Original Research Article

Inverse fourth-power gravity acting between not closed inertial masses

Shinsuke Hamaji[1]*

[1]*Hyama Natural Science Research Institute, Tokyo, Japan*

ABSTRACT

The mass in Einstein's energy-mass equivalence equation has two possible interpretations, whether it is limited to the invariant mass, or it applies to all energy. This paper argues that all of the energy (kg m^2 s^{-2}) has a mass (kg: a degree of weight and inertial resistance). The inertial mass is a mass that was further scaled the gravitational mass to be increased with kinetic energy. The inertial mass of elementary particle in an atomic system also varies similarly by scaling. Thereby the scalable inertial masses of elementary particles constituting the atomic add the gravitation that cannot be ignored as compared with the Coulomb force. We call this effect "Inverse fourth power ($1/r^4$) gravity" to distinguish it from universal gravitation of the universal gravitational constant. Using these mechanisms, we explain the proton radius puzzle and the statistical error found with the muon anomalous magnetic moment. This paper demonstrates a new way of integrating general relativity and quantum theory by separating the scalable inertial mass and the gravitational mass.

Keywords: invariant, mass, weight, resistance, inertial, gravitational, elementary, particle, atomic, scalable, coulomb, force, universal, proton, radius, puzzle, magnetic, moment, general, relativity, quantum, theory.

1. INTRODUCTION

1.1. Muon anomaly and predictions of new physics:

The proton radius (0.84185 fm) obtained from the recent Lamb shift experiment of muonic hydrogen [1] is 4% smaller than the accepted value (0.8751 fm) [2]. This discrepancy is referred to as the proton radius puzzle. Similar measurements (0.84087 fm) have been reported [3] but to date this puzzle has as yet not been resolved. The muon is much heavier having a mass 207 times that of an electron. Therefore, scattering experiments that are currently underway using muonic hydrogen is expected to provide a precise measurement of the proton radius [4]. Also, in the E821 experiment of US Brookhaven National Laboratory in 2001, the anomalous magnetic moment of the muon was measured with accuracy in (a_μ = (g − 2) /2) 0.54 ppm [5]. The experimental values have attracted attention because the predicted value is shifted approximately 3.2σ from the Standard Model, hence offering new physical clues. In the background of these discrepancies and with the improvements in accuracy involving experiments with muons, there is a possibility that hints lurk within the physical phenomena unexplained by the Standard Model, which is the current accepted framework of particle physics [6].

1.2. Unknown relationship between gravity and mass:

* E-mail address: s_hyama@yahoo.co.jp

Indeed, no one has succeeded in observing gravitational phenomenon below 10 μm. Therefore, they say that even the existence of gravity has not yet been confirmed at microscopic scales [7]. The Arkani-Hamed--Dimopoulos--Dvali model [8] argues that extra dimensions might have spread to near 1 mm. Gravity follows the inverse-square law down to distance 1 mm, but a two-dimensional effect from the extra dimensions contributes to the force at distances below 1 mm. Below these distances, the force follows an inverse-fourth law. In contrast, the mass in the energy-mass equivalence equation [9] has two interpretations [10]. Interpretation 1 is that energy and mass are not exactly the same; the energy of an object changes depending on velocity whereas the invariant mass does not change in any way. Interpretation 2 is that, apart from the constant c^2, energy and mass are exactly the same; the energy of a moving object is larger than that at rest. That is, when in motion, an object's relativistic mass is greater than when stationary. The majority of particle physicists have adopted Interpretation 1 [11]. We have here adopted Interpretation 2 with the equivalence principle of the momentum of light. Furthermore, we assume that short-range gravity, which cannot be ignored between interacting elementary particles, acts on the inertial mass and increases on the micro-scale In regard to the muon, quantum gravity is part of a hierarchical structure in which it approaches classical gravity in its dependence on mass.

2. MATERIAL AND METHODS / EXPERIMENTAL DETAILS / METHODOLOGY

2.1. Relationships among invariant, (relativistic,) gravitational, and inertial mass:

The relativistic mass (m_r) of a moving object [12] can be calculated using the Lorentz factor from the invariant mass (m_0). However, Einstein has stated that, apart from its Lorentz factor connection, the "physical meaning of this mass is not known; it therefore is better not to use anything other than invariant mass" [13],

$$m_r = \gamma m_0 = m_0/(1 - v^2/c^2)^{1/2}. \quad (1)$$

The total energy of the object increases with the addition of kinetic energy [12],

$$E = [(m_0c^2)^2 + (pc)^2]^{1/2}. \quad (2)$$

Using a gravitational mass $M = E/c^2$ [9], Eq. (2) is

$$E = Mc^2 = ([m_0c^2]^2 + [pc]^2)^{1/2}. \quad (3)$$

Compared with the speed of light in the invariant system, the wave speed (Hamaji's light equivalence principle [14]) in another inertial system is

$$w = f\lambda = c/\gamma = (c^2 - v^2)^{1/2}. \quad (4)$$

If this increased kinetic energy (pc) is by interpretation 2 [11] a gravitational mass then the mass associated with the total energy is also a gravitational mass M. The inertial mass m is the mass of the combined action of the change in gravitational mass with the energy increase (physical action), and the change in relativistic mass by scale conversion (mathematical action),

$$m = M(c/w) = \gamma M. \quad (5)$$

Hence, the inertial mass reverts to Eq. (1) if $M = m_0$ (see Appendix for details). This frees up the limitation that inertial mass = gravitational mass, but retains the essential equivalence principle of energy and momentum. This enables the mass and speed of another inertial system to be given without the need to perform a coordinate transformation,

$$E = ([m_0c^2]^2 + [pc]^2)^{1/2} = Mc^2 = \gamma m w^2. \quad (6)$$

This preserves the relationship that energy is mass times the square of the speed.

2.2. The ratio of the nuclear force and gravity, the relationship to the universal gravitational constant:

As indicated in Fig. 1, the squared ratio of the average covalent radii (1 Å = 100 pm) or the Bohr diameter of the electronic hydrogen to the charge radius (0.8751 fm) of the proton almost equals the universal gravitational constant.

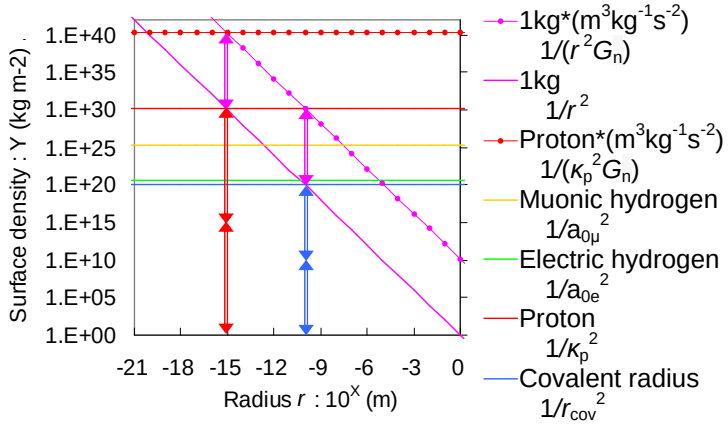

FIG. 1. Ratios among quantum size, nuclear force, and gravity. The pink solid line is the surface density line of 1 kg of substance. The pink dotted line is the line, obtained by dividing the pink solid line with the universal gravitational constant. The red horizontal dotted line is the proton line, scaled to a proton mass of 1 kg at one meter. This line represents the strength of the proton force relative to gravity. The proton force is 1×10^{40} times higher than gravity, of which 1×10^{10} is contributed by the universal gravitational constant (G_n), and $1 \times 10^{(15 \times 2)}$ is contributed by the spin radius. The solid red line is the surface density line of proton, of which 1×10^{10} is contributed by G_n, and $1 \times 10^{(10 \times 2)}$ is contributed by the covalent radius (blue solid line).

The material density of the average covalent radii or the Bohr diameter of the electronic hydrogen may have been scaled by the universal gravitational constant because the mass of the nucleons approximates the mass at the average covalent radius or the Bohr diameter of the electronic hydrogen. That is,

$$G_n/(1\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}) \approx (r_p/r_{cov})^2 \approx (r_p/2a_0)^2. \quad (7)$$

The ratio of the gravitational and nuclear forces, which relates the proton radius to the average covalent radius or the Bohr diameter of electronic hydrogen, is given by

$$r_p^2 G_n/(1\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}) \approx r_p^4/r_{cov}^2 \approx r_p^4/(2a_0)^2. \quad (8)$$

2.3. *1/r*^{4} gravity acting between fermions instead universal gravitation:

The universal gravitation acting between proton and electron (or muon) constituting the electronic (or muonic) hydrogen atom is negligibly small when compared with the Coulomb force,

$$(e^2/4\pi\varepsilon_0 + G_n m_{0p} m_{0x})/a_0^2 \cong m_{0x} v_x^2/a_0. \quad (9)$$

Using Eq. (4) and the coordinate system at rest for the proton ($m_p = m_{0p}$), the inertial mass of the electron (muon) orbiting is

$$m_x = \gamma_x m_{0x} = m_{0x} c/(c^2 - v_x^2)^{1/2}. \quad (10)$$

For the Yukawa meson theory with scalar potential $\alpha e[-r/\kappa]/r$, massive particles mediate the force acting between particles. This force, which falls off with distance and also inversely proportional to mass follows from a scalar interaction field of relativistic quantum theory. We learn that the chiral condensate occupies about 2/3 of the vacuum centered on the nucleus [15]. The confinement radius for the inertial mass is given by the condition

$$\kappa_x = (2/3)\lambda_x = (2/3)h/(m_x c) = \hbar/([1\mathrm{m}]^3 c\rho_x). \quad (11)$$

Substituting the inertial mass for the α coefficient of the Yukawa-type potential, the scalable inertial mass to be differentially coupled to the scalar 1(m)/r(m) is

$$m_{xx} = m_x(1 - e[-r/\kappa_x])/r. \quad (12)$$

This is a function of wavelength λ and radial coordinate r.

$$m(r, \lambda) = h(1 - e[-3r/2\lambda])/(rc\lambda). \quad (13)$$

Replacing the universal gravitational constant of Eq. (7), *1/r*^{4} gravitational constant is

$$G_4 = 2 \approx 8G_n a_0^2/r_p^2 \qquad (\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}). \quad (14)$$

Introducing *1/r*^{4} gravity instead of the universal theory of gravitation, Eq. (9) becomes

$$[e^2/(4\pi\varepsilon_0) + G_4 m_{px} m_{xp}]/a_{0x}^2 = m_x v_x^2/a_{0x}. \quad (15)$$

From Eqs. (10)–(16), we can derive the inertial mass m_x, the orbital velocity v_x, and the orbital radius a_{0x}

$$\hbar = m_x v_x a_{0x}. \quad (16)$$

The effective fine-structure constant is

$$\alpha_x = v_x/c. \quad (17)$$

3. RESULTS AND DISCUSSION

Fig. 2 depicts the model of the ground state for the hydrogen atom and the relational expressions of each physical quantity when considering $1/r^4$ gravity.

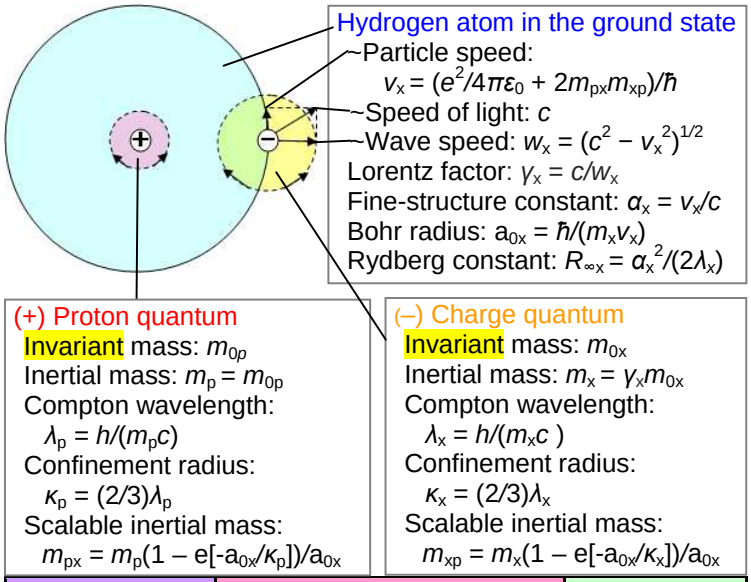

Fig. 2. Model of the internal relationships among the structures of a ground-state hydrogen atom. The blue area denotes the region occupied by the hydrogen atom model in its ground state. The red area indicates the protons residing at the center, where they spin at the speed of light. The yellow area shows the (−) charge quantum (electron or muon) moving around the proton at the Bohr radius. The orbital angular velocity (particle speed) and spin angular velocity (wave speed) move at light speed at right angles to each other (so a complex representation is appropriate). The real and imaginary parts of the complex wave deflection represent the bosons and fermions, respectively.

Table 1 provides values of the specific calculation results.

Table 1. Physical parameters of a ground-state hydrogen atom.

Mag. constant / 4 pi: $\mu_0/4\pi$ (N A^{-2})	1.000000000E–07	Constant CODATA-2014 [2]	
Elementary charge: e (C)	1.602176621E–19		
Planck constant / 2 pi: $\hbar$ (m^2 kg s^{-1})	1.054571800E–34		
Speed of light in vacuum: c (m s^{-1})	2.997924580E+08		
Rest mass: m_0 (kg)	1.672621898E–27	9.109383560E–31	1.883531594E–28
Quantum	**Proton**	**Electron-Proton**	**Muon-Proton**
Particle speed: v_x (m s^{-1})		2.187691273E+06	2.287426913E+06
Inertial mass: m_x (kg)	1.672621898E–27	9.109626113E–31	1.883586424E–28
Bohr radius: a_{0x} (m)	5.291772107E–11	5.291631180E–11	2.447616520E–13

Compton wavelength: λ_x (m)	1.321409854E–15	2.426245633E–12	1.173409953E–14
Confinement radius: κ_x (m)	8.809399024E–16	1.617497089E–12	7.822733022E–15
(+)Proton inertial mass: m_{px} (kg)		3.160881477E–17	6.833676289E–15
(–)Charge inertial mass: m_{xp} (kg)		1.721515692E–20	7.695594501E–16
$1/r^4$ gravity: $2m_{px}m_{xp}/a_{0x}^2$ (N)		3.886601905E–16	1.755657605E–04
Coulomb force: $(e^2/4\pi\varepsilon_0)/a_{0x}^2$ (N)		8.239162197E–08	3.851016988E–03
Inverse fine-structure constant: $1/\alpha_x$	1.370359991E+02	1.370359985E+02	1.310609997E+02
Rydberg constant: $R_{\infty x}$ (m^{-1})	1.097373157E+07	1.097402387E+07	2.480694499E+09

1. Blue quantities, such as fermion masses and physical constants appearing in Fig. 2, are documented. 2. Red quantities are newly derived in this paper.

3.1. The proton radius puzzle that appears for the muonic hydrogen atom:

Fig. 3 presents a graph of the change in the radial coordinate from the center of the potential that appears in the model (i.e., inertial mass of each particle, Coulomb force, and $1/r^4$ gravity acting between elementary particles). At Point_A of Fig. 3, the ratio of the Coulomb force and the $1/r^4$ gravity force acting between proton and muon for the μp atom is

$$(e^2/4\pi\varepsilon_0 + 2m_{p\mu}m_{\mu p})/(e^2/4\pi\varepsilon_0) \approx 1.046. \quad (18)$$

The value obtained by dividing the confinement radius of two-thirds of the Compton wavelength of proton by this ratio is

$$0.881 / 1.046 \approx 0.842 \text{ (fm)}. \quad (19)$$

This produces a proton radius for μp of roughly 4% smaller. However, the reason why the proton radius is reduced is not known, nor why μp is smaller by this mechanism. Nevertheless, the effective fine structure constant is changed by $1/r^4$ gravity, which changes depending on the combination of the charge quantum. It increases the orbital energy of the muon, and leads to an increase in the Lamb-shift in energy.

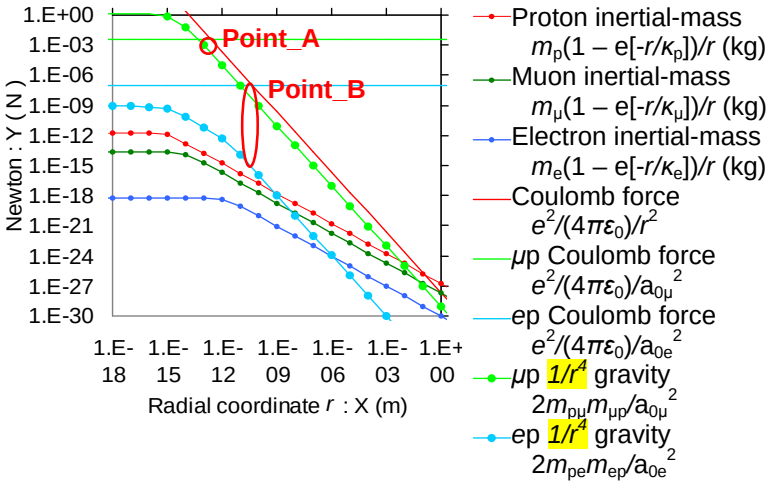

FIG. 3. The scalable inertial mass of fermions, and the Coulomb force and the $1/r^4$ gravity of a combination thereof. The red / green / blue dotted lines are the scalable

inertial mass curves of each fermion. The solid lines are the line that Coulomb force. The Yellow-green / light-blue dotted lines (proton-muon, proton-electron) are the $1/r^4$ gravity line.

3.2. Deviation of the anomalous magnetic moment of the muon from the standard theory:

From Point_B of Fig. 3, the ratio of $1/r^4$ gravity and Coulomb force acting between the proton and electron of an electronic hydrogen atom *e*p is

$$(2m_{pe}m_{ep})/(e^2/4\pi\varepsilon_0) \approx 4.718 \times 10^{-9}. \quad (20)$$

The error associated with the bare fine-structure constant of only the Coulomb force and the effective fine structure constant of the *e*p with added $1/r^4$ gravity effects in Table 1 is

$$1/\alpha - 1/\alpha_e \approx 0.6 \text{ (ppm)}. \quad (21)$$

Given this error in the fine-structure constant for the *e*p, the standard for this physical constant, the actual anomalous magnetic moment is also affected, even if the precision of a local anomalous magnetic moment is high. We believe an error in its standard value appears as the deviation in the anomalous magnetic moment for the muon.

4. CONCLUSION

To summarize, the inertial mass is the degree of resistance to movement that acts the $1/r^4$ gravity by the difference in the scale of energy. The gravitational mass is determined by the strength of the universal gravitational force experienced by an object in the local gravitational field. The two masses are separate physical quantities. When combined with (Hamaji's light equivalence principle) with energy or momentum, which relates these quantities with mass, we are freed from restriction of using only the invariant mass even if the inertial and gravitational masses are different. It explains the interaction of $1/r^4$ gravity between elementary particles. This is achieved without relying on extra dimensions, which to date have not been observed. All of energy tries to diffuse much like the attenuation of light in accordance with the 1/*r* potential. Also, depending on the amount of energy, fermions are confined within a radius determined by the Yukawa-type potential. When the long-range and short-range forces of such a vacuum mechanism act differentially through coupling, the divergence of infinitesimals does not occur because there is a natural cancellation. If this energy mechanism gives rise to mass and gravity in such a background field, there is no choice of whether to consider gravity in special relativity and quantum theory. We have been using the gravitational mass of a stationary object as a measure of its invariant mass. The gravitational mass is generated when photons are confined, and diverges to infinity if the photons are not confined. The Compton wavelength is a measure of the inertial mass and of the energy confined. All energy has a mass equal to the vacuum expectation value generated by the gravitational mass determined from confinement. Hence, all energy referred to in the energy-mass equivalence relation can be replaced by an invariant, gravitation, and inertial masses and Planck's constant based energy representation, and the mass and energy unit are not equal. $1/r^4$ gravity with scalable inertial mass can explain the anomaly of the muon. This paper presented a new way to integrate general relativity and quantum theory by the separation of the scalable inertial mass in $1/r^4$ gravity, and the gravitational mass in universal gravitation.

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APPENDIX

My previous research involved representing energies (gravitational mass, inertial mass, and Planck's constant) of different particle speeds as an equivalence for quantum ($Mc = \Delta m \Delta w = hf/c$). In addition, $E = Mc^2$ (kinetic energy is changed to mass) does not indicate that the total energy change is always proportional to particle speed. Therefore, "energy representation of a mathematical action," and

"energy change of a physical interaction" are not similar. The actual physical phenomenon should distinguish between these actions. Table A1 and A2 show their distinction.

TABLE A1: Differences between the energies computed by the complex notation and by conventional methods

Total energy	Invariant energy [1]	No energy change [2]	Increase of energy	Decrease of energy
Gravitational mass	Mc^2	Mc^2	$\uparrow Mc^2$	$\downarrow Mc^2$
	$Mw_0{}^2$	$M(\Delta v^2 + \Delta w^2)$	$\uparrow M(\uparrow v^2 + \downarrow w^2)$	$\downarrow M(\downarrow v^2 + \uparrow w^2)$
Planck's constant	hf_0	hf	$\uparrow hf$	$\downarrow hf$
	$\hbar\omega_0$	$\hbar\omega$	$\uparrow \hbar\omega$	$\downarrow \hbar\omega$
Inertial mass [3]	m_0cw_0	$(\Delta m)c(\Delta w)$	$\uparrow(\uparrow m)c(\downarrow w)$	$\downarrow(\downarrow m)c(\uparrow w)$
	$m_0c\lambda_0f_0$	$(\Delta m)c(\Delta \lambda)f$	$\uparrow(\uparrow m)c(\downarrow \lambda)f$	$\downarrow(\downarrow m)c(\uparrow \lambda)f$

M: Gravitational mass, *c:* Speed of light, *v:* Particle speed, *w*: Wave speed, *h:* Planck constant, *f:* Frequency, *m:* Inertial mass, *λ:* Wavelength, *ħ:* Dirac's constant, *ω:* Angular velocity, $\Delta\Delta$: Inverse proportion, ↑: Increase, ↓: Decrease.

In the above table, the rows show the energy representation differences.
●: **Gravitational mass** is the weight as defined by universal gravitation.
●: **Planck's constant** is a physical constant of quantum theory.
●: **Inertial mass** quantifies the resistance of an object to the movement.
The columns indicate whether the energy computed in the complex notation has increased or decreased, relative to the standard computation.
● **No energy change** denotes an inverse proportionality between the particle and wave speeds of the physical quantity (4). For example, the particle velocity of an object in free fall increases while its wave speed decreases. In addition, a photon is red (blue) shifted by a change in the gravitational field.
●: (●:) **Increase (Decrease) of energy** denotes that the particle–wave energy relationships of each physical quantity increase or decrease. For example, the kinetic energy increases (decreased) during acceleration (deceleration) of an object. This scenario equally applies to a motionless object seen by a moving observer.

TABLE A2: This was represent the "Case of the total energy change" and "Case of the total energy no change" of the Fermion and the photon.

Fermion	Total energy representation	Photon	Total energy representation
Inertial motion	$E = Mc^2 = M(2\varphi + v^2 + w^2)$	Propagation	$E = Mc^2 = M(2\varphi + w^2)$
	$= hf = \hbar\omega$		$= hf$
	$= mcw = mc\lambda f$		$= mcw = mc\lambda f$
Acceleration by boost	$\uparrow E = \uparrow Mc^2 = \uparrow M(2\varphi + \uparrow v^2 + \downarrow w^2)$	Inverse Compton effect	$\uparrow E = \uparrow Mc^2 = \uparrow M(2\varphi + w^2)$
	$= \uparrow hf = \uparrow \hbar\omega$		$= \uparrow hf$
	$= \uparrow(\uparrow m)c(\downarrow w) = \uparrow(\uparrow m)c(\downarrow \lambda)f$		$= \uparrow mcw = \uparrow mc\lambda f$
Deceleration by friction	$\downarrow E = \downarrow Mc^2 = \downarrow M(2\varphi + \downarrow v^2 + \uparrow w^2)$	Compton effect	$\downarrow E = \downarrow Mc^2 = \downarrow M(2\varphi + w^2)$
	$= \downarrow hf = \downarrow \hbar\omega$		$= \downarrow hf$
	$= \downarrow(\downarrow m)c(\uparrow w) = \downarrow(\downarrow m)c(\uparrow \lambda)f$		$= \downarrow mcw = \downarrow mc\lambda f$
Escape from Gravitational source	$E = Mc^2 = M(\downarrow 2\varphi + \downarrow v^2 + \uparrow w^2)$	Gravitational Red-shift	$E = Mc^2 = M(\downarrow 2\varphi + \uparrow w^2)$
	$= hf = \hbar\omega$		$= hf$
	$= (\downarrow m)c(\uparrow w) = (\downarrow m)c(\uparrow \lambda)f$		$= (\downarrow m)c(\uparrow w) = (\downarrow m)c(\uparrow \lambda)f$
Free-fall to	$E = Mc^2 = M(\uparrow 2\varphi + \uparrow v^2 + \downarrow w^2)$	Gravitational	$E = Mc^2 = M(\uparrow 2\varphi + \downarrow w^2)$

Gravitational source	$= hf = \hbar\omega$		Blue-shift	$= hf$
	$= (\uparrow m)c(\downarrow w) = (\uparrow m)c(\downarrow \lambda)f$			$= (\uparrow m)c(\downarrow w) = (\uparrow m)c(\downarrow \lambda)f$

↑↓: Inverse proportion, ↑: Increase, ↓: Decrease.

1 The invariant system is the same inertial system of the Lorentz factor ($\gamma = 1$) that the wave speed is at the same frequency and the speed of light.

2 The wavelength is inversely proportional to the inertial mass. They are also proportional to the wave velocity and inversely proportional to the energy.

3 The transversal Doppler Effect is determined by the wave speed, and is offset by the difference of translational energy.

4 v^2 includes a gravitational potential (2φ). The wave speed inversely varies with v^2, and the speed of light is constant. The gravitational field is integral to the fermions. Graviton exchange does not change the total energy of the quantum.