

BOUNDARY INTEGRAL EQUATION MODELING OF 2D FUNCTIONALLY GRADED ANISOTROPIC STRUCTURES

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ABSTRACT

Aims: The main purpose of this paper is to present a boundary integral equation modeling to obtain the solution for the problems of magneto-thermoelastic functionally graded anisotropic (FGA) structures in the context of the Green and Naghdi theory of type III.

Study design: Original research paper

Place and Duration of Study: Jamoum laboratory, June 2017

Methodology: a new practical shape optimization technique based on a predictor-corrector implicit-implicit staggered algorithm, which was implemented for use with the time-stepping dual reciprocity boundary element method (DRBEM) to obtain the final coupled linear system of equations for temperature and displacements that describe the magneto-thermoelastic structural analysis problem of functionally graded anisotropic (FGA) structures in the context of the Green and Naghdi theory of type III.

Results: A numerical results demonstrate validity, accuracy, and efficiency of the presented technique.

Conclusion: Our results thus confirm that our technique is efficient and precise. It is noted that the obtained DRBEM results are more accurate than the FEM results, the DRBEM is more efficient and easy to use than FEM because it only needs the boundary of the domain needs to be discretized. The results of DRBEM at any point are more precise, because DRBEM's integration operation is smoother than FEM's differentiation operation. Also from this knowledge of the variation of the displacements and temperature sensitivities with time for multilayered FGA structures, we can design various structures to meet specific engineering requirements and utilize within which to place new information can be more effective. the presented technique can be applied to a wide range of

Keywords: *Predictor-Corrector; Shape optimization; Magneto-thermoelasticity; Functionally graded Anisotropic structures; Dual Reciprocity Boundary Element Method.*

2010 mathematics subject classification: 65M38 - 65K05 - 74B05 - 74E05 - 74F05 - 74H05 - 74H15 - 74S20 - 90C31.

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1. Introduction

An understanding of behaviour of functionally graded anisotropic magneto-thermoelastic materials has great practical applications in applied sciences and engineering. In recent years, many researchers have studied the overall behavior of such materials. With the new advances in computer hardware and software, it is now possible to solve complex magneto-thermoelastic problems. One of the most commonly used techniques for converting the domain integral into a boundary is the so-called dual reciprocity boundary element method (DRBEM). This method was initially developed by Nardini and Brebbia [1]. The interested readers can find more details in the following references [2-6].

The main objective of this article is to present a numerical technique based on a predictor-corrector implicit-implicit staggered algorithm, which was implemented for use with the time-stepping dual reciprocity boundary element method (DRBEM) to obtain the final coupled linear system of equations for temperature and displacements that describe the magneto-thermoelastic structural analysis problem of multilayered FGA structures in the context of the Green and Naghdi theory of type III. The numerical boundary element results obtained are presented in tables and very good agreement is obtained with the finite element results. The tabulated results show our technique is efficient and precise.

2. Formulation of the problem

Consider a functionally graded anisotropic plate placed in a primary magnetic field H_0 acting in the direction of the y -axis, and occupies the region $R = \{(x, y, z): 0 < x < h, 0 < y < b, 0 < z < a\}$ which is bounded by a simple closed curve C . At each and every point on the boundary C , the temperature and displacement are suitably specified. According to Green and Naghdi theory of type III, the governing equations of generalized magneto-thermoelasticity in an initially stressed functionally graded multilayered anisotropic plates for the i th layer, can be written in the following form [7-10]:

$$\sigma_{ab,b} + \tau_{ab,b} = \rho^i(x+1)^m \ddot{u}_a^i \quad (1)$$

$$\sigma_{ab} = (x+1)^m [C_{abfg}^i u_{f,g}^i - \beta_{ab}^i (T^i - T_0 + \tau_1 \dot{T}^i)] \quad (2)$$

$$\tau_{ab} = \mu^i(x+1)^m (\tilde{h}_a H_b + \tilde{h}_b H_a - \delta_{ba} (\tilde{h}_f H_f)) \quad (3)$$

$$\left[\left(k_{ab}^{i*} + k_{ab}^i \frac{\partial}{\partial \tau} \right) T_{,ab}^i + \rho^i \dot{\mathfrak{X}} = \beta_{ab}^i T_0 \ddot{u}_{a,b}^i + \rho^i c^i (x+1)^m \ddot{T}^i \right] \quad (4)$$

where σ_{ab} , τ_{ab} , u_k^i and T^i are the mechanical stress tensor, electromagnetic stress tensor, displacement and temperature, respectively, T_0 , C_{abfg}^i , β_{ab}^i , μ^i , $\tilde{h}$, k_{ab}^{i*} and k_{ab}^{i*} are the reference temperature, constant elastic moduli, stress-temperature coefficients, magnetic permeability, perturbed magnetic field, thermal conductivity coefficients and second order tensor associated with the GN theories, respectively, ρ^i is the density, c^i is the specific heat capacity, τ is the time, τ_1 is the relaxation times, $\mathfrak{X}$ is the heat source, ; $i = 1, 2, \dots, n-1$. represents the parameters in multilayered plate, respectively, and $f(x)$ is a given nondimensional function of space variable x . We take $f(x) = (x+1)^m$, where m is a dimensionless constant.

3. DRBEM implementation

Using the same technique of Fahmy [11-13] for the current problem and implementing the DRBEM, we can write the boundary integral representation formula of coupled thermoelasticity as follows

$$U_D^i(\xi) = \int_C (U_{DA}^{i*} T_A^i - \tilde{T}_{DA}^{i*} U_A^i) dC + \sum_{q=1}^E \left(U_{DE}^{iq}(\xi) + \int_C (T_{DA}^{i*} U_{AE}^{iq} - U_{DA}^{i*} T_{AE}^{iq}) dC \right) \alpha_E^q \quad (5)$$

According to Fahmy [14], the dual reciprocity boundary integral equation (5) can be expressed as

$$\zeta U - \eta T = (\zeta \tilde{U} - \eta \tilde{\varphi}) \alpha \quad (6)$$

66 An implicit-implicit staggered algorithm for the integration of the governing equations was
 67 implemented for use with the DRBEM to solve the governing equations which can be written using
 68 (6) as follows

$$\tilde{M} \ddot{U}^i + \tilde{\Gamma} \dot{U}^i + \tilde{K} U^i = \tilde{\Phi}^i \quad (7)$$

$$\tilde{X} \dot{T}^i + \tilde{A} \dot{T}^i + \tilde{B} T^i = \tilde{Z} \dot{U}^i + \tilde{R} \quad (8)$$

69 where the matrices in (7) and (8) are as follows

$$\begin{aligned} V &= (\eta \ddot{\psi} - \zeta \ddot{U}) J^{-1}, & \tilde{M} &= V(\bar{\Phi} + \delta_{AF}), & \tilde{\Gamma} &= V \bar{\Gamma}_{AF}, \\ \tilde{K} &= -\zeta + V(\mathcal{J} J^{-1} + \bar{\psi}), & \tilde{\Phi}^i &= -\eta \dot{T}^i + V \dot{S}^0, & \tilde{X} &= -\rho^i c^i (x+1)^m, \\ \tilde{A} &= k_{ab}^i \frac{\partial}{\partial x_a} \frac{\partial}{\partial x_b}, & \tilde{B} &= k_{ab}^{i*} \frac{\partial}{\partial x_a} \frac{\partial}{\partial x_b}, & \tilde{Z} &= \beta_{ab}^i T_0, \\ \tilde{R} &= -\rho^i \dot{\mathcal{X}}. \end{aligned}$$

70 where $\ddot{U}^i, \dot{U}^i, U^i, T^i$ and $\tilde{\Phi}^i$ represent the acceleration, velocity, displacement, temperature and
 71 external force vectors, respectively.
 72 Equations (7) and (8) lead to the following coupled system of differential-algebraic equations (DAEs)
 73 [15]:

$$\tilde{M} \ddot{U}_{n+1}^i + \tilde{\Gamma} \dot{U}_{n+1}^i + \tilde{K} U_{n+1}^i = \tilde{\Phi}_{n+1}^{ip} \quad (9)$$

$$\tilde{X} \dot{T}_{n+1}^i + \tilde{A} \dot{T}_{n+1}^i + \tilde{B} T_{n+1}^i = \tilde{Z} \dot{U}_{n+1}^i + \tilde{R} \quad (10)$$

74 where $\tilde{\Phi}_{n+1}^{ip} = \eta T_{n+1}^{ip} + V \dot{S}^0$ and T_{n+1}^{ip} is the predicted temperature.

75 Integrating Eq. (7) with the use of trapezoidal rule and Eq. (9), we obtain

$$\begin{aligned} \dot{U}_{n+1}^i &= \dot{U}_n^i + \frac{\Delta\tau}{2} (\ddot{U}_{n+1}^i + \ddot{U}_n^i) \\ &= \dot{U}_n^i + \frac{\Delta\tau}{2} \left[\ddot{U}_n^i + \tilde{M}^{-1} \left(\tilde{\Phi}_{n+1}^{ip} - \tilde{\Gamma} \dot{U}_{n+1}^i - \tilde{K} U_{n+1}^i \right) \right] \end{aligned} \quad (11)$$

$$\begin{aligned} U_{n+1}^i &= U_n^i + \frac{\Delta\tau}{2} (\dot{U}_{n+1}^i + \dot{U}_n^i) \\ &= U_n^i + \Delta\tau \dot{U}_n^i \\ &\quad + \frac{\Delta\tau^2}{4} \left[\ddot{U}_n^i + \tilde{M}^{-1} \left(\tilde{\Phi}_{n+1}^{ip} - \tilde{\Gamma} \dot{U}_{n+1}^i - \tilde{K} U_{n+1}^i \right) \right] \end{aligned} \quad (12)$$

76 From Eq. (11) we have

$$\dot{U}_{n+1}^i = \bar{Y}^{-1} \left[\dot{U}_n^i + \frac{\Delta\tau}{2} \left[\ddot{U}_n^i + \tilde{M}^{-1} \left(\tilde{\Phi}_{n+1}^{ip} - \tilde{K} U_{n+1}^i \right) \right] \right] \quad (13)$$

77 where $\bar{Y} = \left(I + \frac{\Delta\tau}{2} \tilde{M}^{-1} \tilde{\Gamma} \right)$

78 Substituting from Eq. (13) into Eq. (12), we derive

$$\begin{aligned} U_{n+1}^i &= U_n^i + \Delta\tau \dot{U}_n^i + \frac{\Delta\tau^2}{4} \\ &\quad \left[\ddot{U}_n^i + \tilde{M}^{-1} \left(\tilde{\Phi}_{n+1}^{ip} - \tilde{\Gamma} \bar{Y}^{-1} \left[\dot{U}_n^i + \frac{\Delta\tau}{2} \left[\ddot{U}_n^i + \tilde{M}^{-1} \left(\tilde{\Phi}_{n+1}^{ip} - \tilde{K} U_{n+1}^i \right) \right] \right] - \tilde{K} U_{n+1}^i \right) \right] \end{aligned} \quad (14)$$

79 Substituting $\dot{U}_{n+1}^i$ from Eq. (13) into Eq. (9) we obtain

$$\ddot{U}_{n+1}^i = \tilde{M}^{-1} \left[\tilde{\Phi}_{n+1}^{ip} - \tilde{\Gamma} \bar{Y}^{-1} \left[\dot{U}_n^i + \frac{\Delta\tau}{2} \left[\ddot{U}_n^i + \tilde{M}^{-1} \left(\tilde{\Phi}_{n+1}^{ip} - \tilde{K} U_{n+1}^i \right) \right] \right] - \tilde{K} U_{n+1}^i \right] \quad (15)$$

80 Integrating the heat equation (8) using the trapezoidal rule, and Eq. (10) we get

$$\begin{aligned} \dot{T}_{n+1}^i &= \dot{T}_n^i + \frac{\Delta\tau}{2} (\ddot{T}_{n+1}^i + \ddot{T}_n^i) \\ &= \dot{T}_n^i + \frac{\Delta\tau}{2} \left(\tilde{X}^{-1} \left[\tilde{Z} \dot{U}_{n+1}^i + \tilde{R} - \tilde{A} \dot{T}_{n+1}^i - \tilde{B} T_{n+1}^i \right] + \ddot{T}_n^i \right) \\ T_{n+1}^i &= T_n^i + \frac{\Delta\tau}{2} (\dot{T}_{n+1}^i + \dot{T}_n^i) \end{aligned} \quad (16)$$

$$= T_n^i + \Delta\tau \dot{T}_n^i + \frac{\Delta\tau^2}{4} \left(\ddot{T}_n^i + \tilde{\mathbf{X}}^{-1} \left[\tilde{\mathbf{Z}} \ddot{U}_{n+1}^i + \tilde{\mathbf{R}} - \tilde{\mathbf{A}} \dot{T}_{n+1}^i - \tilde{\mathbf{B}} T_{n+1}^i \right] \right) \quad (17)$$

81 From Eq. (16) we get

$$\dot{T}_{n+1}^i = \mathbf{Y}^{-1} \left[\dot{T}_n^i + \frac{\Delta\tau}{2} \left(\tilde{\mathbf{X}}^{-1} \left[\tilde{\mathbf{Z}} \ddot{U}_{n+1}^i + \tilde{\mathbf{R}} - \tilde{\mathbf{B}} T_{n+1}^i \right] + \ddot{T}_n^i \right) \right] \quad (18)$$

82 where $\mathbf{Y} = \left(\mathbf{I} + \frac{1}{2} \tilde{\mathbf{A}} \Delta\tau \tilde{\mathbf{X}}^{-1} \right)$

83 Substituting from Eq. (18) into Eq. (17), we have

$$T_{n+1}^i = T_n^i + \Delta\tau \dot{T}_n^i + \frac{\Delta\tau^2}{4} \left(\ddot{T}_n^i + \tilde{\mathbf{X}}^{-1} \left[\tilde{\mathbf{Z}} \ddot{U}_{n+1}^i + \tilde{\mathbf{R}} - \tilde{\mathbf{A}} \left(\mathbf{Y}^{-1} \left[\dot{T}_n^i + \frac{\Delta\tau}{2} \left(\tilde{\mathbf{X}}^{-1} \left[\tilde{\mathbf{Z}} \ddot{U}_{n+1}^i + \tilde{\mathbf{R}} - \tilde{\mathbf{B}} T_{n+1}^i \right] + \ddot{T}_n^i \right) \right) \right] - \tilde{\mathbf{B}} T_{n+1}^i \right] \right) \quad (19)$$

84 Substituting $\dot{T}_{n+1}^i$ from Eq. (18) into Eq. (10) we obtain

$$\ddot{T}_{n+1}^i = \tilde{\mathbf{X}}^{-1} \left[\tilde{\mathbf{Z}} \ddot{U}_{n+1}^i + \tilde{\mathbf{R}} - \tilde{\mathbf{A}} \left(\mathbf{Y}^{-1} \left[\dot{T}_n^i + \frac{\Delta\tau}{2} \left(\tilde{\mathbf{X}}^{-1} \left[\tilde{\mathbf{Z}} \ddot{U}_{n+1}^i + \tilde{\mathbf{R}} - \tilde{\mathbf{B}} T_{n+1}^i \right] + \ddot{T}_n^i \right) \right) \right] - \tilde{\mathbf{B}} T_{n+1}^i \right] \quad (20)$$

85 Using the algorithm of Fahmy [20-22], we have the temperature and the displacements.

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88 5. Numerical results and discussion

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90 With a view to illustrate the numerical implementation presented earlier, a prismatic material with the
91 following physical constants

92 Elasticity tensor

$$93 \quad C_{pjkl} = \begin{bmatrix} 60.23 & 18.67 & 18.96 & -7.69 & 15.60 & -25.28 \\ 18.67 & 21.26 & 9.36 & -3.74 & 4.21 & -8.47 \\ 18.96 & 9.36 & 47.04 & -8.82 & 15.28 & -8.31 \\ -7.69 & -3.74 & -8.82 & 10.18 & -9.54 & 5.69 \\ 15.60 & 4.21 & 15.28 & -9.54 & 21.19 & -8.54 \\ -25.28 & -8.47 & -8.31 & 5.69 & -8.54 & 20.75 \end{bmatrix} \text{ GPa}$$

94 Mechanical temperature coefficient

$$95 \quad \beta_{pj} = \begin{bmatrix} 0.002 & 0.02 & 0.03 \\ 0.02 & 0.004 & 0.04 \\ 0.03 & 0.04 & 0.05 \end{bmatrix} \cdot 10^6 \text{ N/Km}^2$$

96 Tensor of thermal conductivity

$$97 \quad k_{pj} = \begin{bmatrix} 0.8 & 0.1 & 0.15 \\ 0.1 & 0.9 & 0.12 \\ 0.15 & 0.12 & 0.7 \end{bmatrix} \text{ W/Km}$$

98 Mass density $\rho = 1600 \text{ kg/m}^3$ and heat capacity $c = 0.1 \text{ J/kg K}$, $H_0 = 1000000 \text{ Oersted}$, $\mu = 0.5$

99 Gauss/Oersted, $\tau_0 = 0.5$, $m = 0.5$, $\Delta\tau = 0.0001$, $Q_0 = 0.5$, $\nu = 1$.

100 It can be seen from these tables that the DRBEM results are in excellent agreement with those
101 obtained using the finite element method (FEM) of Gao and Yao [23]. Our results thus confirm that
102 our technique is efficient and precise. Also from this knowledge of the variation of the displacements
103 and temperature with time for FGA structures, we can design various structures to meet specific
104 engineering requirements and utilize within which to place new information can be more effective.
105 Also, variation of the temperature T with time τ is presented in Figure 1 in comparison with analytical
106 solution of [24], FDM results of [25] and FEM results from software packages FlexPDE 6, to confirm
107 the validity of our presented technique. For further finite difference method details, we refer the
108 reader to [26-29]. Also, for more boundary element method details we refer the reader to [30-49].

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Table 1. Boundary element results of the considered problem.

t	T	u_1	u_2	σ_{11}	σ_{12}	σ_{22}
0.0	1.00000000	0.76353454	-0.78353535	0.04578125	0.16489345	-0.59482854
0.1	0.97564321	0.77345252	-0.75434333	0.04892613	0.21839467	-0.54293714
0.2	0.94523223	0.78455654	-0.73424244	0.05173821	0.24485945	-0.50923845
0.3	0.91254367	0.79565787	-0.70419252	0.05347621	0.29384673	-0.46892341
0.4	0.88453923	0.80253642	-0.68343465	0.05826364	0.33845683	-0.43928143
0.5	0.85646342	0.81343556	-0.64959587	0.06182828	0.37693481	-0.40287456
0.6	0.82436734	0.82435678	-0.61746567	0.06372737	0.39278476	-0.37494983
0.7	0.79653245	0.83569012	-0.57564653	0.06734242	0.42578923	-0.32658739
0.8	0.76434531	0.84563535	-0.53673213	0.07315241	0.47283601	-0.26294879
0.9	0.73456518	0.85363627	-0.46278342	0.07593080	0.52683246	-0.23345634
1.0	0.69283829	0.86345345	-0.41632342	0.07853454	0.55374852	-0.17659682

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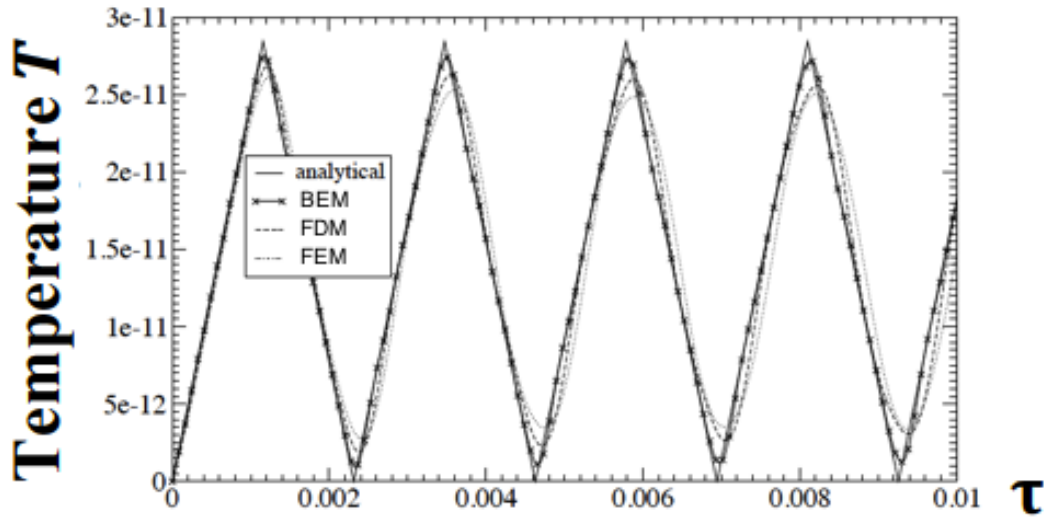
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Table 2. Finite element results of the considered problem.

t	T	u_1	u_2	σ_{11}	σ_{12}	σ_{22}
0.0	1.00000000	0.76353452	-0.78353533	0.04578123	0.16489342	-0.59482851
0.1	0.97564319	0.77345250	-0.75434331	0.04892611	0.21839465	-0.54293712
0.2	0.94523221	0.78455651	-0.73424242	0.05173819	0.24485943	-0.50923843
0.3	0.91254365	0.79565784	-0.70419249	0.05347620	0.29384671	-0.46892338
0.4	0.88453921	0.80253640	-0.68343462	0.05826362	0.33845682	-0.43928140
0.5	0.85646340	0.81343553	-0.64959584	0.06182826	0.37693480	-0.40287453
0.6	0.82436732	0.82435675	-0.61746563	0.06372735	0.39278474	-0.37494981
0.7	0.79653241	0.83569010	-0.57564651	0.06734240	0.42578921	-0.32658737
0.8	0.76434529	0.84563533	-0.53673211	0.07315239	0.47283600	-0.26294878
0.9	0.73456516	0.85363627	-0.46278340	0.07593077	0.52683245	-0.23345632
1.0	0.69283827	0.86345343	-0.41632340	0.07853452	0.55374851	-0.17659680

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Fig. 1. Variation of the temperature T with time for different methods.

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