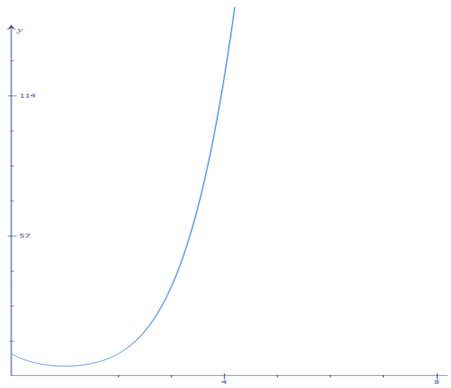

Fig. 1 Plot of Average scale factor versus time

From fig. 1, in the earlier stage, the scale factor is slightly decreasing ($\dot{R}(t) < 0$) and in the expanding phase the scale factor increases rapidly ($\dot{R}(t) > 0$). Hence our model is bouncing at $t = t_0$ ($\dot{R}(t) = 0$).

The spatial volume is given by

$$V = R^4 = \left[(t - t_0)^2 + \frac{t_0}{1 - \beta} \right]^{\frac{4}{1-\beta}} . \qquad (21)$$

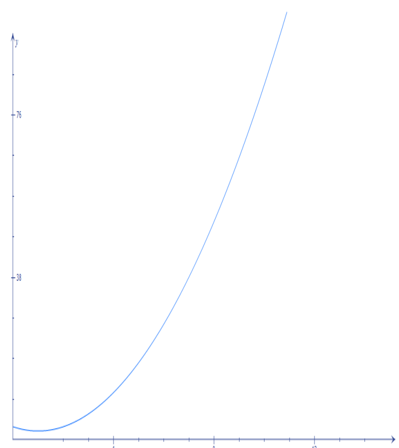

Fig. 2 Plot of Volume versus Time

The spatial volume is finite at time $t = 0$ and increases with increasing value of time hence the model starts expanding with finite volume.

The Hubble parameter is given by

$$H = \frac{2(t - t_0)}{(1 - \beta)(t - t_0)^2 + t_0} . \qquad (22)$$

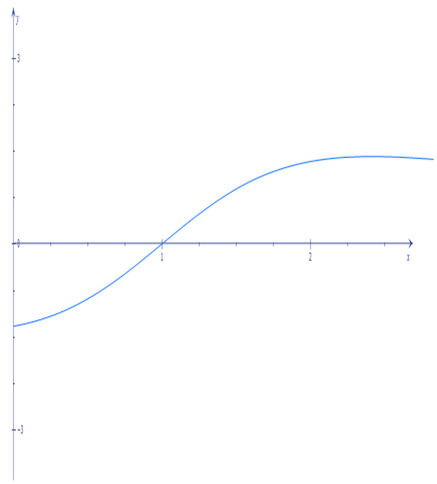

Fig. 3 Plot of Hubble Parameter versus Time

From fig. 3, the Hubble parameter $H<0$, for $t<t_0$ and $H>0$, for $t>t_0$ indicating that H passes across zero $(H=0)$ at $t=t_0$ showing the bouncing cosmology.

The expansion scalar is

$$\theta=\frac{32(t-t_0)}{(1-\beta)(t-t_0)^2+\frac{t_0}{1-\beta}}. \qquad (23)$$

The mean anisotropy parameter A_m is

$$Am=3\frac{(m-1)^2}{(m+3)^2}=cons\tan t\neq 0, \quad \text{for } m\neq 1 \qquad (24)$$

The shear scalar is

$$\sigma^2=24\frac{(m-1)^2}{(m+3)^2(1-\beta)^2}\frac{(t-t_0)^2}{\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^2}. \qquad (25)$$

We observe that

$$\lim_{t\to\infty}\frac{\sigma^2}{\theta^2}=\frac{3}{128}\frac{(m-1^2)}{(m+3^2)}\neq 0, \text{ for } (m\neq 1) \quad . \qquad (26)$$

The matter energy density is given by

$$\rho=\frac{192m(m+1)(t-t_0)^2}{(1-\beta)^2(m+3)^2\left[(t-t_0)^2+\frac{t_0}{1-\beta}\right]^2}. \qquad (27)$$

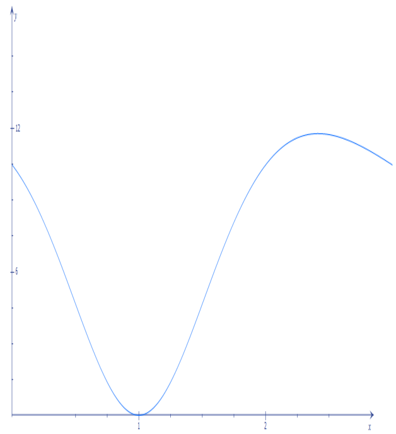

Fig. 4 Plot of Energy Density versus Time

From fig. 4, the energy density decreases at the early stage of evolution when $t < t_0$ and goes into the hot Big-bang era. The model bounces at $t = t_0$ and after bouncing the energy density rapidly increases for $t > t_0$.

The Eos parameter ω is given by

$$\omega = \frac{-2}{m+1} + \frac{(1-\beta)(m+3)}{4(m+1)} - \frac{(1-\beta)(m+3)}{24(m+1)(t-t_0)^2}\left[(t-t_0)^2 + \frac{t_0}{1-\beta}\right]. \qquad (28)$$

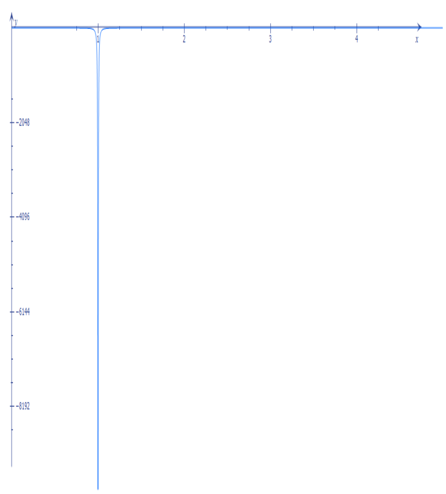

Fig. 5 Plot of EoS parameter versus Time

A bouncing universe model has an initial narrow state by a non-zero minimal radius and then develops to an expanding phase. For the universe going into the hot Big Bang era after the bouncing, the equation of state parameter of the universe should crossing from $\omega < -1$ to $\omega > -1$. From fig. 5, before bouncing point at $t = t_0$, we see that the skewness parameter $\omega < -1$ and after the bounce, the universe requires to enter into the hot Big Bang era and occurs the big rip singularity. Further the Eos parameter $\omega > -1$ for $t > t_0$. It is observed that the skewness parameter $\omega < -1$ for $t < t_0$ and $\omega > -1$ for $t > t_0$. Hence our model is bouncing at $t = t_0$.

CONCLUSION

Kaluza-Klein cosmological model has been investigated in the general theory of relativity. The source for energy momentum tensor is a perfect fluid. The field equations have been solved by using time dependent deceleration parameter. It is observed that the model starts expanding with finite volume and there is no Big-bang singularity. The mean anisotropy parameter A_m is constant and $\lim_{t \to \infty} \frac{\sigma^2}{\theta^2} (\neq 0)$ is also constant, hence the model is anisotropic throughout the evolution of the universe except at $m = 1$ *i.e.* the model does not approach isotropy. In early phase of universe, the value of deceleration parameter is positive while as $t \to \infty$, the value of $q = -1$. Hence the universe had a decelerated expansion in the past and has accelerated expansion at present which is in good agreement with the recent observations of SN Ia (Reiss *et al.* [2-52], Perlmutter *et al.* [53-54]). It is interesting to note that the behavior of the model is bouncing as the Hubble parameter H passes across zero $(H = 0)$ from $H < 0$ to $H > 0$ for some finite time $t = t_0$. Also the energy density decreases at the early stage of evolution and rapidly increases showing big bounce $t = t_0$.

REFERENCES

[1] Perlmutter S., Aldering G., Goldhaber G., *et al.*, "Measurement of Ω and Λ from 42 high-redshift supernovae", *The Astrophysical Journal*, Vol. 517, No. 2, 1999, pp. 565-586. doi:10.1086/307221.

[2] Riess A. G., Filippenko Al. V., Challis P., *et al.*, "Observational evidence from supernovae for an accelerating universe and a cosmological constant", Astron. J., 116: 1009-1038, 1998.

[3] Tegmark M., Strauss M., Blanton M., *et al.* , "Cosmological parameters from SDSS and WMAP", Phys. Rev. D, 69: 103501 (2004).

[4] Seljak U., Makarov A., McDonald P. *et al., "Cosmological parameter analysis including SDSS Lyalpha forest and galaxy bias: constraints on the primordial spectrum of fluctuations, neutrino mass and dark energy", ibid*. 71, 103515, 2005; [arXiv:astro-ph/0407372].

[5] Eisenstein D. J., Zehavi I., Hogg D. W. *et al., "Detection of the Baryon Acoustic peak in the large scale correlation function of SDSS luminous red galaxies",* Astrophys. J. 633, 560, 2005; [arXiv:astro-ph/0501171].

[6] Spergel D. N., Verde L., Peiris H. V. *et al.* "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters", Astrophys. J. Suppl., 148: 175–194 (2003).

[7] Komatsu E., Dunkley, J., Nolta, M. R., *et al.*: **"**Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Interpretation", Astrophys. J. Suppl. 180, 330-376, 2009.

[8] Spergel D. N., Bean R., Dore O. *et al.*, "Wilkinson microwave anisotropy probe (WMAP) three year results: implications for cosmology" *ibid*. 170, 377, 2007; [arXiv:astro-ph/0603449].

[9] Komatsu E., Smith K. M., Dunkley J. *et al.*, "Seven-year Wilkinson microwave anisotropy probe (WMAP) observations: cosmological interpretation", Astrophys. J. Suppl. 192, 18, 2011; DOI: 10.1088/0067-0049/192/2/18, [arXiv:1001.4538v3 [astro-ph.CO]].

[10] Hinshaw G., Larson D., Komatsu E. *et al.*, “ Nine-Year Wilkinson microwave anisotropy probe (WMAP) observations: cosmological parameter results”, arXiv:1212.5226v3 [astro-ph.CO]. DOI:10.1088/0067-0049/208/2/19.

[11] Ade P. A. R., Aghanim N., Armitage-Caplan C. *et al.*, “Plank 2013 results. XVI. Cosmological parameters”, [Planck Collaboration], arXiv:1303.5076v3 [astro-ph.CO], DOI: 10.1051/0004-6361/201321591.

[12] Jain B., Taylor A., “Cross-correlation tomography: Mesuring dark energy evolution with weak lensing”, Phys. Rev. Lett. 91, 141302, 2003; [arXiv:astro-ph/0306046], DOI:10.1103/PhysRevLett.91.141302.

[13] Sahni V., Starobinsky A. A., “The case for a positive cosmological Lambda-term”, Int. J. Mod.Phys. D 9, 373-444, 2000; [astro-ph/9904398]. DOI:10.1142/S0218271800000542.

[14] Peebles P. J. E., Ratra B., “The cosmological constant and dark energy”, Rev. Mod.Phys. 75, 559-606, 2003; [astro-ph/0207347]. DOI: 10.1103/RevModPhys.75.559.

[15] Sahni V., “Reconstructing the properties of dark energy”, Prog. Theor. Phys. Suppl. 172, 110-120, 2008.

[16] Caldwell R. R., Kamionkowski M., “The Physics of cosmic acceleration”, Ann. Rev. Nucl. Part. Sci. 59, 397-429, 2009; [arXiv:0903.0866 [astro-ph.CO]].

[17] Cai Y. F., Saridakis E. N., Setare M. R., Xia J. Q., “Quintom cosmology: Theoretical implications and observations”, Phys. Rept. 493, 1-60, 2010; [arXiv:0909.2776 [hep-th]].

[18] Li, M., Li X. D., Wang S., Wang Y., “Dark Energy”, Commun. Theor. Phys. 56, 525-604, 2011; [arXiv:1103.5870 [astro-ph.CO]].

[19] Bamba K., Capozziello S., Nojiri S., Odintsov S. D., “Dark energy cosmology: the equivalent description via different theoretical models and cosmography tests”, Astrophys. Space Sci. 342, 155-28, 2012; [arXiv:1205.3421 [gr-qc]].

[20] Capozziello S., “Curvature Quintessence”, Int. J. Mod. Phys. D 11, 483-492, 2002; [gr-qc/0201033].

[21] Carroll S. M., Duvvuri V., Trodden M., Turner M. S. “Is cosmic speed-up due to new gravitational Physics?”, Phys. Rev. D 70, 043528, 2004; [astro-ph/0306438].

[22] Nojiri S., Odintsov S. D., “Modified gravity with negative and positive powers of the curvature: unification of the inflation and of the cosmic acceleration”, Phys. Rev. D 68, 123512, 2003; [arxiv:hep-th/0307288].

[23] Nojiri S., Odintsov S. D., “Unified cosmic history in modified gravity:from F® theory to Lorentz non-invariant models”, Phys. Rept. 505, 59144, 2011; [arXiv:1011.0544 [gr-qc]].

[24] Capozziello S., Faraoni V., *Beyond Einstein Gravity* (Springer, 2010).

[25] Capozziello S., De Laurentis M., “Extended theories of gravity”, Phys. Rept. 509, 167-321, 2011; [arXiv:1108.6266 [gr-qc]].

[26] Cruz-Dombriz A. de la, Saez-Gomez D., “Black holes, cosmological solutions, future singularities and their thermodynamical properties in modified gravity theories”, Entropy 14, 1717-1770, 2012; [arXiv:1207.2663 [gr-qc]]. DOI: 10.3390/e14091717.

[27] Capozziello S., De Laurentis M., Faraoni V., “A bird’s eye view of f(R) gravity”, arXiv:0909.4672 [gr-qc], DOI:10.2174/1874381101003010049.

[28] Francisco S. N. Lobo, “The dark side of gravity: Modified theories of gravity”, Dark Energy-Current Advances and Ideas, 173-204, Research Signpost, ISBN 978-81-308-0341-8, 2009; [arXiv:0807.1640 [gr-qc]].

[29] Cai. Y. F., Qiu. T., Piao. Y. S., Li. M., Zhang X., “Bouncing universe with quintom matter”, J. of High Energy Phy., 0710, 071, 2007. [arxiv. 0704.1090v1 [gr-qc]]. DOI: 10.1088/1126-6708/2007/10/071.

[30] Sadatian S. D., , Rip singularity scenario and bouncing universe in a Chaplygin gas dark energy model”, Int. J. Theo. Phys., 53, 675-684, 2014; DOI 10.1007/s10773-013-1855-1.

[31] Bamba K., Makarenko A. N., “Bounce cosmology from F(R) gravity and F(R) bigravity”, arXiv. 1309.3748v2 [hep-th] 2013.

[32] Astashenok A. V., “Effective dark energy models and dark energy models with bounce in frames of F(T) gravity”, Astrophys. Spa. Sci., 351, 377-383, 2014. DOI: 10.1007/s10509-014-1846-6.

[33] Solomans D., Dunsby P. K. S., Ellis G. F. R., “Bounce beheviour in Kantowski-Sachs and Bianchi Cosmologies” Classical and Quan. Grav., 23 (23), :arXiv : [gr-qc] 0103087v2, 2006], DOI: 10.1088/0264-9381/23/23/001.

[34] Silva G. F., Piattella O. F., Fabris J. C., Casarini L., Barbosa T. O., “Bouncing solutions in Rastall's theory with a barotropic fluid”, Grav. Cosm., 19, 156-162, 2013. arXiv: 1212.6954v3 [gr-qc] 2014. 10.1142/S0217732314500783.

[35] Brevik I., Obukhov V. V., Timoshkin A. V., “Bounce universe induced by an inhomogeneous dark fluid coupled with dark matter”, Mod. Phys. Lett. A 29, Issue 15, 1450078, 2014. DOI: (2015) Universe, 1, 24-37, 2014; arxiv 1404.11887v1 [gr-qc].

[36] Singh T., Chaubuy R., Singh A., “k-essence cosmologies in Kantowski-Sachs and Bianchi space-times”, Canadian J. of Physics, 93 (11) 1319-1323, 2015. DOI: 10.1139/cjp-2015-0001.

[37] Kaluza T.: “Zum Unit at sproblem der Physik”, Sitz . ber. Preuss. Akad Wiss Berlin (Phys. Math) K 1, 966, 1921.

[38] Klein O., “Quantentheorie und fün f dimensionale Relativitäts theorie”, *Zeits. Phys.* 37, 895-906, 1926.

[39] Ponce de Leon J., “Cosmological models in a Kaluza-Klein theory with variable rest mass”, *Gen. Rel. Grav.* 20, 539-550, 1988.

[40] Chi L. K., “New cosmological models in the five dimensional space time mass gravity theory”, *Gen. Rel. Grav.* 22, 1347-1350, 1990.

[41] Fukui T., “5D geometrical property and 4D property of matter”, *Gen. Rel. Grav.* 25, 931-938, 1993.

[42] Liu H., Wesson P. S., “Cosmological solutions and their effective properties of matter in Kaluza-Klein theory”, *Int. J. Mod. Phys.* D 3, 627-637, 1994.

[43] Coley A. A., “Higher dimensional vacuum cosmologies”, *Astrophys. J.*, 427, 585, 1994.

[44] Adhav K. S., “Kaluza-Klein inflationary universe in general relativity”, *Prespacetime J.*, Vol. 2, Issue 11, pp. 1828-1834, 2011.

[45] Reddy D. R. K., Satyanarayana B., Naidu R. L., “Five dimensional dark energy model in a sclar tensor theory of gravitation”, *Astrophys. Spa. Sci.*, 339, 401-404, 2012.

[46] Ranjit C., Chakraborty S., Debnath U., “Observational constraints of homogeneous higher dimensional cosmology with modified Chaplygin gas,” The European Physical Journal Plus, vol. 128, article 53, 2013.

[47] Katore S. D., Sancheti M. M., Bhaskar S. A., “ Kaluza-Klein cosmological models for perfect fluid and dark energy”, *Bulg. J. Phys.*, 40, No. 1, pp. 17-32, 2013.

[48] Ram S., Priyanka, “Some Kaluza-Klein cosmological models in f(R,T) gravity theory”, Astrophys. Space Sci., 347, 389-397 (2013)

[49] Sahoo P. K., Mishra B., Tripathy S. K., “Kaluza-Klein cosmological model in f(R, T) gravity with l(T)”, *Indian Journal of Physics*, DOI 10.1007/s12648-015-0759-8, arXiv: 1411.4735v2 [gr-qc], 2014. Impact Factor: 1.337.

[50] Reddy D. R. K., Vijaya Lakshmi G. V., “Kaluza-Klein minimally interacting holographic dark energy model in a scalar tensor theory of gravitation”, *Prespacetime J.* , Vol. 06, Issue 04 pp. 295-304, 2015.

[51] Ghate H. R., Mhaske S. S., “ Kaluza-Klein barotropic cosmological model with varying gravitational constant in creation field theory of gravitation”, Global Journal of Science Frontier Research: F Mathematics & Decision Sciences, Vol. 15, Issue 3, 2015.

[52] Riess A. G., Sirolger L. G., Tonry J., *et al.*, “Type Ia Supernova discoveries at z >1 from the *Hubble Space Telescope:* Evidence for the past deceleration and constraints on dark energy evolution,” *The Astrophysical Journal*, Vol. 607, No. 2, 2004, pp. 665-678. doi:10.1086/383612.

[53] Perlmutter S., *et al.*, “Measurement of the cosmological parameters Ω and Λ from the first seven supernovae at $z \geq 0.35$ ”,*The Astrophysical Journal*, Vol. 483, No. 2, pp. 565, 1997. doi:10.1086/304265.

[54] Perlmutter S., *et al.*, “Discovery of Supernovae explosion at half the age of the Universe”, *Nature*, Vol. 391, No. 2, pp. 51-54, 1998. doi:10.1038/34124.

[55] Thorne K. S., “Primordial Element Formation, Primordial Magnetic Fields, and the Isotropy of the Universe”, *Astrophys. J.*, Vol. 148, pp. 51, 1967.

[56] Kantowski R., Sachs R. K., “Some spatially homogeneous anisotropic relativistic cosmological models”, *J. Math. Phys.* Vol. 7, No. 3, pp. 443, 1966.

[57] Kristian J., Sachs R. K., “Observations in Cosmology”, *Astrophysical Journal*, Vol. 143, pp. 379, 1966

[58] Collins C. B., Glass E. N., Wilkinson D. A., “Exact spatially homogeneous cosmologies”, *Gen. Relat. Grav.*, Vol. 12, No. 10, pp. 805-823, 1980.

[59] Scheerer R. J., “Purely kinetic k essence as unified dark matter”, Phys. Rev. Lett., 93, 011301, 2004.