Answers to FINAL EVALUATOR's comments

PART 2:

FINAL EVALUATOR'S comments on revised paper (if any)	**Authors' response to final evaluator's comments**
1) The authors have corrected and completed the main parts of the paper having problems. **2) P.5, point 3): the constant c should be written smaller.** **3) P.10, point 5): please verify and prove the second formula for $\overline{v}(\psi)$.** **4) P. 18, point 1) of the section 3.2: ε_0 should be written smaller.** **5) P. 20: use justify.** **P. 28: the usual way of writing is: "the function $\bar{c}$ is (strictly) decreasing on the interval $(-\infty, 0)$, from the value Φ^{-1} to the value 0".** **6) P. 39, 47-50: please use justify.**	All the FINAL EVALUATOR'S comments have revised (see below Separate List of corrections) All the corrections have been inserted into the manuscript (see the revised manuscript).

LIST of CORRECTIONS

1)The authors have corrected and completed the main parts of the paper having problems.

REVISED

Authors fully answered all the questions of the previous reviewers

2)P.5, point 3): the constant c should be written smaller

REVISED: On P. 5 (point 3) we gave the following answer (see yellow)

3). $\overline{v} = \dfrac{v}{c}$ *is a normalized Lorentzian speed of the light source (dimensionless),* $|\overline{v}| < 1 \Leftrightarrow |v| < c$.

3)P.10, point 5): please verify and prove the second formula for $\overline{v}(\psi)$.

REVISED

The following text is inserted at p. 10-11:

3). The dimensionless parameter $\bar{c}(\psi)=\sqrt{\frac{sF(\psi)}{sF(\psi-2)}}=\frac{\sqrt{[cF(\psi-1)]^2-1}}{|sF(\psi-2)|}$ is called the *normalized Fibonacci speed of light in vacuum.*

4). The parameter $v(\psi)=c(\psi)\bullet\bar{v}(\psi)=c_0\bullet\frac{sF(\psi)}{cF(\psi-1)}\left[\frac{m}{\sec}\right]$ is called the *Fibonacci speed of the light source in vacuum*.

5). The dimensionless parameter $\bar{v}(\psi)=\frac{1}{\bar{c}(\psi)}\bullet\frac{sF(\psi)}{cF(\psi-1)}=\frac{|\psi|}{\psi}\bullet\frac{\sqrt{[cF(\psi-1)]^2-1}}{cF(\psi-1)}$

is called the *normalized Fibonacci speed of the light source in vacuum.*

Proof. From coincidence of the conjugating matrices in (2.3), we get:

$$\frac{1}{\sqrt{1-(\bar{v})^2}}=cF(\psi-1),\quad \bar{c}(\psi)\frac{\bar{v}}{\sqrt{1-(\bar{v})^2}}=sF(\psi).$$

Hence, we have: $\bar{v}=\bar{v}(\psi)$.

To find the explicit form of the dependence $\bar{v}=\bar{v}(\psi)$, we divide the second above-mentioned relation on the first relation. Then we have:

$$\frac{\bar{c}(\psi)\dfrac{\bar{v}(\psi)}{\sqrt{1-[\bar{v}(\psi)]^2}}}{\dfrac{1}{\sqrt{1-[\bar{v}(\psi)]^2}}}=\frac{sF(\psi)}{cF(\psi-1)}.$$

Hence, after the reduction by $\sqrt{1-[\bar{v}(\psi)]^2}$, we get:

$$\bar{v}(\psi)=\frac{1}{\bar{c}(\psi)}\bullet\frac{sF(\psi)}{cF(\psi-1)}.$$

Since

$$\bar{c}(\psi)=\sqrt{\frac{sF(\psi)}{sF(\psi-2)}}=\frac{\sqrt{[cF(\psi-1)]^2-1}}{|sF(\psi-2)|},$$

then

$$\bar{v}(\psi)=\frac{1}{\bar{c}(\psi)}\bullet\frac{sF(\psi)}{cF(\psi-1)}=\sqrt{\frac{sF(\psi-2)}{sF(\psi)}}\bullet\frac{sF(\psi)}{cF(\psi-1)}=$$

$$=\sigma(\psi)\bullet\sqrt{\frac{[sFs(\psi-2)][sF(\psi)]^2}{sFs(\psi)}}\bullet\frac{1}{cF(\psi-1)}==\sigma(\psi)\bullet\sqrt{sF(\psi-2)\bullet sF(\psi)}\bullet\frac{1}{cF(\psi-1)}.$$

Here $\sigma(\psi)$ is the sign $\sigma(\psi)=$ sign $[sF(\psi)]$ of the function:

$$sF(\psi)=\frac{\Phi^{\psi}-\Phi^{-\psi}}{\sqrt{5}}=\frac{2}{\sqrt{5}}sh(\psi\ln\Phi).$$

Hence, it follows that

$$\sigma(\psi)=\frac{|\psi|}{\psi},$$

Furthermore, from the basic relation (2.1)

$$[cF(\psi-1)]^2-sF(\psi-2)sF(\psi)=1$$

we get:

$$sF(\psi-2)sF(\psi)=[cF(\psi-1)]^2-1.$$

But then for $\bar{v}(\psi)$ we have:

$$\bar{v}(\psi)=\frac{1}{\bar{c}(\psi)}\bullet\frac{sF(\psi)}{cF(\psi-1)}=\sigma(\psi)\bullet\sqrt{sF(\psi-2)\bullet sF(\psi)}\bullet\frac{1}{cF(\psi-1)}=\frac{|\psi|}{\psi}\frac{\sqrt{[cF(\psi-1)-1]^2}}{cF(\psi-1)}$$

,

what it is required to prove.

4) P. 18, point 1) of the section 3.2: ε_0 should be written smaller.

REVISED

The following answer is given on p.19:

The numerical value of this dimensionless constant α is:

$$\alpha=\frac{e^2}{2\varepsilon_0 c\bullet\bar{h}}=\frac{1}{137.035999679}=7.2973525376\times10^{-3}, \qquad (3.1)$$

where ε_0 is *electric constant* ; $\hbar$ is *Dirac's constant*; c =const $[\frac{m}{\sec}]$ is the speed of light in vacuum; e is *elementary charge* .

According to CODATA-2014 (http://www.codata.org), the recommended value of the *fine-structure constant* is equal:

$$\alpha = \frac{1}{137.035999139} = 7.2973525664 \times 10^{-3}.$$

Among these problems, the problem of *the fine-structure constant* is included by David Gross into his formulation of the First Physics MILLENNIA PROBLEM [5,6, 24-29].

3.2. The significances of symbols for the fine-structure constant α

1). *Electric constant* (or in other terminology *dielectric permeability of vacuum*)

$$\varepsilon_0 = 8.854187817620 \times 10^{-12} \left[\frac{A^2 \bullet c^4}{m^3 \bullet Kg}\right], \tag{3.2}$$

5)(a) P. 20: use justify.

REVISED

We gave the detailed answer to the question about the comparison of experimental data about changes of the fine structure constant with the age of the Universe with our theoretical model of the Fibonacci special theory of relativity. See inequality (3.30) of the article.

We obtained the following result from our theoretical model:

$$\alpha_{\min} = 0.0072973519973773 62 < \alpha_0 = 0.0072973547331940 72 <$$
$$< \alpha_{\max} = 0.0072973563757885 605$$

Thus, the above inequality holds up to tenth decimal places.

5) (b)P. 28: the usual way of writing is: "the function $\bar{c}$ is (strictly) decreasing on the interval $(-\infty, 0)$, from the value Φ^{-1} to the value 0".

REVISED

On page 29, we have written:

Consequently, the function $\bar{c}(\psi)$ is (strictly) decreasing on the interval $(-\infty,0)$ from the value Φ^{-1} to the value 0.

6) (a)**P. 39: please use justify.**

REVISED

On pages 40-41, we have written:

2). we use as the initial value of the *fine-structure constant* α the following value, given by Kosinov's formula [24]:

$$\alpha = 10^{-\frac{43}{20}} \times \pi^{\frac{1}{260}} \times |\Phi|^{\frac{7}{130}} = 0.0072973519973773 62 \,.$$

Since 2014, the recommended SODATA (http://www.codata.org) value of the fine-structure constant is as follows

$$\alpha = \frac{1}{137.035999139} = 7.2973525664 \times 10^{-3} \quad \text{(dimensionless).}$$

Thus, the absolute error $\Delta\alpha$ between the true and estimated values α is equal to:

$$\Delta\alpha = \left| 7.2973525664 \times 10^{-3} - 7.2973519973 \times 10^{-3} \right| =$$
$$= 5.691 \times 10^{-10} = 0.0000000005691$$

The relative error $\dfrac{\Delta\alpha}{\alpha}$ in this case, equal to:

$$\frac{\Delta\alpha}{\alpha} = \frac{5.691 \times 10^{-10}}{7.2973525376 \times 10^{-3}} = 7.779872 \times 10^{-8} = 0.0000000779872 \,,$$

that is, Kosinov's formula [24] actually coincides with the recommended CODATA value of the fine-structure constant.

6) **(b)47-50: please use justify.**

REVISED

On p. 48, we have written:

We have proved the high coincidence of theoretical data for the value of the fine structure constant α with the experimental data for the *Light Ages* of the Universe (see also Fig. 3.1 and Tables 3.5-3.6).

A substantiation of the coincidence between the theoretical and experimental data for the *Black Hole* and the *Dark Ages* is not possible. Such experimental data in physics and astronomy do not exist yet. However, we have pointed out both theoretical and numerical picture of the change of the fine-structure constant for the *Black Hole*, and for the *Dark Ages* (Fig. 3.1, Tables 3.1-3.2, 3.3-34).