A natural solution to the dark energy problem

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Abstract

Recent observations indicate that the acceleration of our universe expansion is slowing down, which challenge the standard ΛCDM model. In this article, we show that the existence of particles with imaginary energy density can explain the observed slowing down acceleration and provide a complete solution to the origin of dark energy. This model can give good agreements with observational constraints by 28 data from supernovae between redshift $z = 0.07 - 2.3$.

A. Introduction

The ΛCDM model is the most robust scenario to describe the evolution of our universe. The existence of dark energy or cosmological constant provides the acceleration of universe expansion which is confirmed by the data from supernovae [1, 2]. This model can also sucessfully explain the large-scale structure of the universe and the flatness of the universe that revealed from the Cosmic Microwave Background (CMB) [3, 4].

However, some recent observations begin to challenge this standard model in cosmology [5, 6]. For example, the current value of the cosmological constant is so small, which is 120 orders of magnitude smaller than the theoretical anticipated value. Also, we are so surprised that the dark energy is just starting to dominate the total energy density. These two problems are known as the Fine Tuning Problem and the Coincidence Problem respectively [7]. Although some tiny modifications such as the quintessence model and the modified gravity model may help to alleviate the problem, the complexity of the required models is further increased. Moreover, the ΛCDM model predicts that the deceleration parameter should approach to a constant ($q \approx -0.55$) while recent observations from supernovae indicate that the acceleration of expansion of our universe is decreasing [8–11]. For example, numerical calculations from SNIa data obtain $q = -0.38 \pm 0.05$ [12], and recently $q = -0.2086^{+0.0374}_{-0.0380}$ [13]. Some calculations even obtain a positive deceleration parameter [6, 14]. All the above problems indicate that the ΛCDM model may not be the ultimate model to describe our universe.

Furthermore, the origin of the dark energy is still unknown. The acceleration of universe expansion may be driven by some unknown scalar fields (see [15] for a review), effect from higher dimensions [16] or some other recent proposed models [17–22]. In particular, the existence of Tachyonic field is one of the popular models to explain the origin of dark energy [23–26]. The existence of tachyon is first derived from quantum field theory. Generally speaking, tachyon is a hypothetical particle that always moves faster than the speed of light. Although this kind of particles can never be directly discovered, they can be inferred indirectly from observations. For example, some works have already shown that neutrino can be a tachyon [27–29]. Many theoretical works show that tachyonic particles could be stable for a long period of time [30–32]. The energy of a tachyon with rest mass m is given by $E = \gamma mc^2$, where γ is the Lorentz factor. Since a tachyonic particle travels faster than light,

γ would be an imaginary number. Traditional theory of tachyon suggests that m is also an imaginary number so that the energy E would be real to observe [33]. However, it is also possible that m is a real number so that E is an imaginary number [34, 35]. In this article, I suggest that if there exists some tachyons with imaginary value of energy, the evolution of our universe might be affected. In the following sections, I show that the existence of tachyons in our universe can naturally explain the acceleration of universe expansion. If the energy density of tachyons is a complex number, the spatial dimensions would also be a complex number so that the Friedmann equation is self-consistent [35]. If the real part of the spatial dimensions is our observable universe, it can be shown that the evolution of the imaginary part of the spatial dimensions could affect the real universe and contribute to a dark energy term in the evolution. I also show that the new tachyonic Friedmann equation can satisfy current observations from 28 supernovae data and explain the slowing down acceleration in the universe expansion.

Although there are plenty of theoretical researches showing that the tachyonic field can explain the origin of dark energy, most of the models depend strongly on the arbitariness of the tachyonic potential functions [5]. In our model, we do not require any tachyonic potential functions to explain the origin of dark energy.

B. The Tachyonic Friedmann Equations

The original Friedmann equation without cosmological constant in a flat universe is given by

$$\dot{a}^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_r\right) a^2, \tag{1}$$

where a is the cosmic scale factor, ρ_m and ρ_r are the energy density of matter and radiation respectively. In the following discussion, we neglect the radiation term for simplicity because it would not have a significant effect on the late time of universe expansion. If the total matter energy density includes tachyonic particles with imaginary value of energy density, we can write $\rho_m = \rho_n + \rho_t i$, where ρ_n and ρ_t are the energy density of normal matter and imaginary part of the energy density of tachyonic matter respectively. Since the right hand side of the Friedmann equation is a complex number, the only way for it to be consistent is that the cosmic scale factor is also a complex number, which is given by $a = a_r + a_i i$.

The evolution of the energy density is controlled by the fluid equation, which is given by

$$\frac{d}{dt}(a^3\rho) = -\frac{P}{c^2}\frac{d(a^3)}{dt}, \tag{2}$$

where ρ and P are the density and pressure due to every component of the universe. Since $P \approx 0$ for normal matter (including dark matter), we get $\rho_n = \rho_{n0}a^{-3}$, where ρ_{n0} is the present energy density of normal matter. For tachyonic matter, although the 'pressure' may be unphysical in nature, the pressure term serves an effect of gravity in the Friedmann equation. From statistical mechanics, the pressure would be $P = \rho_t/3$ if the speed of the tachyonic particles is just exceeding the speed of light (ultra-relativistic regime). Therefore, Eq. (2) gives $\rho_t = \rho_{t0}a^{-4}$, where ρ_{t0} is the imaginary part of the present energy density of tachyonic matter.

By writing $y = \rho_{t0}/\rho_{n0}$, $\Omega_{t0} = \rho_{t0}/\rho_{re}$, $\Omega_{n0} = \rho_{n0}/\rho_{re}$, $x = a_i/a_r$, and using Eq. (1), we can get

$$\dot{a}_r = \sqrt{\frac{4\pi G\rho_{re}}{3}\Omega_{n0}a_r^{-1}\left(f(x,y) + \sqrt{[f(x,y)]^2 + [g(x,y)]^2}\right)}, \tag{3}$$

where

$$f(x,y) = \frac{1}{1+x^2} + \frac{2xy}{a_r(1+x^2)^2} \tag{4}$$

and

$$g(x,y) = \frac{x}{1+x^2} + \frac{y(1-x^2)}{a_r(1+x^2)^2}. \tag{5}$$

Here, we call ρ_{re} the reference density, which determines the value of Hubble constant. Also, we have the evolution of the imaginary part:

$$\dot{a}_i = \frac{4\pi G\Omega_{n0}\rho_{re}}{3\dot{a}_r a_r}\left[\frac{-x}{1+x^2} + \frac{y(1-x^2)}{a_r(1+x^2)^2}\right]. \tag{6}$$

By differentiating Eq. (1) and combining with Eq. (2), the acceleration equation is given by

$$\ddot{a}_r = -\frac{4\pi G\Omega_{m0}\rho_{re}}{3a_r^2}\left[\frac{1-x^2}{(1+x^2)^2} + \frac{2y(3x-x^3)}{a_r(x^6+3x^4+3x^2+1)}\right]. \tag{7}$$

Assuming $a_r = 0$ initially and $a_r = 1$ at present, the only free parameters in this model are Ω_{t0}, Ω_{m0} and the initial a_i. The reference density ρ_{re} can be calculated from these free parameters when the model is compared with observational data.

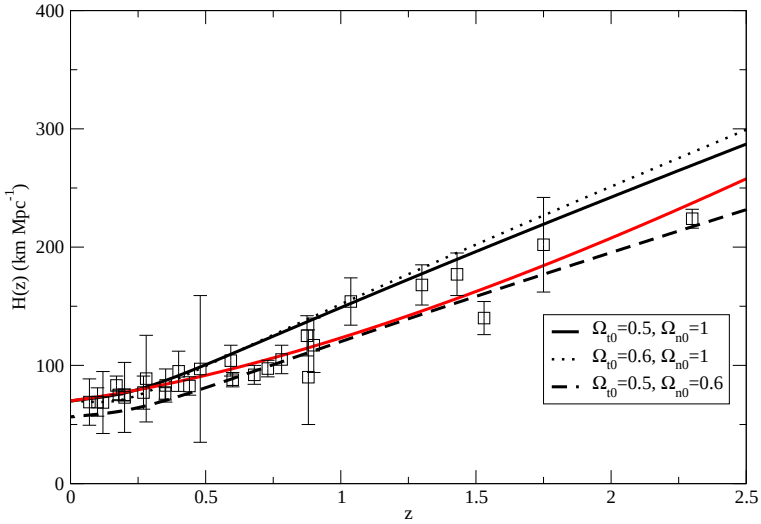

FIG. 1: Our model (black lines) is fitted with $H(z)$ data [36]. We set $a_i(t=0) = -1$ in all fits. The ΛCDM model with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ is shown in red line.

C. Comparing the results with observations

There are a few constraints we need to satisfy. They are the Hubble constant as a function of redshift $H(z)$, the deceleration parameter $q = -a_r\ddot{a}_r/\dot{a}_r$ and the minimum age of universe t_0. In particular, we do not compare with the data obtained from CMB because the spectrum calculated is highly dependent on the form of the Friedmann equation. Therefore, in this section, we mainly compare the results with observations from the data of supernovae.

In Fig. 1 and Fig. 2, we fit our model with 28 observational data from supernovae. We find that our model gives good agreements with both $H(z)$ data for $z \leq 2.5$ [36] and $q(z)$ data for $z \leq 0.4$ [13, 37] (high-redshift data for deceleration parameter are not reliable). Moreover, we find that the initial a_i must be a negative value if Ω_{t0} is positive. Otherwise, no acceleration of the universe would be resulted. For the ΛCDM model, it generally agrees with the $H(z)$ data. However, it does not match the present deceleration parameter, which indicates a slowing down acceleration.

In Fig. 3, we plot the real and imaginary parts of the cosmic scale factor as a function of

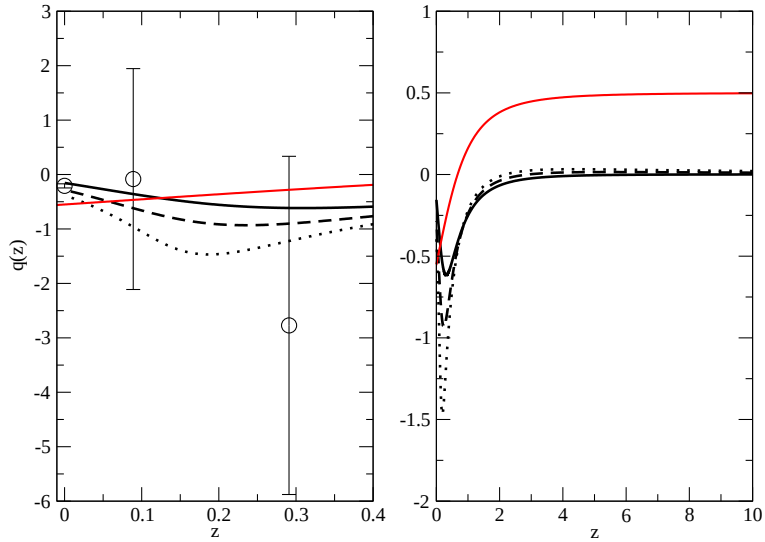

FIG. 2: Left: Our model (black lines) is fitted with the $q(z)$ data for $0 \leq z \leq 0.4$ [13, 37]. Right: $q(z)$ vs z for $0 \leq z \leq 10$ in our model. We set $\Omega_{n0} = 1$ and $a_i = -1$ in all fits. Different lines in both figures correspond to $\Omega_{t0} = 0.5$ (solid line), $\Omega_{t0} = 0.6$ (dashed line) and $\Omega_{t0} = 0.7$ (dotted line). The ΛCDM model with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ is shown in red line.

time. For $\Omega_{t0} = 0.5$, $\Omega_{n0} = 1$ and initial $a_i = -1$, the age of universe obtained in our model is about 13.4 Gyr, which is very closed to that predicted by CMB (13.7 Gyr). Nevertheless, it is still compatible with the minimum age of universe estimated from the ages of the oldest star HE 1523-0901, which is $(13.4 \pm 0.8 \pm 1.8)$ Gyr [38]. Besides, the reference density is $\rho_{re} \approx 1.7 \times 10^{-29}$ g cm^{-3}, which is about twice the critical density in the ΛCDM model. In fact, the difference of the evolution between our model and the ΛCDM model is very small except for high redshift ($z \geq 3$). Therefore, observations from high redshift quasars may be able to further identify which model is better.

D. Discussion

The claim of having tachyons with imaginary value of energy density in this model is very controversial. We have no promising experimental evidence for the existence of tachyon and the matter with imaginary energy density. This assumption is only speculative but not yet confirmed. However, the origin of dark energy and dark energy pressure in the ΛCDM model is also a mystery in standard physics. Strictly speaking, both models possess similar simplicity. The ΛCDM model requires the existence of positive dark energy and

negative dark energy pressure while our model assumes the existence of imaginary value of energy density and pressure. Moreover, both models can generate an acceleration of universe expansion and give good agreements with $H(z)$ data from supernovae.

However, recent observations from supernovae indicate that the acceleration of universe expansion is slowing down. Our model sucessfully predicts this phenomenon while the ΛCDM model predicts that a constant deceleration parameter should be approaching. Although there are some solutions suggested such as assuming a time-dependent cosmological constant or redshift-dependent equation of state of dark energy to reconcile the tension [6], these assumptions simultaneously increase the complexity of the model. Also, we need to explain the origin of the time-dependent variation. In our model, the acceleration of our universe is slowing down because a_i is a restoring function which finally gives $x = 0$ in a long run. By Eq. (7), we can see that our universe will enter to the deceleration regime again when x becomes very small.

Furthermore, both models obtain the same order of magnitude of energy density ratios ($\rho_{t0}/\rho_{n0} \approx 0.5$ and $\rho_\Lambda/\rho_m \approx 2.3$) at present. However, we have $\rho_{t0}/\rho_{n0} \approx 500$ and $\rho_\Lambda/\rho_m \approx 10^{-9}$ at $z = 1000$. The extremely small ρ_Λ/ρ_m at high redshift suggests that the ΛCDM model requires serious fine-tuning at high redshift and thus suffers much more from the Coincidence Problem. Therefore, based on the above facts, it can be concluded that our model is better than the ΛCDM model to describe our universe.

We haven't tested our model with the CMB spectrum because the CMB calculations are dependent on the form of the Friedmann equation. This can be done if the standard Friedmann equation in the spectrum calculation is fully replaced by our equations in this model. Further verification and investigation by using the CMB data including empirical fits with the free parameters in this model are needed.

In this model, we assume that the scale factor is a complex number in order to make the Friedmann equation consistent. This does not affect the Einstein equation and the matter-energy tensor in General Relativity because the derivation of the Friedmann equation follows from these equations. However, the line element would be different from the standard one. In this model, we assume that the real part of the scale factor a_r describes our observable universe. Therefore, based on this assumption, the scale factor in the line element is a_r, but not a. As a consequence, the line element is real, but not complex.

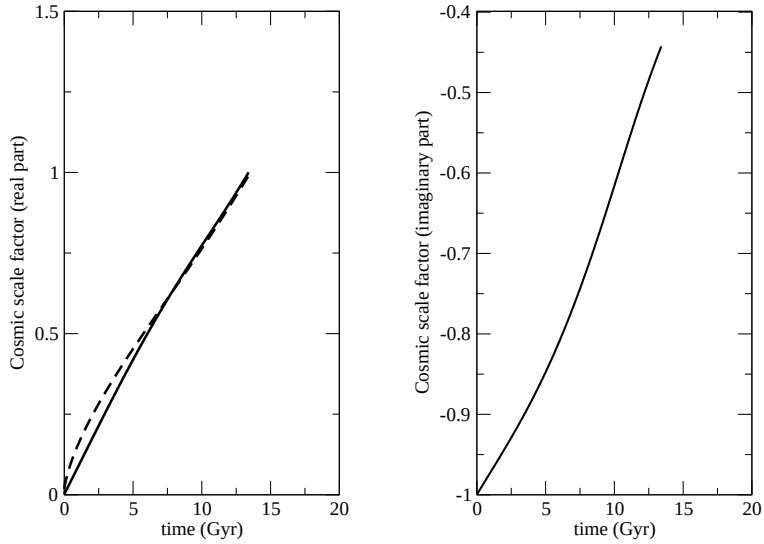

FIG. 3: The time evolution of a_r and a_i in our model. The solid line and dashed line represent our model ($\Omega_{t0} = 0.5$, $\Omega_{n0} = 1$, $a_i = -1$) and the ΛCDM model respectively ($\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$).

E. Conclusion

The dark energy term can be obtained naturally if there exists some matter containing imaginary energy density. This result also agrees with the recent observations of the slowing down acceleration of our universe expansion and the Hubble constant $H(z)$ from 28 supernovae data. This model can be tested in the future by using the data from Cosmic Microwave Background (CMB) if the model is incorporated into the CMB spectrum calculations.

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