A first order phase transition and self-organizing states in single-domain ferromagnet

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Abstract

Analytical consideration of uni-axial single-domain ferromagnet during the first order phase transition induced by a magnetic field is performed. Field is directed along the symmetry axis antiparallel to initial magnetization direction. For samples of the flat

shape, besides the known change of the magnetization direction on 180° , at definite relations between values of magnetic field, the magnetization and the anisotropy of a crystall, there is continuous spectrum of states with intermediate magnetization directions. In these states, a precession frequency $\omega = 0$. For samples of spherical shape, a process of the phase transition does not depend on the demagnetization field. At addition action of high frequency field perpendicular to the main magnetic field, there are dynamic equilibrium states, i.e. "self-organizing states" of ferromagnet, when the entropy increase connected with dissipation is compensated by the negative entropy flow due to the periodic field. It is shown that under these conditions, by varying the frequency of the periodic field, we can control the self-organising system, i.e. decrease or increase the system energy and, correspondingly, change the direction of magnetisation in ferromagnet.

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1. Introduction

A very large number of phenomena and processes are known, which can be classified as "self-organizing systems" or "dissipative structures" [1-4]. Processes belonging to this category are, for example, sounding of wind and stringed musical instruments, a whistle sound, existence of proteins, development of plants, functioning of animals and humans. Generally, the life itself in all its forms is an example of such "self-organizing systems". It may seem surprising that, unlike nature, the man himself was able to invent so limited number of such systems. This could include such examples that can be reproduced on the laboratory table: the chemical "Belousov-Zhabotinsky reaction" [5, 6], "Benar cells" at liquid boiling [7]. Precessing ball solitons during the magnetic phase transition in ferromagnet could also be considered as "a self-organizing system" or "self-organizing states" (SOS) [8]. Some of these systems are structures periodic in space or in time. Others are more complex. But the common

feature of all these processes is that the loss of energy in the system associated with dissipation, is fully offset by the influx of energy from external sources, i.e. inflow of entropy due to the dissipation is compensated by the negative flow of entropy due to the coupling to an external source.

In this article, the SOS arising at the first order phase transition in uni-axial singledomain ferromagnet under the action of a magnetic field directed along the symmetry axis are considered.

At first, in the second part of this article, peculiarities of the first-order phase transition in a single-domain ferromagnet has been analyzed. The sole purpose of the single-domain condition for this article is to exclude extraneous sources of nucleation of a new phase, such as domain walls or external boundaries of the crystal. (For example, 2π -degree boundaries themselves are nuclei of a new phase.) In such conditions, the phase transition under the action of the magnetic field is determined by the process of coherent magnetization change.

In the third part of the article, the changes in the phase transition of ferromagnet under the action of additional high frequency magnetic field perpendicular to the main field have been considered. In such conditions, SOS of ferromagnet arise. Features of these states have been investigated.

2. Phase transition in single-domain ferromagnet

Analysis scheme of ferromagnetic is presented in Fig. 1. Initially, the sample is magnetized to saturation along the direction (-z) – see in Fig. 1(a). For this it is necessary that the applied field $H_{dsat} < 0$ was in absolute value greater than arising in the sample the demagnetizing field, i.e. $|H_{zsat}| > H_d$. Fig. 1(b) shows the ferromagnet magnetized to saturation along the axis (z) under the action of the field $H_z > |H_d|$. Thus, Fig. 1 corresponds to the final states of the ferromagnet. In a given article the process of $(a) \rightarrow (b)$ transition is analyzed.

(a) $\int z H_z$ (b)

To analyse magnetic phase transition in the ferromagnet with uni-axial anisotropy, we use the Landau–Lifshitz equation [9] in the Gilbert form:

$$\frac{\partial \mathbf{m}}{\partial t} = \gamma \mathbf{m} \times \frac{\partial W}{\partial \mathbf{m}} + \kappa \left(\mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} \right) (\kappa > 0)$$
(1)

and the following expression for the density of energy:

$$W = \frac{K_1}{2} |m_{\perp}|^2 - m_z H_z + E_d.$$
⁽²⁾

 H_z is an external magnetic field directed along the anisotropy axis $Z(H_z > 0)$; $K_1 > 0$, $\gamma = 2\mu_B/\hbar$; **m** is a non-dimensional vector of ferromagnetism equal (in the absolute value) to $I, m_{\perp} = m_x + im_y$, initial magnetizaton is along the (-z) direction, in present paper $m_z = \pm \sqrt{1 - |m_{\perp}|^2}$; E_d is energy of demagnezation for the sample. We consider only two cases: sample of the flat shape, moreover, the thickness of such sample is much smaller than the dimensions in other directions and the symmetry axis is perpendicular to the plane of a sample; in the second case the sample is of the spherical form.

2.1 Sample of the flat shape

In this case the energy of demagnetization is

$$E_d = 4\pi M_0 m_z^2 \tag{3}$$

where M_0 is the magnetization of a crystal. In such case, equation (1) can be written as:

$$i\frac{\partial m_{\perp}}{\partial \tau} = -hm_{\perp} - \left(1 - 2h_d\right)m_z m_{\perp} + \kappa \left(m_{\perp}\frac{\partial m_z}{\partial \tau} - m_z\frac{\partial m_{\perp}}{\partial \tau}\right).$$
(4)

Here the differentiation is carried out with respect to the dimensionless time $\tau = 2\mu_B K_1 \hbar^{-1} t$; $h = H_z/K_1$; parameter of demagnetization field $h_d = 4\pi M_0/K_1$.

The solutions of Eq. (4) have the following form:

$$m_{\perp}(\tau) = p(\tau)e^{i\omega(\tau)\tau}$$
(5)

From (4), the equations, which define the correspondence between m_z and ω and the time changes of these parameters are the following:

$$(1 - 2h_d)m_z + h = \omega(1 + \kappa^2),$$
(6)

(here $\kappa^2 \ll 1$, therefore we neglect this value.)

$$\frac{dm_z}{d\tau} = \kappa\omega(1 - m_z^2). \tag{7}$$

For the energy density relative to initial state we have:

$$e_{fl} = (1 - 2h_d) \frac{(1 - m_z^2)}{2} - h(1 + m_z),$$
(8)

and

$$\frac{de}{d\tau} = -\kappa\omega^2 \left(1 - m_z^2\right). \tag{9}$$

In what follows we consider the process of changing of parameters of a ferromagnet in the transition from the initial state when $m_z = -1$. In this process, the energy decreases, respectively m_z increases from the initial value, and precession frequency also changes. The character of changes in a ferromagnet during the phase transition depends strongly on the shape of the sample.

In Figs. 2 for the sample of flat form, limit values of main parameters are given as functions of h_d value for different values of acting field h. Initial energy is $e_1 = 0$, final energy is e_2 . Correspondingly, we have initial $m_{z1} = -1$ and final m_{z2} , initial ω_1 and final ω_2 . These limiting values are determined from equations (6) and (8).

If h > 0, there are two ranges for limit values of the parameters. In the first of them, a completed reorientation (CR) takes place if

$$0 \le h \le (2h_d - 1): \ m_{z^2} = +1, \ \omega_1 = h - (1 - 2h_d), \ \omega_2 = h + (1 - 2h_d), \ e_2 = -2h.$$
(10)

In the second range, the transitions into intermediateie states, where $m_{z2} < (+1)$, occur if

$$h \ge (2h_d - 1): m_{z_2} = -h/(1 - 2h_d), \ \omega_1 = h - (1 - 2h_d), \ \omega_2 = 0, \ e_2 = \frac{(1 - h - 2h_d)^2}{2(1 - 2h_d)}.$$
 (11)

If $h \le 0$, there is not completed phase reorientation, but only the transitions into intermediate states. In latter case, the limit parameters, as in (11), are the following:

$$m_{z2} = -h/(1-2h_d), \ \omega_1 = h - (1-2h_d), \ \omega_2 = 0, \ e_2 = \frac{(1-h-2h_d)^2}{2(1-2h_d)}.$$
 (12)

As can be seen, for a given value of a field, completed phase reorientation occurs only at sufficiently small value of h_d . At a higher value of h_d , the final value $\omega_2 = 0$ and as can be seen in these Figs. 2, the values $m_{z2} < (+1)$. The field value becomes insufficient to overcome the demagnetizing fields. Fig. 2(e) corresponds to h = 0, i.e. when the field $H_{zsat} < 0$, which magnetizes the sample to saturation, is simply removed. In this case $m_{z2} = 0$. In this case the magnetization in final state is perpendicular to the axis of anisotropy. Note that for multi-domain sample, zero magnetic field corresponds to the state, when the magnetic moments of



domains are directed with equal probability along or against the anisotropy axis and averaged $m_z = 0$, as m_{z2} in our case.

Figures 2: Limit values of main parameters for flat sample vs demagnetisation field (energy is denoted by filled circles, m_z value – empty rectangle, frequency – continuous line): (a) h = 1.5, (b) h = 1, (c) h = 0.5, (d) h = 0.1, (e) h = 0, (f) h = -0.5.

If the current field is negative, i.e. h < 0, the value $m_{z2} < 0$, as is shown in Fig. 2(f).

So there are continuous spectrum of intermediate states, as though frozen states of a ferromagnet. System tends in each of these "frozen states" (FS) asymptotically, wherein the precession frequency $\omega \rightarrow 0$.

In Fig. 3, in dependence of the critical field on the parameter of demagnetization field, the boundaries of considered above areas are shown.



Figure 3: The dependence of the critical field on the parameter of demagnetization field showing the boundaries between areas with different characters of the phase reconstruction in the case of the thin flat sample.

Note that the expressions (10), (11) and (12) are also valid for the phase transition in the case of a ferromagnet with the easy magnetization plane, i.e. at $K_1 < 0$.



Figures 4: The time dependences of e_{fl} , m_z and ω for flat sample: (a) h = 0.5, $h_d = 0.6283$ ($M_0/K_1 = 0.05$); (b) h = 0.1, $h_d = 1.6336$ (0.13); (c) h = 0, $h_d = 0.5655$ (0.045); (d) h = -1.4, $h_d = 1.2566$ (0.1).

Let us consider the time dependence of parameters during the phase transition. In correspondence with equations (6) and (7), the time dependence of m_z can be obtained:

$$\tau = \frac{1}{\kappa} \int_{m_{z_0}}^{m_z} \frac{dm_z}{(1 - m_z^2) [h + (1 - 2h_d)m_z]}.$$
(13)

In Figs. 4, the time dependences of main parameters for flat sample are presented, according to (13) and (6), (8). In all these and in subsequent examples, the dissipation parameter is $\kappa = 5 \times 10^{-4}$. These time changes correctly correspond to dependences of the type shown in Figs. 2.

For a given h_d value, a minimum field, in which a change in orientation occurs, is: $h_{\min} = 1 - 2h_d$. If for flat sample $h_d = 1.2566$, i.e. $M_0/K_1 = 0.1$, this field equals to $h_{\min} = -1.513274$ (an approach to this value can be seen in Fig. 4(d)).

In Fig. 5, field dependences of energy and m_{zFS} parameter of FS for flat sample at $h_d = 1.2566$ are presented.



Figure 5: Field dependences of energy (full circles) and m_z parameter (empty rectangles) of FS for flat sample at $h_d = 1.2566$. Here, the frequency for all states is $\omega \rightarrow 0$.

2.2 Sample of a spherical shape

In this case

$$E_{d} = N_{dz} M_{0} m_{z}^{2} + N_{d\perp} M_{0} |m_{\perp}|^{2} = \frac{4\pi}{3} M_{0}, \qquad (14)$$

i_e_ for spherical shape of the sample, the parameters m_z , ω and e and their time changes do not depend on the h_d value, but the value of energy contains a constant component

$$\left(+\frac{4\pi M_0}{3K_1}\right)$$
. Therefore, instead of (6) and (8), we have the following:
 $m_z + h = \omega(1 + \kappa^2)$ ($\kappa^2 \ll 1$), (15)

$$e_{sph} = \frac{(1 - m_z^2)}{2} - h(1 + m_z).$$
(16)

In Fig. 6, dependences of energy and frequency on m_z at three field values, h = 0.5, 1.0 and 1.5, for spherical sample are presented, in correspondence with (15) and (16). Of course, the transition from $m_z = -1$ to $m_z = +1$ is possible only if $h \ge 1$.



Figure 6: Dependences of energy and frequency on m_{τ} at three field values for spherical sample.

In the case of spherical sample, according to (7) and (14):

$$\tau = \frac{1}{\kappa} \int_{m_{z_0}}^{m_z} \frac{dm_z}{(1 - m_z^2)(h + m_z)} \,. \tag{17}$$

In Fig. 7, the time dependences of energy and m_z value are shown at h = 1,1.1 and 1.5 for spherical sample. In these cases the change of m_z is from (-0.999) to (+0.999).



Figures 7: Time dependences of energy and m_z for spherical sample: (a) h=1, (b) h=1.1, (c) h=1.5. All changes are from $m_{z1} = -0.999$ to $m_{z2} = +0.999$.

Reorientation of the magnetization in single-domain ferromagnets under the action magnetic field pulses has been studied in many papers. These investigations were carried out mainly in connection with the creation of high-speed elements for computers. We give here only a few articles related to the issues addressed in this paper: [10 - 16]. In these studies were considered single-domain ferromagnets (Stoner particles) on the basis of numerical solutions

of Landau-Lifshitz. It takes into account the different direction of the external magnetic field, but was not considered the symmetric case, when the field is acting along the symmetry axis of the crystal with uniaxial symmetry. Moreover, calculations were made at conditions corresponding to high values of dissipation parameter $\kappa \approx 1$ in equation Landau-Lifshitz, i_e_ in the area of Stoner-Wohlfatrth limit [10]. In this region, the time by coherent switching takes a minimum value in the range near to 10^{-9} s.

In the present paper, for the analysis of self-organizing states in the next part of this article, a particular case considered when the magnetic field is directed along the axis of anisotropy. Such geometry allowed to enter the frequency of precession relatively to the axis of symmetry and make an exact analytical consideration of the phase transition (as opposed to numerical calculations in [10 - 16]). Moreover, some features of reorientation (see above) not detected in [10 - 16] manifested.

Comparing our results with [10 - 16], we see that by taking into account κ^2 relative to a unit in the equation (15), we have instead of (17) the ratio (in this case for the spherical sample):

$$\tau = \left(\frac{1}{\kappa} + \kappa\right) \int_{m_{z0}}^{m_z} \frac{dm_z}{(1 - m_z^2)(h + m_z)}.$$
(18)

In Fig. 8, dependency of $\kappa/(1+\kappa^2) \equiv \left(\kappa + \frac{1}{\kappa}\right)^{-1}$ on κ is shown. It can be seen that the rate of change of parameters of the ferromagnet, i_e_ a speed of coherent switching is maximum at $\kappa = 1$, in corresponding with [10].



Figure 8: Dependency of effective parameter κ_{eff} of coherent switching on κ .

Using the value $\kappa_{eff} \equiv \frac{\kappa}{1+\kappa^2} = 0.5$ corresponding to the maximum speed, instead of the value $\kappa = 0.5 \times 10^{-3}$ taken from the researches of magnetic resonance, as it was in Fig. 4 and Fig. 7, we have the same curves for the change of parameters, but with the transformation

 $t(\mu s) \rightarrow t(ns)$ of the timeline, i.e. accordance with [10 - 16] about quick coherent switching in ferromagnets.

3. Self-organizing states

Using an additional external high frequency magnetic field, we can fix the precession frequency and thereby stabilize the intermediate states of the ferromagnet. If the added periodic field is perpendicular to a main field, and

$$\mathbf{H}_{\perp} = K_1 h_{\perp} e^{i\omega_0 \tau},\tag{19}$$

we can express the magnetic component of magnetization in the form

$$m_{\perp}(\tau) = p(\tau)e^{i(\omega_0\tau - \beta(\tau))},$$
(20)

i.e. the precession phase of magnetic moments differs from the phase of periodic field. In this case, the equations for m_z in the case of flat sample take the following form:

$$\left(1 - m_z^2\right)\left[\left(1 - 2h_d\right)m_z + h - \left(\omega_0 - \frac{d\beta}{d\tau}\right)\right] = +\kappa \frac{dm_z}{d\tau} + h_\perp m_z \sqrt{1 - m_z^2} \cos\beta, \qquad (21)$$

$$\frac{dm_z}{d\tau} = \sqrt{1 - m_z^2} \left[\kappa \sqrt{1 - m_z^2} \left(\omega_0 - \frac{d\beta}{d\tau} \right) - h_\perp \sin\beta \right]$$
(22)

From (8), we obtain expressions for energy density relative to the initial state, together with the energy of interaction with the periodic field (see, for example, [8]):

$$e_0 = (1 - 2h_d) \frac{(1 - m_z^2)}{2} - h(1 + m_z) - h_\perp \sqrt{1 - m_z^2} \cos\beta$$
(23)

and for the change of this energy connected with dissipation and the action of external periodic field:

$$\frac{de}{d\tau} = -\kappa \left[\frac{1}{1 - m_z^2} \left(\frac{dm_z}{d\tau} \right)^2 + \left(1 - m_z^2 \right) \left(\omega_0 - \frac{d\beta}{d\tau} \right)^2 \right] + h_\perp \sqrt{1 - m_z^2} \omega_0 \sin\beta.$$
(24)

The equations (19) – (24) constitute a complete description of the system, including its time transformation. However, in the present paper we consider only dynamic equilibrium state of ferromagnet, i.e. when the decrease of energy caused by dissipation is compensated by energy flow from the external periodic field, i.e. $de_0(\tau)/d\tau = 0$. Furthermore, in this case $dm_{z0}/d\tau = 0$ and $d\beta_0/d\tau = 0$. Therefore, for this equilibrium state of ferromagnet, i.e. for self-organizing state (SOS), we obtain the following expressions:

$$\frac{dm_z}{d\tau} = \sqrt{1 - m_{z0}^2} \left(\kappa \omega_0 \sqrt{1 - m_{z0}^2} - h_\perp \sin \beta_0 \right) = 0, \qquad (25)$$

$$\frac{de_0}{d\tau} = -\omega_0 \sqrt{1 - m_{z0}^2} \left(\kappa \omega_0 \sqrt{1 - m_{z0}^2} - h_\perp \sin \beta_0 \right) = 0.$$
⁽²⁶⁾

From these expressions, we obtain the relation:

$$\sin\beta_0 = \left(\kappa\omega_0 \sqrt{1 - m_{z0}^2} / h_\perp\right). \tag{27}$$

Correspondingly, the corrected equation for SOS takes the following form (instead of (6)):

$$\sqrt{1 - m_{z0}^2} \left[\left(1 - 2h_d \right) m_{z0} + h - \omega_0 \right] = m_{z0} \sqrt{h_\perp^2 - \kappa^2 \omega_0^2 (1 - m_{z0}^2)} .$$
⁽²⁸⁾

From equations (25) and (26), it can also be seen that the energy compensation and consequently the origin of SOS is possible only if

$$h_{\perp} \ge h_{\perp \min} = \kappa \omega_0 \sqrt{1 - m_{z0}^2} .$$
⁽²⁹⁾

For such a system, the entropy increase connected with dissipation is compensated by the negative flow of the entropy, which is the result of external periodic field. It can be expressed as follows:

$$\frac{ds}{d\tau} = \frac{ds_{diss}}{d\tau} + \frac{ds_{h_{\perp}}}{d\tau} = 0, \qquad (30)$$

where

$$\frac{ds_{h_{\perp}}}{d\tau} = -\frac{ds_{diss}}{d\tau} = \frac{1}{T} \frac{de_{diss}}{d\tau} = -\frac{\kappa\omega_0^2}{T} \left(1 - m_z^2\right) < 0.$$
(31)

Examples of the frequency dependences of energy, value of m_{z0} and the change in entropy for the flat and spherical samples in SOS at $h_{\perp} = 2 \times 10^{-3}$ are presented in Fig. 9 and Fig. 10. It should be noted that in the examples shown, the quantities $e_0(\omega_0)$ and $m_{z0}(\omega_0)$ differ very little, not more than 1-2%, from such values in the absence of \mathbf{H}_{\perp} field.







Figure 10: The frequency dependences of energy, m_{z0} value and the negative flow of the entropy due the external periodic field in SOS for spherical sample, if h = 1.5, $h_{\perp} = 0.002$.

Figure 11: The frequency dependences of energy, m_z value and minimum amplitude of periodic field h_{\perp} for SOS in the case of flat sample, at h = 0.1, $h_d = 1.2566$.

In Fig. 11, the frequency dependence of minimum amplitude of periodic field h_{\perp} for SOS in flat sample, according to (27, is shown.

However, in addition to obtaining SOS, you can change these states. Assume that the precession frequency varies slowly enough. In this case, in Equations (24) - (27) instead of ω_0 , we have $(\omega_0 + \frac{d\omega}{dt}t)$, where ω_0 is the initial frequency. We can in all equations simply replace $(\omega_0 + \frac{d\omega}{dt}t)$ on $\omega_0(t)$. In result, we obtain the characteristics of self-organising state which depend on time, i.e. $e_0(t)$, $m_{z0}(t)$ and

$$\sin\beta_0(t) \cong \kappa\omega_0(t) h_{\perp}^{-1} \sqrt{1 - \frac{(\omega_0(t) - h)^2}{(1 - 2h_d)^2}}$$
(32)

- in the case of flat sample, and

$$\sin\beta_0(t) \cong \kappa\omega_0(t)h_{\perp}^{-1}\sqrt{1 - (\omega_0(t) - h)^2}$$
(33)

- for spherical sample.

and, correspondingly, the change of entropy depends on the time too, according to Fig. 9 and Fig. 10. As a result, changing the frequency of external field, and consequently the energy

 $e_{\perp}(t) = h_{\perp}\sqrt{1 - m_{z0}^2} \cos\beta_0$ too, we can control the self-organising system, and not only reduce the system energy, but also increase it, decreasing m_{z0} value and returning the ferromagnetic in direction to initial phase state.

Further, we can compare the soliton SOS described in [8] with those presented here. Precessing ball solitons of paper [8] may also occur at the first-order transition in a ferromagnet. But their origin is spontaneous and is connected with significant fluctuations in the system configuration. Moreover, the probability of such SOS is strongly dependent on the temperature and the distance from the bifurcation point, in which their energy relative to the initial state is zero.

The SOS presented here, in contrast to [8], are not localized in space, but distributed throughout all volume of the crystal; their appearance is not associated with fluctuations, they do not have a random, probability character, and do not depend on temperature.

Conclusions

- 1 A single-domain ferromagnet with uniaxial anisotropy at the first-order phase transition under the action of a magnetic field directed along the anisotropy axis has been considered. Analytical analysis of the entire process of phase transition is performed for two configurations of crystalline samples: a thin sample with the anisotropy axis perpendicular to the surface of the plate, and the spherical shape of the sample.
- 2 Characters of phase reconstruction are significantly different in the two cases. In the first case the phase transition depends essentially on the relation between the sample magnetization and anisotropy of the crystal and thus of the demagnetisation field. There are two areas for the parameter of demagnetisation field. In the first of these areas, completed reorientation is carried out in the crystal, i.e. change in the magnitude m_z from $m_{z1} = -1$ to $m_{z2} = +1$. In the second area, "unfinished" phase reconstruction is carried out, i.e. transition in "frozen states" (FS) where $m_{z2} < 1$. These states have a continuous spectrum, for each such state precession frequency $\omega \rightarrow 0$.
- 3 Phase transition for sample of spherical form is described in the same variables as for flat sample, but in this case the process of transition is not dependent on the demagnetization field. In spherical samples, FS does not arise.

- 4 At simultaneous action of high-frequency magnetic field perpendicular to the direction of the main field, a self-organizing state (SOS) of a ferromagnetic arises, in which the ferromagnetic is in dynamic equilibrium. In this equilibrium state, the entropy increase connected with dissipation is compensated by the negative flow of the entropy that is the result of external periodic field.
- 5 Relations between the main parameters of SOS, i.e. between the values of fields, energy, precession frequency, and the angle between ferromagnetism vector and the anisotropy axis, have been analysed.
- 6 Changing the frequency of the alternating field, and thereby, the flow of the entropy, can be a continuous method to change all parameters of SOS, including reduction or increase of the system energy.

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