UNDER PEER REVIEW

Original Research Article

Effects of the Magnetic Moments of the Interacting Particles on the Coulomb Potential. Application to Hydrogen Atom.

ABSTRACT

By using a Coulomb potential, modified by the interaction between the magnetic moments of the electron and proton, we have calculated the energy levels of a hydrogen atom. We have obtained fine and hyperfine structure as well as Lamb shift. All these are obtained from a simple formula which is a direct solution of the Schrödinger equation The obtained results are in a good agreement with experimental data For example, the hyperfine splitting between the energy levels of the states $1S_{1/2,1}$ and $1S_{1/2,0}$, is of the order of 5.6×10^{-6} eV, which is the source of the famous "21 cm line" which is strongly useful to radio astronomers for tracking hydrogen in the interstellar medium of galaxies. The energy of the states $nP_{1/2}$ is lower than those of the states $nS_{1/2}$ (Lamb shift) because, in the first case, the interaction between the magnetic moments of the proton and the electron spin is canceled by the spin-orbit coupling.

Keywords: Magnetic moments;; fine and hyperfine structure; Lamb shift of hydrogen atom.

1. INTRODUCTION

With the usual Hamiltonian for the hydrogen-like atom we have the n^2-fold degeneracy of states with the same principal quantum number, or $2n^2$-fold once the spin degree of freedom is included. In this real world, however, the degeneracy is lifted by corrections that arise due to the special relativity. These corrections (known as fine structure) derive from three (superficially) different sources: (*a*) relativistic corrections to the kinetic energy, (*b*) coupling between the spin and orbital degree of freedom, (*c*) and a contribution knowing as a Darwin term. Relativistic corrections split degenerate multiplets leading to small shift in energy, ca $10^{-4} – 10^{-5}$ eV. In addition, nucleus has a spin which leads to a nuclear magnetic moment. Interaction of electronic magnetic moment with field generated by nuclear magnetic moment leads to further splitting of multiplets (hyperfine structure), ca. $10^{-7} – 10^{-8}$ eV. In 1947, an experimental study by W. Lamb discovered that $2P_{1/2}$ state is slightly lower than $2S_{1/2}$ state- Lamb shift [1]. The effect is explained in the theory of quantum electrodynamics[2], in which the electromagnetic interaction itself is quantized. Some of the effects of this theory which cause the Lamb shift are as follows: vacuum polarization, electron mass renormalization, anomalous magnetic moment. On the basis of this theory, we have studied in a previous paper [3], the Lamb shift without taking into account the electron charge. Famous fine structure was first gotten by Bohr-Sommerfeld model in 1916[4]. The fine structure used formally now is the hydrogen solution by Dirac equation[5]. Surprisingly, these solutions by Dirac equations are just equal to those of Sommerfeld model. However, Dirac's hydrogen includes a lot of wrong states (= $1P_{1/2}$, $2D_{3/2}$, $3F_{5/2}$,...). The interpretation of very tiny Lamb shift depends completely on the interpretation that Dirac's hydrogen is right. Quantum electrodynamics Lamb shift is much more complicated and filled with artificial tricks. Lamb shift measurements is too difficult and vague in respect of accuracy. We cannot see what is really happening in the key small effect (= 0.000004372 eV , 1068 MHz) hyperfine level. Though the Lamb shift is very small, the author tried to measure this value believing $2S_{1/2}$ state is "metastable" and the collision between excited hydrogen atom and plates is a precise method for Lamb shift. In this experiment there is no guarantee that modified Zeeman effect is always linearly effective, and excited metastable states really mean $2S_{1/2}$. There are only assumptions. And, of course, the collision method is rough and not precise to measure this very tiny value. Even the latest optic methods, cannot confirm these states really express the energy difference between $2S_{1/2}$ and $2P_{1/2}$. They just estimate it. Considering Lamb shift is almost same as nuclear hyperfine structure some nuclear or electron's vibrations may influence very tiny data. In this paper we calculate the hydrogen energy levels by solving Schrödinger equation with a modified Coulomb potential by interaction between magnetic moments of nucleus and electron's respectively, as we have proceed to study ferromagnetism [6]. Also, we have used this modified Coulomb potential to evaluate high excitation energy levels of helium [7], deuteron energy states [8], and energy levels of a pionic atom[9]. As we will see below, Lamb shift appears as a natural result for the energy eigenvalues of Schrödinger equation .

2. EFFECTS OF THE INTERACTION BETWEEN THE MAGNETIC MOMENTS ON THE COULOMB'S POTENTIAL

In a previous paper [10] we have found the following expression for the energy of interaction between two electrons via bosons

$$E_I = \frac{-\hbar^2 D^2}{32 m^2 R^2 (\rho_o + \frac{DR}{c^2})^2} \frac{\sum_{k,q,qo} \quad (qq_o)^2}{\omega_q^2 \omega_{qo}^2} \frac{1}{2} |\sum_n \quad e^{iq_o R_n}|^2 \frac{1}{\varepsilon_k - \varepsilon_{k-q} - \omega_q} n_k n_{k-q} (n_q + 1)(n_{qo} + 1) \qquad (1)$$

where D is a coupling constant, m is the mass of an electron, R is the distance between the two electrons, ρ_o is the massive density of the interacting field, DR/c^2 is the "mass less density" of the interacting field, $\omega_q = cq$ is the classical oscillation frequency of the interacting field, ω_{qo} is the oscillation frequency of an electron, $\boldsymbol{q}$ is the wave vector of the interacting field, $\mathbf{q}_o$ is the wave vector of the boson associated with the electron, k is the wave vector of the electron , $\varepsilon_\kappa = \hbar k^2/2m$, n_q is the occupation number of the bosons associated with the magnetic field, n_{qo} is the occupation number of the bosons associated with the electrons, and n_k is the occupation number of the electrons. When the interacting field is a photon field, then $\rho_o = 0$. For a quasi free electron $\varepsilon_k - \varepsilon_{k-q} = 0$, $\omega_{qo} = \hbar q_o^2/2m$. The Coulomb interaction occurs via photons, so that we may assume that the interacting electron oscillate with ω_{qo}. By using that n_q, $n_{qo} = 0$, n_k, $n_{k-q} = 1$, Eq. (1) becomes

$$E_I = \frac{\hbar^3 c^4}{32 m^2 R^4} \frac{\sum_{k,q,qo} \quad (qq_o)^2}{\omega_q^3 \omega_{qo}^2} \frac{1}{2} |\sum_n \quad e^{iq_o R_n}|^2 \qquad (2)$$

$\sum_k \quad 1 = 1$. Further,

$$\frac{\sum_q \quad (qq_o)^2}{\omega_q^3 \omega_{qo}^2} = (\frac{2m}{\hbar})^2 \frac{1}{q_o^2 c^3} \frac{\Omega}{(2\pi)^2} \int_0^{\pi} \quad (\cos(\alpha))^2 \sin(\alpha) d\alpha \int_0^{qo} \quad q dq = (\frac{2m}{\hbar})^2 \frac{R^3}{9\pi c^3}$$

For $q_{o1} = q_{o2} = q_o$ and $\boldsymbol{R_2} - \mathbf{R}_{\boldsymbol{1}} = \boldsymbol{R}$, we have

$$\sum_{qo} \quad |\sum_n \quad e^{iq_o R_n}|^2 = \sum_{qo} \quad 2(1 + cos(\Gamma)) = 2 \sum_{qo} [1 + \cos(q_o R)] = \\ 2 + 2 \frac{\Omega}{(2\pi)^2} \int_0^{0.94\pi/a} \quad q_o^2 dq_o \int_0^{\pi} \quad cos(q_o R cos\theta) sin\theta d\theta = 3.3 \qquad (3)$$

where $\Gamma = \mathbf{q_o R_2} - \mathbf{q_o R_1}$, for $n = 1{,}2$. The interaction energy becomes

$$E_I = 0.00729 \frac{\hbar c}{R} = \alpha \frac{\hbar c}{R} \qquad (4)$$

Taking the upper limit of q_o as $0.94\pi/R$, which is with 6% lower than π/R, one obtains the value of α just as for experimental value. The relation (4) represents the Coulomb's law, which now is obtained without taking into account the electron charge concept. It was shown[10] that for charges of opposite sign the interaction energy (4) has the sign minus.

In presence of a magnetic field in the above equation we introduce and thus we substitute $\boldsymbol{q_o R}$ by

$\boldsymbol{q_o R} - \frac{e}{\hbar c}\oint A dR$ and

$$|\sum_{n=1,2} e^{i q_o R_n}|^2 = 2[1 + cos(\Gamma)]$$
$$\Gamma = \overrightarrow{q_o}R_2 - q_o R_1 - \frac{e}{\hbar c}(\oint A(R_2)dR_2 - \oint A(R_1)dR_1) \qquad (5)$$

We consider the potential vector **A**= ($\boldsymbol{\mu} \times$ **R**)/R^3 where $\boldsymbol{\mu}$ is the magnetic dipole moment and **R** is a vector from the middle of the loop to the observation point. The theory and experiment demonstrate that the free electron has a magnetic moment equal to the Bohr magneton μ_B and a spin momentum **s**, the projection of which on a specified direction are $s_z = \pm\hbar/2 = \hbar m_s$ where $m_s = \pm 1/2$ is the spin quantum number. For $\mu_z^{(s)} = \mu_B g m_s$, with g = 2, we obtain

$$\Gamma = \frac{1}{2}(q_1 + q_2)(R_2 - R_1) + \frac{1}{2}(q_2 - q_1)(R_2 + R_1)$$
$$\frac{-e}{\hbar c^2}\frac{e^2}{4\pi m}\frac{h}{e}\oint 2\frac{m_{s2}\times R_{21}}{R_{21}^3}dR_{21} - \frac{e}{\hbar c^2}\frac{e^2}{4\pi m}\frac{h}{e}\oint 2\frac{m_{s1}\times R_{12}}{R_{12}^3}dR_{12} \qquad (6)$$

where (h/e)m_{s2} and (h/e)$\mathbf{m_{s1}}$ are the flux vectors. For $\mathbf{q_1 = q_2 = q_o}$**,** one obtains

$$\Gamma = q_o R cos\theta - \Gamma_o$$
$$\Gamma_o = \frac{e^2}{mc^2}(\frac{4\pi m_{s2}}{R} - \frac{4\pi m_{s1}}{R}) \qquad (7)$$

We have used the relation $q_o' = q_o - \frac{e^2}{2mc^2}\frac{2m_s}{R^2}x$ where **x** is a unit vector which is perpendicular to **R** and $\boldsymbol{\mu}$. The interaction energy between the two electrons when we take into account their magnetic moments is given by the expression

$$E_C = \frac{\hbar c}{144\pi R}[2 + 1.3\cos(\Gamma_o)] \qquad (8)$$

where Γ_o is given by Eq. (7). For $m_{s1} = m_{s2} = ½$, one obtains $\Gamma_o = 0$ so that Eq. (8) reduces to Eq. (4), that is when the spins of the two electrons are oriented in the same direction there is not a modification of the Coulomb's potential. When $m_{s1} = ½$, $m_{s2} = -1/2$, one obtains $\Gamma_o = 2\pi e^2/mc^2R$, so that for a certain value of R results $\Gamma_o = \pi$, and the interaction energy between the two electrons is reduced by a factor of 0.7/3.3 ~ 1/5.

However, like the electron, the proton has spin angular momentum with $s_p = ½$, and associated with this angular momentum is an intrinsic dipole moment

$$\mu_p = \frac{\gamma_p e}{Mc}s_p$$

where M is the proton mass and γ_p is a numerical factor known experimentally to be 2.7928. The magnetic moment of the electron moving around the proton is

$$\mu_e = \frac{e}{2Mc}(L + 2S)$$

where **L** is the orbital angular momentum and **S** is the spin angular momentum. For the hydrogen atom

$$\Gamma_o = \frac{e}{\hbar c}\frac{\oint (\mu_p - \mu_e)\times r}{r^3}dl = \frac{a}{r} \qquad (9)$$

$$a = \frac{\pi e^2}{mc^2}\left(\frac{2m}{M}\gamma_p s_p - m_l - 2m_s\right)$$

$s_p = \pm 1/2$ is the proton quantum number, $m_s = \pm 1/2$ is the electron spin quantum number and m_l is the magnetic quantum number. In Table I are given the values of parameter *a* for different states of the electron in hydrogen atom.

Table I

The values of the parameter *a* for the hydrogen atom energy states

State	m_l	m_s	s_p	a, 10^{-15}m
$nS_{1/2,1}$	0	½	½	8.83910580399
$nS_{1/2,0}$	0	½	-1/2	8.86601104677
$nP_{1/2}$	1	-1/2	±1/2	0.01345262138974
$nP_{3/2,2}$	1	½	½	17.69166422937
$nP_{3/2,-1}$	1	½	-1/2	17.718569447215
$nD_{3/2,2}$	2	-1/2	½	8.83910680399
$nD_{3/2,1}$	2	-1/2	-1/2	8.86601104677
$nD_{5/2,3}$	2	½	½	26.54422265475
$nD_{5/2,2}$	2	½	-1/2	26.57112789753
$nF_{5/2,3}$	3	-1/2	½	17.69166422937
$nF_{5/2,2}$	3	-1/2	-1/2	17.718569447215
$nF_{7/2,4}$	3	½	½	35.39678392921
$nF_{7/2,3}$	3	½	-1/2	35.42368632291

3 THE ELECTRON ENERGY LEVELS IN THE HYDROGEN ATOM

For the radial wave function $\Psi = R(r)Y_l^m exp(-iEt/\hbar)$, the non-relativistic Schrödinger equation for the hydrogen atom becomes

$$\left(\frac{\hbar^2}{2m}\left(\frac{-d^2}{dr^2} - \frac{2}{r}\frac{d}{dr} + \frac{l(l+1)}{r^2}\right) - V_c(r)\right)R = ER \qquad (10)$$

Now we write $R(r) = r^l\rho(r)$ where $\rho(0) = 0$. Eq. (10) is now

$$(\frac{-\hbar^2}{2m}(\frac{d^2}{dr^2}+\frac{2(l+1)}{r}\frac{d}{dr})-\frac{2\hbar c}{144\pi r}(1+0.650132\cos(\frac{a}{r})))\rho = E\rho \qquad (11)$$

We are interested in the bound state solutions and therefore $\rho(r) \sim e^{-\beta r}$ for $r\to\infty$, so that we try the solution $\rho(r)=f(r)exp(-\beta r[1+0.65013266\cos(a/r)+0.65013266\sin(a/r)])$, Eq. (4) becomes

$$f'' - 2\beta[1+0.65013266cos(a/r)+0.65013266sin(a/r)]f' + \frac{1.30026532\beta a}{r}(-sin(a/r)+cos(a/r))f'$$
$$+0.65013266\frac{\beta a}{r^2}(cos(a/r)-sin(a/r))f + \beta^2[1+0.65013266\cos(a/r)+0.65013266\sin(a/r)]^2 f$$
$$\frac{-1.30026532\beta^a a}{r}\langle cos(a/r)-sin(a/r)\rangle[1+0.65013266cos(a/r)+0.65023266sin(a/r)]f$$
$$+0.65013266^2\beta^2\frac{a^2}{r^2}[-\sin(a/r)+\cos(a/r)]^2 f - 065013266\frac{\beta a^2}{r^3}(-cos(a/r)-sin(a/r))f$$
$$\frac{+2(l+1)}{r}f' - \frac{2(l+1)}{r}\beta[1+0.65013266cos(a/r)+0.65013266sin(a/r)]f$$
$$+1.30026532(l+1)\frac{\beta a}{r^2}(cos(a/r)-sin(a/r))f + \frac{2\hbar c}{144\pi r}\frac{2m}{\hbar^2}[1+0.65013266cos(a/r)]f = \frac{-2m}{\hbar^2}Ef$$

To avoid $f(r)$ to diverge at infinity to overcome the wanted exponential suppression, we require $f(r)$ to be a polynomial in r

$$f(r) = \sum_k \quad c_k r^k \qquad (12)$$

The differential equation then becomes

$$
\begin{aligned}
&\sum_k c_k k(k-1)r^{k-2} - 2\beta[1 + 0.65013266cos(a/r) + 0.65013266sin(a/r)]\sum_k c_k k r^{k-1} \\
&+1.30026532\beta a(cos(a/r) - sin(a/r))\sum_k c_k k r^{k-2} \\
&+\beta^2[1 + 0.65013266\cos(a/r) + 0.65013266\sin(a/r)]^2\sum_k c_k r^k \\
&-1.30026532\beta^2 a(cos(a/r) - sin(a/r))[1 + 0.65013266cos(a/r) + 0.65013266sin(a/r)]\sum_k c_k r^{k-1} \\
&+(0.65013266)^2\beta^2 a^2(\cos(a/r) - \sin(a/r))^2\sum_k c_k r^{k-2} \\
&-1.30026532\beta a^2(-cos(a/r) - sin(a/r))\sum_k c_k r^{k-3} + 2(l+1)\sum_k c_k r^{k-2} \\
&-2(l+1)\beta[1 + 0.650cos(a/r) + 0.65013266sin(a/r)]\sum_k c_k r^{k-1} \\
&+1.30026532(l+1)\beta a(cos(a/r) - sin(a/r))\sum_k c_k r^{k-2} \\
&\frac{+4mc}{144\pi\hbar}[1 + 0.65013266cos(a/r)]\sum_k c_k r^{k-1} + \frac{2m}{\hbar^2}E\sum_k c_k r^k = 0
\end{aligned}
$$

At this stage we assume the constraint condition that the argument of *sine* and *cosine*, $a/r = a/n^2 a_o$, where $n = l+k+1$ is the principal quantum number and a_o is the Bohr radius.

Collecting coefficients of r^{k-1} the above equation gives us the recursion relation

$$
\begin{gathered}
c_{k+1}k(k+1) + c_{k+1}1.30026532\beta a(cos(a/n^2\,a_o) - sin(a/n^2\,a_o))(k+1) \\
+c_{k+1}0.65013266\beta a(cos(a/n^2\,a_o) - sin(a/n^2\,a_o)) + c_{k+!}2(k+1)(l+1) \\
+c_{k+1}0.65013266^2\beta^2 a^2(-\sin(a/n^2\,a_o) + \cos(a/n^2\,a_o))^2 \\
+c_{k+1}\{1.30026532(l+1)\beta a(cos\{a/n^2\,a_o\} - sin(a/n^2\,a_o))\} \\
-c_k\{2\beta[1 + 0.65013266cos(a/n^2\,a_o) + 0.65013266sin(a/n^2\,a_o)]k\} \\
-c_k 1.30026532\beta^2 a(cos(a/n^2\,a_o) - sin(a/n^2\,a_o))[1 + 0.65013266cos(a/n^2\,a_o) + sin(a/n^2\,a_o)] \\
-c_k 2(l+1)\beta[1 + 0.65013266cos(a/n^2\,a_o) + 0.65013266sin(a/n^2\,a_o)] \\
+c_k\{\frac{4mc}{144\pi\hbar}[1 + 0.65013266cos(cos(a/n^2\,a_o)) + 0.65013266sin(a/n^2\,a_o)]\} \\
+c_{k-1}\{\beta^2[1 + 0.65013266cos(a/n^2\,a_o) + 0.65013266sin(a/n^2\,a_o)] + \frac{2m}{\hbar 2}E\} \\
-c_{k+2}\{1.30026532\beta a^2(-cos(a/n^2\,a_o) - sin(a/n^2\,a_o))\} = 0
\end{gathered}
$$

We assume $c_{k+1} = 0$, $c_{k+2} = 0$ and

$$
\begin{gathered}
2\beta(k+l+1) + 1.30026532\beta^2 a(cos(a/n^2\,a_o) - sin(a/n^2\,a_o)) \\
\frac{-4mc}{144\pi\hbar}\frac{1 + 0.65013266\cos(a/n^2\,a_o)}{1 + 0.65013266\cos(a/n^2\,a_o) + 0.65013266\sin(a/n^2\,a_o)} = 0
\end{gathered}
$$

$$E = \frac{-\hbar^2}{2m}\beta^2[1 + 0.65013266\cos(a/n^2\,a_o) + 0.65013266\sin(a/n^2\,a_o)]^2$$

whence

$$\beta = \frac{\sqrt{n^2 + \frac{4mca}{144\pi\hbar} \times 1.30026532(\cos(a/n^2\,a_o) - \sin(a/n^2)a_o)\frac{1 + 0.65013266\cos(a/n^2\,a_o)}{1 + 0.65013266(\cos(a/n^2\,a_o) + \sin(a/n^2\,a}}}{1.30026532a(\cos(a/n^2\,a_o) - \sin(a/n^2\,a_o))}$$

Further,

$$E = \frac{-\hbar^2}{2m}[1 + 0.65013266\cos(a/n^2\,a_o) + 0.65013266\sin(a/n^2\,a_o)]^2$$

$$\frac{[\sqrt{n^2 + \frac{4mca}{144\pi\hbar} \times 1.30026532(\cos(a/n^2 a_o) - \sin(a/n^2 a_o))N} - n]^2}{(1.30026532)^2 a^2(\cos(a/n^2 a_o) - \sin(a/n^2 a_o))^2} \qquad (13)$$

$$N = \frac{1 + 0.65013266\cos(a/n^2 a_o)}{1 + 0.65013266\cos(a/n^2 a_o) + \sin(a/n^2 a_o)}$$

The values of parameter a are given in Table I. For $a \to 0$, one obtains the usual formula

$$E = \frac{-\alpha^2 mc^2}{2n^2} \qquad (14)$$

By using series expansions

$$(1+x)^{1/2} = 1 + \frac{x}{2} - \frac{x^2}{8} + \ldots$$
$$cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$
$$sin(x) = x - \frac{x^3}{3!} + \ldots$$

Eq.(13) reduces to

$$E = \frac{-\alpha^2 mc^2}{2n^2}\left[1 - \frac{1.30026532}{3.30026532}\frac{\alpha mca}{n^3\hbar}\left(1 - \frac{a}{n^2 a_o}\right)\frac{1.65013266}{1.65013266 + 0.65013266\frac{a}{n^2 a_o}}\right]^2 \qquad (15)$$

where $\alpha = 2\times1.65013266/144\pi$. For hydrogen-like atoms, α is replaced by αZ. In Table II are presented the values of the hydrogen energy levels, which are calculated by using Eq. (15).

Table II

Theoretical values of the hydrogen energy levels.

State	- *E* eV	State	-*E* eV
$1S_{1/2,1}$	13.596644180791	$3D_{5/2,2}$	1.510914931481
$1S_{1/2,0}$	13.59663873496	$4S_{1/2,1}$	0.8499003548622
$2P_{1/2}$	3.3996083257396	$4S_{1/2,0}$	0.8499003494741
$2S_{1/2,1}$	3.3995523113018	$4P_{1/2}$	0.8499021000549
$2S_{1/2,0}$	3.399552481535752	$4P_{3/2,2}$	0.8498985045103
$2P_{3/2,2}$	3.399496472329	$4P_{3/2,1}$	0.8498985991906
$2P_{3/3,1}$	3.399496302014	$4D_{3/2,2}$	0.8499003548622
$3P_{1/2}$	1.5109370602835	$4D_{3/2,1}$	0.8499003494741
$3S_{1/2,1}$	1.5109297061228	$4D_{5/2,3}$	0.8498968542270
$3S_{1/2,0}$	1.5109664833516	$4D_{5/2,2}$	0.8498968489076
$3P_{3/2,2}$	1.5109223298343	$4F_{5/2,3}$	0.8498985045103
$3P_{3/2,1}$	1.5109223074163	$4F_{5/2,2}$	0.8498985991906
$3D_{3/2,2}$	1.5109297061228	$4F_{7/2,4}$	0.8498951040425
$3D_{3/2,1}$	1.5109664833516	$4F_{7/2,3}$	0.8498950987239

$3D_{5/2,3}$ 1.5109149538382

We have used the following values of the constants: $m = 9.109389\times10^{-31}$ kg, $c = 2.997925\times10^{8}$ m/s, $\hbar = 1.054572\times10^{-34}$Js, $a_o = 0529177\times10^{-10}$m. With these values of the constants one obtains $E_1 =\alpha^2 mc^2/2 = 13.605703346973$ eV. The obtained results are in a good agreement with experimental data.
For the specific case of the ground state of the hydrogen atom ($n = 1$), the energy separation between the states $1S_{1/2,0}$ and $1S_{1/2,1}$ is 5.6×10^{-6} eV The photon corresponding to the transition between these states has wavelength of 21.1 cm. This is the source of the famous "21 cm line", which is extremely useful to radio astronomers for tracking hydrogen in the interstellar medium of galaxies. The separation between $2P_{3/2}$ and $2P_{1/2}$ is 10^{-4} eV and is generated by the spin-orbit coupling. Lamb shift appears also as a natural result in our model. The difference in energy between two energy levels $2S_{1/2}$ and $2P_{1/2}$ is 5.6×10^{-5} eV, and so on.

4. CONCLUSIONS

We have presented a theory which explain fine and hyperfine structure as well as the Lamb shift for the hydrogen atom. The theory is based on the modification of the Coulomb potential due to the interaction between the magnetic moments of the electron and proton, respectively. Every energy level associated with a particular set of quantum numbers *n, l,* and *j*, is split into two levels of slightly different energy depending on the relative orientation of the proton magnetic dipole with the electron state. The obtained results are in a good agreement with experimental data. For example the separation energy between the two states of the ground state corresponds to the famous wavelength of a photon of 21.1 cm. The energy of the states $nP_{1/2}$ is lower than the energy of the states $nS_{1/2}$ because in the first case the contribution of the interaction between the magnetic moments of the proton and neutron is canceled by the spin-orbit coupling.

REFERENCES

1. Lamb E, Rutherford RC. Fine Structure of the Hydrogen Atomby a Microwave Method Phys. Rev. 1947;72;241-243
2. Greiner W, Reinhardt J, Quantum Electrodynamics, Springer,2003
3. Dolocan V, Dolocan A, Dolocan VO , Quantum Mechanical Treatment of the Lamb Shift without taking into Account the Electric Charge, Nonlinear and Complex Dynamic s, Eds. J A Tenreiro Machado, D Baleanu and C J Luo, pp.150, Springer, 2011
4. Sommerfeld A. Zur Quantentheorie der Spectrallinien. Annalen der Physik 1916;51;125-167.
5. Dirac PAM. The Quantum Theory of the Electron. Proc. Roy. Soc. of London 1928;A117;610-624.
6. Dolocan V, Dolocan VO. Effect of the Spin-Spin Interaction on the Coulomb's Law. Application to Ferromagnetism. World Journal of Condensed Matter Physics, 2012;2;117- 123; revised verssion arXiv: 1206.0537
7..Dolocan V, Evaluation of the Coulomb and Exchange Integrals for Higher Excited States of Helium Atom, taking into Account the Interaction between the Magnetic Moments of the Electrons. arXiv:1304.2988
8. Dolocan V, Dolocan VO, Dolocan A. Interaction between Fermions via Mass less Bosons and Massive Particles. Journal of Physical Science and Applications, 2013;5;332-340; revised version arXiv:1406.1311
9. Dolocan V. Effects of the Interaction between the Magnetic Moments on the Energy Levels of a Pionic Atom, arXiv:1503.09125
10. Dolocan V, Dolocan A, Dolocan VO. Quantum Mechanical Treatment of the Electron-Boson Interaction viewed as a Coupling through Flux Lines.Polarons. Int. J. Mod. Phys. 2010;B24; 479-

495; On the Chemical Bindings in Solids. Rom. J. Phys. **2010;**55; 153172.
11. A. E.Kramida Atomic data and nuclear data tables **96,** 586(2010)