

Original Research Article

Combined Analysis of Safety and Optimal Efficacy in UV-light-activated Corneal Collagen Crosslinking

ABSTRACT

Aims: To analyze the combined factors for safety and optimal efficacy for UV-light-initiated corneal collagen cross-linking (CXL).

Study design: modeling and theory for CXL

Place and Duration of Study: New Vision Inc, Taipei, between Oct. 2015 and July 2016.

Methodology: Analytic formulas are derived based on the coupled dynamic equations for the critical parameters: the safety dose (E^*), minimum corneal thickness and concentration of the riboflavin, and the cornea stiffness increase after the CXL. The critical parameters influencing the efficacy of CXL include: various absorption coefficients, initial concentration (C_0) and diffusion depth (D) of the riboflavin solution, the quantum yield, the UV light intensity (I_0) and dose (E), irradiation duration (t), the cytotoxic energy threshold of endothelial cells(E') and the corneal thickness (z).

Conclusion: The safety dose (E^*) is an increasing function of the parameter set (D , z , C_0) and has a range of 5.3 to 10.1 J/cm² for cytotoxicity threshold 0.63 to 1.26 J/cm². Minimum corneal thickness $z^*=(300, 400)$ um for dose of $E^*=(5.0, 10.1)$ J/cm² which is much higher than the conventional dose 5.4 J/cm² (with $z^*=400$ um). For maximum efficacy, optimal dose is 0.7 to 1.3 J/cm². However to achieve crosslink depth of 220 to 300 um, higher dose of 0.2 to 3.0 J/cm² is recommended.

Keywords: [Modeling; Corneal keratoconus, safety dose, UV-light, corneal collagen crosslinking, CXL, efficacy, stiffness increase]

1. INTRODUCTION

Photopolymerization has been widely used in many applications ranging from chemical engineering to biomedical engineering and biomaterials[1-4]. It has been used for production of catheters, hearing aids, surgical masks, medical filters, and blood analysis sensors². Photopolymers have also been explored for uses in dentistry, drug delivery, tissue engineering and cell encapsulation systems³. Both ultraviolet, visible and infrared lights (360 to 1000 nm) have been used as the photoinitiators for various photosensitizers[3,4].

For ophthalmology applications, corneal collagen cross-linking (CXL) systems have been commercialized for years for human clinical uses [5-10]. Photochemical kinetics of CXL and the biomechanical properties of corneal tissue after CXL are reported[11,12]. However, much less efforts have been invested in basic theoretical studies of photopolymerization¹³⁻²¹, where Lin et al presented the first dynamic modeling for the safety of CXL[17,18]. The safety and efficacy issues of CXL have been explored clinically and theoretically[10, 19-27]. To increase the speed of CXL procedure, they accelerated CXL using high UV power (9 to 45 mW/cm²)[28-34]. In addition, pulsed mode of the UV light was proposed for potential improvement on CXL efficacy[53,36] as well as femtosecond-laser-assisted pocket was proposed[37]. More recently, a corneal topography-guided CXL was commercialized by Avedro based on a pending US patent[38]. The safety of CXL would be significantly improved if the epithelium could be left in situ. Several methods have been reported for this purpose including the use of benzalkonium chloride EDTA, gentamicin, iontophoresis, as well as minimal trauma (through epithelial poke marks) to the epithelium[10]. A sufficient concentration of riboflavin is pre-required and can be achieved by several methods⁵ including: diffusion in the de-epithelialized stroma (standard method); diffusion through the epithelium into the stroma (transepithelial method); or direct introduction of riboflavin into the stroma (pocket technique, ring technique, needle technique); and enrichment of riboflavin in the stroma by iontophoresis[39].

In CXL, both type-I and type-II photochemical reactions occur [11], The photosensitizer riboflavin absorbs the UV energy and is excited to its triplet state ($3Rf^*$). In type-I mechanism, the excited triplet state reacts directly with the collagen proteins and creating substrate free radicals or radical ions of superoxide anion (O_2^-) and OH, respectively, by hydrogen atoms or electron transfer and the sensitizer is consumed during the reaction. In type-II mechanism, the excited sensitizer reacts with oxygen to form reactive singlet oxygen (RSO), $^1O_2$ and regenerates ground state sensitizer. These RSO then further reacts with the collagen covalent bonds between the collagen molecules and proteoglycans to produce additional cross-linked bonds [11]. Unlike type-I process, type II process does not consume sensitizers during the reaction, however type-II mechanism is favored at high oxygen concentrations. CXL can harden the collagen tissue but also damage viable cells when the UV dose is higher than the damage threshold or when the antioxidant defense system is overwhelmed.

It has been reported that oxygen concentration in the cornea is modulated by UV irradiance and temperature and quickly decreases at the beginning of UV light exposure [11]. The oxygen concentration tends to deplete within about 10-15 seconds for irradiance of 3 mW/cm^2 and within about 3-5 seconds for irradiance of 30 mW/cm^2 [11]. By using pulsed UV light of a specific duty cycle, frequency, and irradiance, input from both Type I and Type II photochemical kinetic mechanisms may be optimized to achieve the greatest amount of photochemical efficiency. The rate of reactions may either be increased or decreased by regulating one of the parameters such as the irradiance, the dose, the on/off duty cycle, riboflavin concentration, soak time, and others [11, 34, 37].

This study will demonstrate that the conventionally accepted criterion having a safety dose $E^*=5.4 \text{ J/cm}^2$ and a corneal minimum thickness $z^*>400 \mu\text{m}$ is just one of the special case meeting our safety criteria. Without specifying the parameters of riboflavin (RF) concentration and its diffusion depth, the conventional safety dose (E^*) and minimum corneal depth (z^*) of $(E^*, z^*) = (5.4 \text{ J/cm}^2, 400 \mu\text{m})$ as the safety condition is meaningless. Furthermore, the safety dose E^* is proportional to the cytotoxic threshold of endothelial cells. Therefore, the accurate safety dose relies upon the accurate threshold dose which was estimated (or proposed) to be 0.63 J/cm^2 by prior work based on the measured animal data[25,26]. Moreover, the conventionally accepted UV light absorption in RF solution[22,23] is based only on its initial (at $t=0$) intensity and ignoring the RF dynamic depletion causing the steady-state intensity higher than the initial value. Therefore, the prior work[22,24] underestimates the dynamic light intensity or overestimate the safety dose.

A recent article of Mooren et al [40] evaluated the cytotoxicity threshold for human endothelium with setting from the endothelial side and concluded a much higher value (at least eight times higher) than that of the previously reported animal models (0.63 J/cm^2)[25,26]. Based on their measured human data, the corresponding safety surface dose (based on our calculation) is as high as 27 J/cm^2 . This value is much higher than any of the clinically reported data[21-26]. Therefore further validation of the cytotoxicity threshold for human endothelium is required. Furthermore, it was reported by Lombardo et al[40] that thin cornea with $325 \mu\text{m}$ thickness after the CXL did not shown endothelium damage under a high dose of 5.4 J/cm^2 . These reported data indicated that the conventional safety dose[25,26] 5.4 J/cm^2 and minimum corneal thickness of $400\mu\text{m}$ are underestimated.

This study will demonstrate the new finding that thin human cornea of $300\mu\text{m}$ (much less than the conventional animal model of $400 \mu\text{m}$) is still allowed in CXL-Lasik process for endothelium threshold dose higher than 1.8 J/cm^2 . Our new theory will show that the safety dose (E^*) and the corneal safety thickness (z^*) cannot be set as a constant as that of conventional protocols. Instead, they are variable functions defined by the combined parameter set. This study will use the currently available (or measured) data in the calculations, whereas the unknown parameters will be treated as free parameters within the clinically recognized ranges.

2. MATERIAL AND METHODS

2.1 The Modeling System

As shown in Figure 1, a corneal model consists of its epithelial layer and the underlying stroma collagen. The UV light is normal incident to the corneal surface. A typical CXL protocol is to administer RF on the

corneal surface 5 to 10 times at 2 to 5 minute intervals and wait until the RF solution diffuses into the top layer at approximately 200 to 300 μm . The CXL procedures could be conducted (as shown by Figure1) either with epithelium removed (epi-off) with a 0.1% riboflavin-dextran solution or with epithelium intact (epi-on) with a 0.25% riboflavin aqueous solution. It was known that riboflavin diffusion depth in the epi-on case is normally less than that of epi-off and therefore the epi-on is less efficient[5].

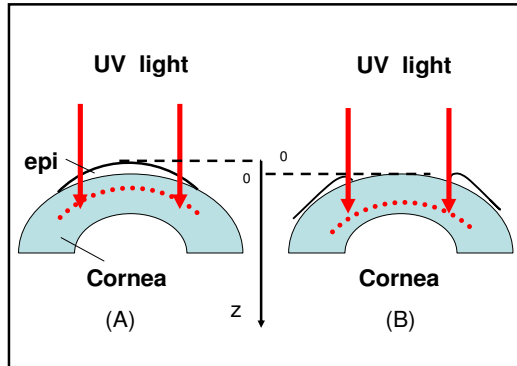


Fig. 1. A corneal model system under UV light cross-linking for the epi-on (A) and epi-off (B) case, where z is along the corneal thickness direction ($z=0$ for the corneal surface); the red dotted curves define the riboflavin diffusion boundaries [18].

In the above described CXL model, the UV light intensity in the corneal stroma is given by a revised time-dependent Lambert-Beer law [18]

$$I(z,t) = I_0 \exp[-A(t)z] \quad (1)$$

where the time-dependent extinction coefficient $A(t)$ shows the dynamic feature of the UV light absorption due to the RF concentration depletion. Without the RF, $A(t)$ becomes a constant given by the absorption coefficient of the corneal stroma tissue reported to be²⁷ $A=2.3Q$, with $Q=13.9$ (1/cm). With the RF in the stroma, the initial (at $t=0$) overall absorption has an extra absorption defined by the extinction coefficient and initial concentration of the RF, i.e., $A(t=0) = 2.3(Q + \epsilon_1 C_0)$, with the reported data [20,28] $\epsilon_1 = 204$ ($\% \cdot \text{cm}$)⁻¹. For $t>0$, $A(t)$ is an increasing function due to the deletion of C_0 in time and defined by both the extinction coefficient of the RF (ϵ_1) and its photolysis product (ϵ_2) which is not yet available for human, but was estimated to be about 80 to 120 ($\% \cdot \text{cm}$)⁻¹, based on measured data in RF solution under UV light irradiation[18].

In the CXL process, the UV light intensity $I(z,t)$ and RF concentration $C(z,t)$ in the corneal stroma may be described by a set of coupled first-order differential equations, or by the integral equation [16-20]

$$I(z,t) = I_0 \exp \left[-2.3 \int_0^z [(\epsilon_1 - \epsilon_2)C(z',t) + \epsilon_2 C_0 F(z') + Q] dz' \right] \quad (2.a)$$

with the time-dependent RF concentration given by,

$$C(z,t) = C_0 F(z) \exp[-aE(z,t)] \quad (2.b)$$

$$E(z,t) = \int_0^t I(z,t') dt' \quad (2.c)$$

Where $a = 83.6 \lambda \phi \epsilon_1$, with ϕ being the quantum yield and λ being the UV light wavelength; ϵ_1 and ϵ_2 are the extinction coefficients of RF and the photolysis product, respectively. I_0 is the initial UV light surface intensity, or $I(z=t=0)=I_0$; and C_0 is the initial RF surface concentration having a distribution profile given by $F(z) = 1 - 0.5z/D$, or $C(z,t=0)=C_0 F(z)$, with a diffusion depth D in the stroma.

The prior work of Schumacher et al[22] based on a non-depleted RF concentration, i.e, the assumption of $aE=0$ in Eq. (2.b), significantly overestimates the RF concentration for $t>0$. Therefore it also

underestimates the UV light intensity $I(z, t)$ in Eq. (2.a). Moreover, the time-dependent of $I(z, t)$ and $C(z, t)$ in their Eq. (5) and (6) is the pre-treatment time, rather than the actual UV light exposure time. That is, after the pre-treatment time (t'), they treated the CXL as a steady process without solving the dynamics of CXL. Their t' corresponding to our initial time ($t=0$). The profiles shown in their Fig. 1 are just the initial (at $t=0$) profiles of our Eq. (2). The prior work of Schumacher et al[22] also used an oversimplified model to assume $\varepsilon_2 = 0$. Therefore, their calculated profiles, Fig. 3 and 4, significantly deviate from our exact numerical profiles to be shown later.

The reported measurements[27,28] provide the parameters of $\varepsilon_1 = 204 (\% \cdot \text{cm})^{-1}$ and $Q = 13.9 (\text{cm}^{-1})$, whereas ε_2 is not yet available in human, but was estimated to be in the range of 80 to 120 $(\% \cdot \text{cm})^{-1}$ by the RF depletion test[17,18] and will be treated as our free parameters. In Eq. (2), the following units are used: $C(z, t)$ in weight percent (%), $I(z, t)$ in (mW/cm^2) , λ in cm, Q in $(\text{cm})^{-1}$ and ε_j (for $j=1,2$) in $(\% \cdot \text{cm})^{-1}$. As shown by Eq. (2) that there are three major UV absorption components in the CXL process: the absorption of the stroma tissue (Q), which is independent to the RF concentration; the absorption of the unreacted RF solution ($\varepsilon_1 C_0$), and the photolysis product ($\varepsilon_2 C_0$), both are proportional to the initial RF concentration C_0 .

The initial UV light intensity (at $t=0$) is obtained by the integration of Eq. (2.a) with $C(z, 0) = C_0 F(z)$ [18,20]

$$I_1(z, 0) = I_0 \exp(-A_1 z) \quad (3.a)$$

and the steady state light intensity is derived by using $C(z, t = \infty) = 0$ in Eq. (2.a),

$$I_2(z, \infty) = I_0 \exp(-A_2 z) \quad (3.b)$$

$$A_j = 2.3[Q + \varepsilon_j C_0 G(z)] \quad (3.c)$$

$$G(z) = [1 - 0.25z / D] \quad (3.d)$$

where A_1 (for $j=1$) is the initial state (at $t=0$) absorption coefficient, independent to ε_2 ; and A_2 (for $j=2$) is the steady-state absorption coefficient, which is independent to ε_1 because of the complete concentration depletion, $C(z, t = \infty) = 0$. We note that $G(z)$ in Eq. (3.c) is the integration of $F(z)$ over z .

2.2 The effective and safety dose

The effective dose (or fluence) applied to the cornea collagen (at a depth z) for a UV exposure time (t) is defined by [20]

$$E(z, t) = \int_0^t I(z, t') dt' \quad (4)$$

The UV light intensity is a dynamic function of time and z and requires a full numerical simulation of Eq. (2). For comprehensive analytic formulas, various numerically fit techniques were presented earlier¹⁸⁻²⁰. In this study we will use the linear approximation of the light intensity $I(z, t)$ given by the mean value of the initial (I_1) and steady intensity (I_2) defined by $I(z, t) = 0.5[I_1(z) + I_2(z)]$ for $t < T$ and $I(z) = I_2(z) + 0.5[I_1(z) - I_2(z)]$ for steady-state, or $t > T$, where T is the steady-state cross-linking time given by $T = m/(aI_0)$, with $m = 12$ fit numerically with the exact solution of Eq. (2).

Time integration of the light intensity, the effective dose (at z) Eq. (4) becomes [20]

$$E_{eff}(z) = E(1 - B') \exp(-A_2 z) \quad (5)$$

where $E = I_0 t$ is the surface dose (at $z=0$), and B' is a correction factor for the transient state given by $B' = 0.5(I_2 - I_1) \exp(zA_2 T/E)$. For $\varepsilon_1 = 204 (\% \cdot \text{cm})^{-1}$, $\varepsilon_2 = 102 (\% \cdot \text{cm})^{-1}$, and $C_0 = 0.1\%$, we obtain an

approximated value of the correction factor $B=(0.97/E_0)[1-\exp(A_1-A_2)z]$, which is about 18% correction to the steady-state when $E=4.0 \text{ J/cm}^2$ and at $z=400 \text{ }\mu\text{m}$.

Given the cytotoxicity threshold dose of the endothelium E' (at a depth z), the normalized surface safety dose (defined on the corneal surface $z=0$) is given by solving the surface dose of Eq. (5) to obtain an E^* -formula [20]

$$E^* = (E' + g) \exp(A_2 z) \quad (6)$$

where the correction factor $g = 0.97[\exp(-A_2 z) - \exp(-A_1 z)]$.

At the referenced point of $z=400 \text{ }\mu\text{m}$, quantum yield $\phi = 0.1$, and $C_0 = 0.1\%$, E^* (at reference point) $= E_0 = 7.62(E' + 0.069)$ for $\epsilon_1 = 204 (\% \cdot \text{cm})^{-1}$, $\epsilon_2 = 102 (\% \cdot \text{cm})^{-1}$ and $Q = 13.9 (\text{cm}^{-1})$. However, for low concentration $C_0 = 0.02\%$, $E_0 = 4.18(E' + 0.0214)$.

For a given E' and E_0 , the normalized safety dose, E^*/E_0 is given as follows.

For $C_0 = 0.1\%$,

$$E^*/E_0 = 0.133(1 + B) \exp(A_2 z) \quad (7.a)$$

$$B = (g - 0.113) / E' \quad (7.b)$$

For $C_0 = 0.02\%$,

$$E^*/E_0 = 0.24(1 + B) \exp(A_2 z) \quad (7.c)$$

$$B = (g - 0.24) / E' \quad (7.d)$$

We note that B is about 0.012 (for $C_0 = 0.1\%$) which is much less than g in eq. (6) or B' in Eq. (5).

Using Eq. (5), we obtain the analytic formula for the RF concentration, from Eq. (2.b)

$$C(z, t) = C_0 F(z) \exp[-aE_0 \exp(-A_2 z) + ag] \quad (8)$$

Equation (7) provides us the formula for the cross-linking time [18,20] defined by when $C(z, t=T^*)/C_0 = 0.018$, or $E_0 \exp(-A_2 z) - g = M/a$, with $M=4$, which leads to the formula for the cross-linking time

$$T^*(z, t) = T_0(1 + ag/M) \exp(A_2 z) \quad (9)$$

T_0 is the surface cross-linking time given by $T_0 = T^*(z=0) = 1000M/(aI_0)$, for I_0 in mW/cm^2 , and $a = 83.6\lambda\phi\epsilon_1 = 6.2(\phi/0.1)$, with ϕ being the quantum yield. For $M=4$, $T_0 = (644/I_0)(0.01/\phi)$. For example, for a dose $I_0 T_0 = 5.4 \text{ J/cm}^2$ and $I_0 = 10 \text{ mW/cm}^2$, we obtain $T_0 = 64.4$ and 32.2 seconds, for quantum yield $\phi = 0.1$, and 0.2 , respectively. Higher quantum yield, the cross-linking time is proportionally reduced. The validation of the above approximated formulas, Eq. (6) and (7), will be shown later by numerical data.

The safety dose (E^*) is defined by the maximum dose (on the cornea surface, $z=0$) without causing the endothelial cells damage (at a depth of $z=400 \text{ }\mu\text{m}$), which is given by the measured dose in the animal model [24-26], $3\text{mW/cm}^2 \times 60 \times 30 \text{ seconds} = 5.4 \text{ J/cm}^2$. We will show later that the UV intensity at the endothelial cells (at a depth of $z=400 \text{ }\mu\text{m}$) estimated as 0.35 mW/cm^2 and the threshold dose $0.35 \text{ mW/cm}^2 \times 60 \times 30 = 0.63 \text{ J/cm}^2$ are under estimated based on the simplified model [22] assuming a constant RF concentration, or $aE=0$ in Eq. (2). For human cornea, the damage threshold dose of the endothelial cells were reported much higher than the animal model [41]

2.3 The minimum thickness and concentration

Solving Eq. (7.a) for $z=z^*$, using a mean value of $G(z=400\text{ }\mu\text{m})=0.8$, and $B \ll 1$ is neglected, we obtain the safety (or minimum) corneal thickness as follows.

For $C_0 = 0.1\%$

$$z^* = [\ln(E^*/E_0) + 2.03] / A_2 \quad (10.a)$$

At $E^*=E_0$ and $C_0=0.1\%$, we obtain $z^*=400 \text{ }\mu\text{m}$, as expected.

Similarly, one may solve Eq. (7.a) for safety (or minimum) concentration

$$C_0^* = [\ln(E^*/E_0) + 2.03 - 32z]/(235Gz) \quad (10.b)$$

For $C_0 = 0.02\%$, we solve Eq. (7.c) to obtain

$$z^* = [\ln(E^*/E_0) + 1.43]/A_2 \quad (11.a)$$

and the safety (or minimum) concentration

$$C_0^* = [\ln(E^*/E_0) + 1.43 - 32z]/(235Gz) \quad (11.b)$$

2.4 The efficacy profiles

The kinetic equation of the rate of polymerization is mainly determined by the rate of reacting monomers given by [6,16,20]

$$\frac{d[M](z,t)}{dt} = R(z,t) = K_0 [\phi C(z,t) I(z,t)]^{1/2} \quad (12)$$

Above equation shows that photoinitiation rate (R) is a product of two competing factors, the RF concentration and the laser intensity. Therefore, an optimal value of $\varepsilon_1 C_0 I_0$ is expected for a maximum photoinitiation rate.

Using the UV light intensity defined by the mean value Eq. (3) and $C(z,t)$ given by Eq. (8), we obtain analytic formula for the photoinitiation rate (R)

$$R(z,t) \propto K_0 \sqrt{\phi I_0 C_0 F(z) H(z)} \exp(-0.5aE) \quad (13.a)$$

$$H(z) = 0.5 [\exp(-A_1 z) + \exp(-A_2 z)] \quad (13.b)$$

where $H(z)$ is given by the mean intensity $H(z) = 0.5(I_1 + I_2)/I_0$, and E is given by an analytic formula, Eq. (5). Taking $dR(z,t)/dC_0 = 0$, $dR(z,t)/d(I_0) = 0$, we obtain the optimal value $C_0^* \propto 1/(\varepsilon_1 + \varepsilon_2)Gz$, and $I_0^* \propto \exp(A_2 z)/(\varepsilon_1 \phi t)$. Moreover, taking $dR(z,t)/dz = 0$, we also find an optimal z^* resulted from the distribution function $F(z)$ for which is a decreasing function of z (for small $D < 0.5$ cm), whereas there is no optimal for the case of uniform distribution with diffusion depth $D \gg 1$ cm, or when $F(z) = 1.0$.

Integration of Eq. (13) over time (t), and using Eq. (5) for E , we obtain the profile (z-dependence) of the

$$\text{increase in corneal stiffness (S')} \quad S'(z,t) \propto \frac{2K_0}{1-B'} \sqrt{\frac{C_0 F H'}{\phi I_0}} [1 - \exp[-0.5(atI_0) \exp(-A_2 z)]] \quad (14)$$

The efficacy of CXL may be defined by the increase in corneal stiffness (S) after CXL given by the total amount of induced crosslink.

Integration of Eq. (14) over z , and taking averaged over the CXL depth (z), we obtain the normalized increase in stiffness (S)

$$S(z,t)/K_0 \propto \frac{1}{zR_0} \int_0^z S'(z',t) dz' \quad \dots\dots\dots (15)$$

2.5 The scaling laws

From E. (14), for small $(atI_0) \ll 1$, the transient state of $S'(z,t)$ is proportional to (atI_0) , we obtain the scaling law

$$S' \propto (I_0 t) \sqrt{\phi C_0 / I_0} \propto t \sqrt{\phi C_0 I_0} \quad (16.a)$$

which shows that the transient state of S is linearly proportional to the exposure time (t) for a given UV intensity (I_0), and is proportional to the square root of I_0 for a given t. However, the steady-state, when $(atI_0) \gg 1$, $\exp(-0.5atI_0) = 0$ in Eq. (15), has a different scaling law given by

$$S' \propto \sqrt{C_0 I(\phi I_0)} \quad (16.b)$$

The above steady state is proportional to $I_0^{-0.5}$, comparing to the transient state S is proportional to $I_0^{0.5}$. The opposite trend is due to the time integration over time (t) which causes the dependence of S' as $I_0^{-0.5}$. These features will be reconfirmed by our numerical data later.

The above scaling laws based on the approximated Eq. (13) will be used to analyze the exact numerical results be shown elsewhere. The optimal C_0^* and I_0^* and the scaling laws for $S(z,t)$ may be mathematically derived similar to that of $S'(z,t)$. However, analytic formulas are not available.

3. RESULTS AND DISCUSSION

Numerical calculations of the factors influencing the CXL efficacy based on Eq. (2) and (15) will be presented elsewhere. This paper will focus on the results based on the developed analytic equations (6), (7), (10), (11), (14) and (16).

3.1 The effective dose

The conventional CXL procedures are based on the Dresden protocol which requires a surface safety dose of 5.4 J/cm² and a minimum corneal thickness of 400 μm. During the UV exposure, riboflavin drops were applied every few minutes for saturated riboflavin concentration in the stroma and extra protection of the corneal endothelial cells. However, the effective dose of CXL is reduced by the extra absorption of this surface layer. For maximum efficacy, the wasted-dose can be avoided by washing out the extra surface riboflavin layer after its sufficient diffusion into the stroma and prior to the UV light exposure. For RF surface layer (with thickness d), we calculate the reduction factor, $Re=(0.8, 0.64)$, for $d= (100, 200)$ μm and $C_0=0.1\%$, which is further decrease to $Re = (0.64, 0.41)$, for $C_0=0.2\%$. At steady state, or when the RF layer is largely depleted, the UV light surface intensity (I_0) is reduced to $I_0 \exp(-2.3 \mathcal{E}_2 C_0 d)$, at $z=d$, as shown by Eq. (3.b). The effective dose is given by Eq. (5) with $Q=0$, and at $z=d$. For example, the conventional dose 5.4 (J/cm²) is reduced to an effective dose of $5.4 \times 0.8 = 4.3$ (J/cm²), that is 20% of the dose, or 1.08 (J/cm²) dose is wasted in the B2 surface layer having a thickness of 100 μm and concentration of 0.1%. The effective dose is further reduced to $5.4 \times 0.64 = 3.45$ (J/cm²) for a thicker RF layer of 200 μm. In the current Dresden protocol (using a 5.4 dose J/cm²), we estimate the effective dose has a range of 3.5 to 4.5 J/cm², depending on the frequency of RF installation during the UV exposure.

The simplified modeling of prior art²² assuming a constant RF concentration, using $A=51$ (1/cm) in Eq. (2), underestimates the dynamic light intensity and the effective dose. For example at $z=400$ μm, and $E=4.0$ J/cm², Eq. (5) leads to $E_{eff}= 0.81$ J/cm² (with $A_2=34.8$ (1/cm)), which is much higher than that of the simplified model 0.52 J/cm² based on $A_2=51$ (1/cm). It also underestimate the safety

3.2 The safety dose

As shown by Eq. (6), the safety dose $E(z,t)$ is an exponential increasing function of D , C_0 , $\mathcal{E}_2$, Q and the corneal thickness (z), as shown by Eq. (). In a recent article by Mooren et al.,⁴⁰ they evaluated the cytotoxicity threshold for human endothelium (E') and concluded a much higher value (at least 5.4 J/cm²) than that of the previously reported animal models (0.63 J/cm²). Using $E'=5.4$ J/cm² the corresponding safety surface dose, based on Eq. (6), gives $5.4 \times 6.3 = 27$ J/cm² for $C_0=0.04\%$. This value seems to be too high clinically. Therefore, the actual cytotoxic threshold of endothelial cells in human requires further studies.

In the following we will discuss the normalized safety dose defined by E^*/E_0 based on Eq. (7), for the case that cytotoxic threshold of endothelium is known at the referenced point $z=400$ μm, $C_0=0.1\%$ (or 0.02%) $D=500$ μm and quantum yield $\phi=0.1$, with known parameters of $\mathcal{E}_1 = 204$ (%·cm)⁻¹ and $\mathcal{E}_2=102$

(%·cm)⁻¹ and $Q = 13.9 \text{ (cm}^{-1}\text{)}$. The reference dose in Eq. (7), for high concentration $C_0 = 0.1\%$, E^* (at reference point) = $E_0 = 7.62(E' + 0.069) = 5.3$ and 10.1 J/cm^2 , for threshold dose of the endothelial cells $E' = 0.63$ and 1.26 J/cm^2 , respectively. However, for low concentration $C_0 = 0.02\%$, $E_0 = 4.18(E' + 0.0214)$ which reduce to 2.72 and 5.36 J/cm^2 .

Fig. 2 shows the safety dose (E^*) and the normalized dose E^*/E_0 for $C_0 = 0.1\%$, $D = 500 \text{ }\mu\text{m}$ quantum yield $\phi = 0.1$, and various cytotoxic threshold $E' = (0.63, 1.26, 1.9) \text{ J/cm}^2$. The left figure shows E^* is an increasing function of E' (and E_0), whereas the normalized dose is almost independent to the E' , that is the 3 curves of the left figure converged to one “universal” curve shown in the right figure. This universal feature may be seen by Eq. (7.a), in which the contribution from B is very small such that E^*/E_0 is almost independent to E' . As shown by Fig. 2, for a corneal thickness of $400 \text{ }\mu\text{m}$ and $C_0 = 0.1\%$, $D = 500 \text{ }\mu\text{m}$, the safety dose has a range of 5.3 to 10.1 J/cm^2 depending on the cytotoxicity threshold range of 0.63 to 1.26 J/cm^2 . The safety dose based on our theory is much higher than the conventionally quoted value of 5.4 J/cm^2 based on animal model having $E' = 0.63 \text{ J/cm}^2$. Fig. 2 shows that for $E^* = (0.65, 1.0, 1.3)E_0$, the minimum corneal thickness is $z^* = (300, 400, 450) \text{ }\mu\text{m}$. For examples, for $C_0 = 0.1\%$ and $D = 500 \text{ }\mu\text{m}$, $E^* = 10.1 \text{ J/cm}^2$ (for $E' = 0.63 \text{ J/cm}^2$), $z^* = (300, 400) \text{ }\mu\text{m}$, for dose of $E^* = (5.0, 10.1) \text{ J/cm}^2$.

Fig. 3 shows the normalized safety dose (E^*/E_0) versus C_0 for $D = 500 \text{ }\mu\text{m}$ at various corneal thickness $z = (350, 400, 450) \text{ }\mu\text{m}$, Fig. 4 shows E^*/E_0 versus z , for $C_0 = 0.1\%$ at various diffusion depth $D = (200, 300, 500) \text{ }\mu\text{m}$. These data show that E^*/E_0 is a nonlinear increasing function of C_0 , D , and z , as also shown by Eq. (3.c) and (7).

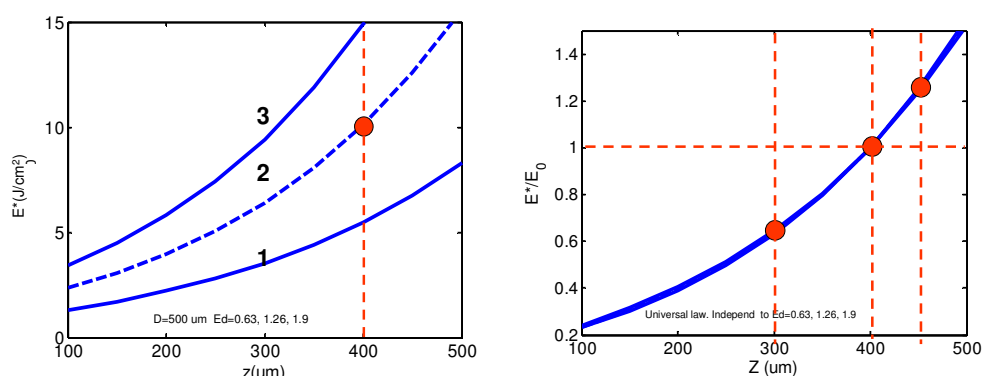


Fig. 2. The safety dose (E^* , left figure) and normalized safety dose (E^*/E_0 , in right figure) versus z for $C_0 = 0.1\%$, $D = 500 \text{ }\mu\text{m}$, quantum yield $\phi = 0.1$ and $E' = (0.63, 1.26, 1.9) \text{ J/cm}^2$, for curves (1,2,3) in left figure, where the red dot represents $E^* = E_0$ at the reference point ($z = 400 \text{ }\mu\text{m}$ and $E' = 1.26 \text{ J/cm}^2$).

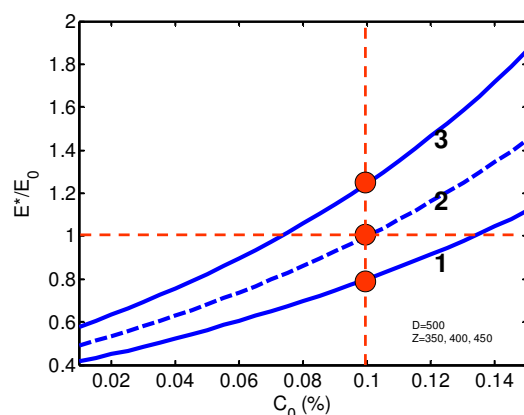


Fig. 3 The normalized safety dose (E^*/E_0) versus C_0 for $D = 500 \text{ }\mu\text{m}$ and $z = (350, 400, 450) \text{ }\mu\text{m}$, for curve (1,2,3).

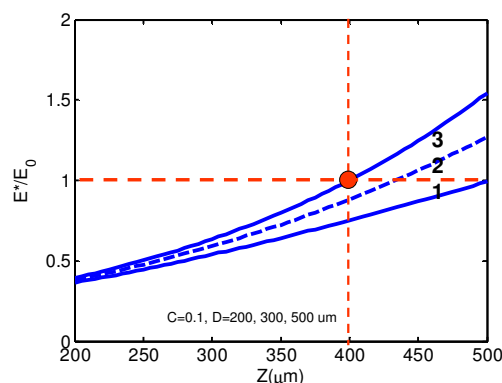


Fig. 4 The normalized safety dose (E^*/E_0) versus z , for $C_0=0.1\%$ and $D=(200, 300, 500)$ μm , for curve (1,2,3).

3.3 The minimum corneal thickness

Fig. 5 shows that for a given safety dose $E^*=E_0$, the safety (minimum) corneal thickness (z^*) is a decreasing function of the RF concentration but is an increasing function of the diffusion depth D as also shown by Eq. (10). For $C_0=0.1\%$, $E_0=(5.3, 10.1)$ J/cm^2 , for cytotoxicity threshold $E'=(0.63, 1.26)$ J/cm^2 . Therefore $z^*=(400, 425)$ μm , for $D=(500, 300)$ μm and $E^*=5.3$ J/cm^2 (for $E'=0.63$ J/cm^2), or $E^*=10.1$ J/cm^2 (for $E'=1.26$ J/cm^2). Therefore the conventional dose 5.4 J/cm^2 is safe for a corneal thickness of 400 μm , but requires $D>500$ μm and $C_0=0.1\%$ when $E'=0.63$ J/cm^2 . However, if $E'=1.26$ J/cm^2 , then the safety dose is much higher, $E^*=10.1$ J/cm^2 .

Fig. 6 shows z^* versus C_0 for $D=500$ μm showing that z^* is an increasing function E^*/E_0 . These calculated data show that small D and C_0 require large corneal thickness to protect the endothelium cells. As shown by Fig. 6, the safety corneal thickness $z^*=(250, 400, 490)$ μm , for $E^*/E_0=(0.5, 1.0, 1.5)$. Therefore the conventional dose $E^*=5.4$ J/cm^2 is safe even in thin corneas of 250 μm if $E_0=10.8$ J/cm^2 or when $E'=1.35$ J/cm^2 , calculated from $E_0=7.62(E'+0.069)$, when $C_0=0.1\%$ and $D=500$ μm .

The above examples demonstrate that thin human cornea of 300 μm (much less than the conventional animal model of 400 μm) is still allowed in CXL-Lasik process as far as the endothelium threshold dose is higher than 1.26 J/cm^2 which was reported recently [40,41].

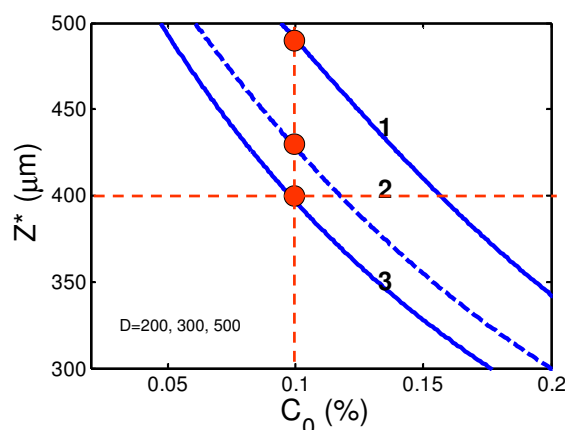


Fig. 5 The safety corneal thickness (z^*) versus concentration for $E^*/E_0=1.0$ and $D=(200, 300, 500)$ μm for curve (1,2,3).

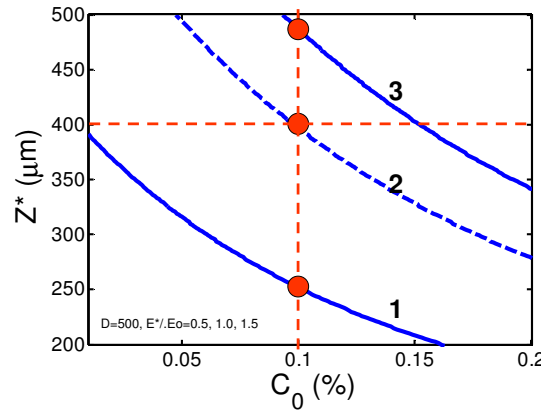


Fig. 6. Same as Fig. 5, but for $D=500 \mu\text{m}$ and $E^*/E_0=(0.5, 1.0, 1.5)$ for curve (1,2,3).

3.4 The stiffness profiles and crosslink depth

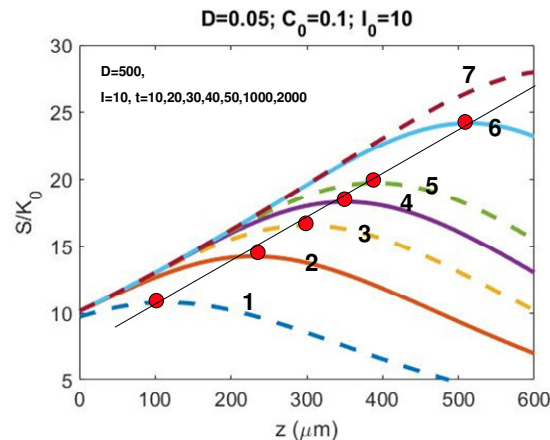
Detailed computer simulations for the influence of each of the parameters of $[C_0, D, I_0, t, \phi]$ in the increase of stiffness defined by Eq. (14) and (15) will be shown elsewhere. This paper is focusing on the features summarized from the numerical simulations, besides typical examples of the stiffness profiles. Fig. 7 shows the typical profiles (z -dependence) of the increase in corneal stiffness (S') for a fixed UV light intensity $I_0=10 \text{ W/cm}^2$, defined by Eq. (14). Each profile has a maximum value S^* (at the crosslink depth Z^*), where both S^* and Z^* are increasing function of the UV exposure time (t), or dose (for a given light intensity).

Fig. 8 shows S' versus z for a fixed exposure time $t=100 \text{ s}$, but for various UV light intensity, where S^* is a decreasing function of the UV light intensity (for a given exposure time), whereas Z^* is an increasing function of I_0 (or dose $E_0=I_0 t$) which is shown in Fig. 9. By more simulation profiles for various D and C_0 (not shown here), we found Z^* versus C_0 and D in Fig.10 and 11. Combining the curves shown by Figs. 9 to 11, we are able to figure out the following approximate analytic equations for S^* and Z^* for the linear regime (for dose between 2.0 and 3.0 J/cm^2 as shown in Fig. 8).

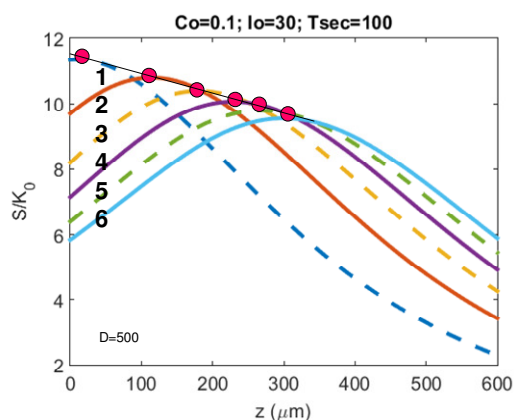
$$S^*=10-0.5(E-2.0)+62(C_0-0.1)+0.005(D-500) \quad (17.a)$$

$$Z^*=229+71(E-2.0)-880(C_0-0.1)+0.21(D-500) \quad (17.b)$$

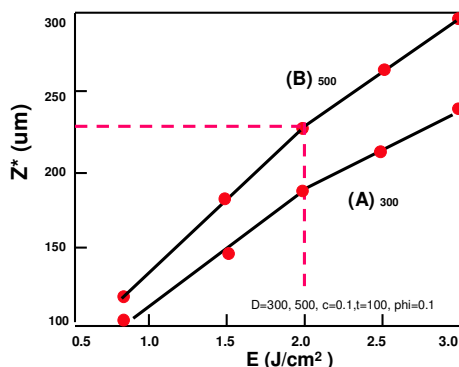
where units are S^* in K_0 , E in J/cm^2 , C_0 in %, Z^* and D in μm . Eq. (17) provides the important feature that S^* is an increasing function of C_0 and D , but it is a decrease function of E . In comparison, Z^* is an increasing function of E and D , but it is a decrease function of C_0 .



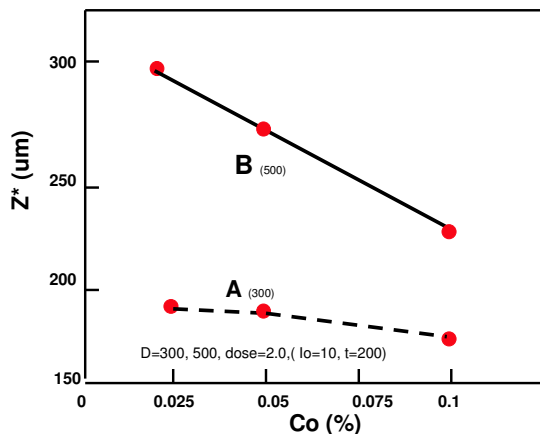
393 **Fig. 7** The profile (z-dependence) of the increase in corneal stiffness (S') for $C_0=0.1\%$, $D=500\mu\text{m}$,
 394 quantum yield $\phi=0.1$ and fixed UV light intensity $I_0=10\text{ W/cm}^2$ for various exposure time $t=(10, 20,$
 395 $30,40,50, 100, 200)$ s, shown by curves (1,2,3,4,5,6,7).



396 **Fig. 8** Same as Fig. 7 but for a fixed exposure time $t=100$ s, and various UV light intensity $I_0=$
 397 $(5,10,15,20,25,30)$ mW/cm² shown by curves (1,2,3,4,5,6,7).
 398



399 **Fig. 9** Crosslink depth (Z^*) versus UV light dose (E) for $D=300\text{ }\mu\text{m}$ (curve (A) and $D=500\text{ }\mu\text{m}$ (curve
 400 (B) for $C_0=0.1\%$ and a fixed exposure time $t=100\text{s}$, for various UV light intensity $I_0=5$ to 30
 401 mW/cm², or dose $E=0.5$ to 3.0 J/cm^2 .
 402
 403



404 **Fig.10** Same as Fig. 9 but for Z^* versus C_0 for UV light intensity of $I_0=10\text{ mW/cm}^2$, and exposure time
 405 $t=200\text{s}$ (or dose of 2.0 J/cm^2)..
 406
 407

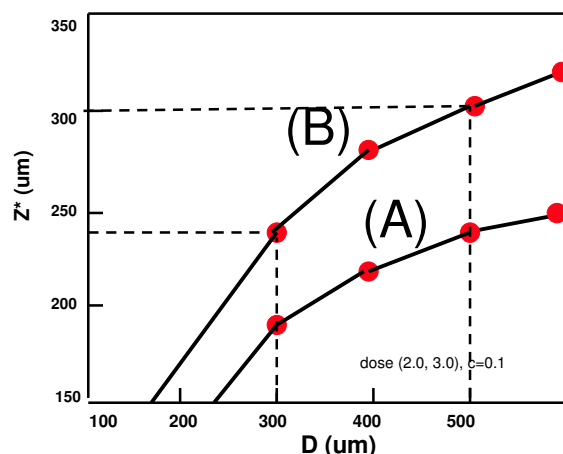


Fig. 11 Cross linking depth (Z^*) versus diffusion depth (D) for dose of 2.0 J/cm^2 , curve (A) and 3.0 J/cm^2 , curve (B), $C_0=0.1\%$ and quantum yield $\phi=0.1$

3.5 The optimal efficacy

CXL efficacy may be described by the normalized increase in stiffness (S/K_0) as shown by Eq. (15). Numerical simulation shows S/K_0 versus dose (E_0) for $C_0 = (0.05, 0.1)\%$ and $D=(200, 300, 500) \text{ um}$, where optimal dose of $E_0^*=(0.7, 1.2, 1.3) \text{ J/cm}^2$ for $D=(200, 300, 500) \text{ um}$ and $C_0 = 0.1\%$. The increase in stiffness (S/K_0) decreases about 10% to 18% from its maximum values for dose of 2.0 to 3.0 J/cm^2 which are higher than the optimal value of 0.7 to 1.3 J/cm^2 .

As shown in Fig. 9, the crosslink depth is only $Z^*=(120 \text{ to } 170) \text{ um}$ at the optimal dose $E_0^*=(0.7 \text{ to } 1.3) \text{ J/cm}^2$. To achieve Z^* in the range of 220 to 300 um , one requires higher dose of $E_0=0.2$ to 3.0 J/cm^2 despite the drop of 10% to 18% from its optimal value.

Numerical simulations to confirm the scaling laws shown by Eq. (16) will be presented elsewhere.

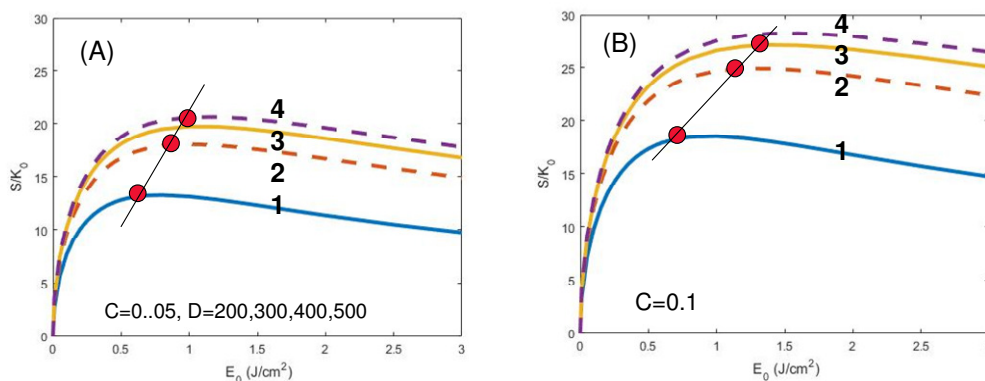


Fig. 12 Normalized maximum stiffness increase (S^*/K_0) versus dose (E_0) for $C_0 = (0.05, 0.1)\%$ (Fig. A,B) and $D=(200, 300, 500) \text{ um}$, for curves (1,2,3,4). The red dots show the optimal dose E_0^* for various D .

4. CONCLUSION

We have presented, for the first time, analytic formulas for the safety dose, minimum corneal thickness and RF concentration characterized by the extinction coefficient (A), concentration and the diffusion depth of the riboflavin, the UV light intensity, dose, irradiation duration (t), and the corneal thickness (z).

The safety dose (E^*) is an increasing function of the parameter set (D , z , C_0) and has a range of 5.3 to 10.1 J/cm² for cytotoxicity threshold 0.63 to 1.26 J/cm². Minimum corneal thickness z^* =(300, 400) μ m for safety dose of E^* =(5.0, 10.1) J/cm² which is much higher than the conventional dose 5.4 J/cm² (with z^* =400 μ m). For maximum efficacy, optimal dose is 0.7 to 1.3 J/cm². However to achieve crosslink depth of 220 to 300 μ m, higher dose of 0.2 to 3.0 J/cm² is recommended.

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