Original Research Article

A three-dimensional statistical model of karst flow conduits

ABSTRACT

It already exists several three-dimensional models dealing with groundwater circulation in karst systems. However, few of them are able either to give a large scale prediction of the repartition of the flow conduits or to make a comparison with real field data. Therefore, our objective is to develop a three-dimensional model about the early formation of karst flow conduits and to compare it with actual field data. This geometric and statistical model is based on percolation and random walks. It is computational and can be run on a personal computer. We examine the influence of fissures (joints and bedding planes) of variable permeability and orientations on the development or early flow conduits. The results presented here correspond to computations up to 2015. Because of long runtimes, we focused on some particular stereotypical situations, corresponding to some particular values of the parameters. Regarding the conduit patterns, the opening and directions of fissures have the same qualitative influence in the model than in actual systems. Two other predictions in good accordance with real karst are that flow conduits can either develop close to the water table or deeper, depending on the distribution of permeable fissures; and that, when viewed in the horizontal plane, conduits don't always develop close to the straight line between inlet and outlet. From a quantitative point of view, in the case of weak dips, our model predicts a realistic relationship between the stratal dip, the length of the system and the averaged depth of the conduits. Eventually, we show that the repartition of conduits depends not only on obvious geometrical parameters such as directions and sizes, but also also on other quantities difficult to measure such as the probability of finding open fissures. The lack of such data doesn't enable, at the present time, a whole comparison between model and reality.

Keywords: Karst, Proto-conduits, Statistical model, Three-dimensional, Water-table, Percolation

1 INTRODUCTION

We develop a three-dimensional model of water circulation in karst systems. This model relies upon other basis than already existing models; it allows to study other features of the systems. We are still far from “the construction of models that describe the complete aquifer including the interactions of all components” regarded by White [20] as an interesting topic. However the present model leads to a view complementary of other already existing models and a view at a larger scale. When they are not devoted to a particular place [21, 22, 25] or to particular properties of already existing conduits [23, 24, 26], most of the pre-existing models deal with a single conduit or fissure, or a few numbers of them. The authors are often interested in how the size of the flow conduits evolves over time, and the global shape of the system is less frequently studied. In these works, a comparison with actual systems is not always done. Attempts to reduce and clarify the number of parameters aren't systematically made. Certain authors don't make any attempt to examine how the predictions of the model vary according to general parameters such as stratal dip. For instance, in [1] Szymczak & Ladd examine a single fissure. One of their results is that during its evolution, such a system is unstable; the larger parts tend to become larger and larger. Water doesn't flow through the entire fissure, but along certain lineaments (in other words one-dimensional objects, proto-conduits). However, at least seven parameters, with precise fixed numerical values, are used in this study. The calculations are quite complex, but the system remains very simple at the scale of a karst. Its shape is fixed prior to the calculations and it cannot be regarded as a result. The results of these authors deal largely with the notion of breakthrough (The breakthrough is the fact that the conduits permeabilities and sizes increase suddenly after a rather slow early evolution). There is no comparisons with actual caves, only with other models. In [2], Perne, Covington and Gabrovsek have studied a two-dimensional network of interconnected conduits. They used a lot of static numerical parameters (Table 1 of their article). The influence of the dip is examined, but the relative directions of the conduits or joints aren't. Their model is a rectangular network with one of its directions along the dip. The evolution of the conduits over time and their transition from phreatic to vadose are studied. However the conduits are constrained, they all belong to a same plane: this doesn't enable any statistical study of their depth. No direct comparison with actual caves is made. In [3], Rongier & Collon-Drouaillet have developed a three-dimensional method of stochastic simulation of conduits. They take in account the geological context (faults etc.) in order to generate conduits having a shape closely corresponding to field observations. These authors mention that their model needs a lot of parameters. They also indicate that the user must adjust some parameters according to what he wants to produce; so this isn't a fully predictive model. No statistical study about the position of the conduits is made. Worthington made such a work in [4]**.** He used data from actual surveys and actual caves and his work deals with the water table controversy, which is explained for instance in [5]. This question, which is also one of ours, can be asked as follows: what are the possible shapes of a cave pattern when one takes in account the initial conditions (nature, direction and dips of the bedding planes, joints and other fissures) and the balance between steepest route and route of maximal hydraulic conductivity? Using regression of the mean depth D against other parameters such as the sinus of the stratal dip θ and the length of the system L, Worthington produces a certain number of scaling laws. However, the relationships are dependent of the system of units chosen to express them. For instance, he proposes in [4] the following equation:

$$D = 0.061\ L^{0.91}\ \theta^{0.72} \qquad (1)$$

This Eq. (1) wouldn't keep the same expression if the unit of length had been changed (or equivalently if the mean distance between fissures had been different). In other words, the coefficient 0.061 is not dimensionless and behaves as a length at power 0.09; which is quite dim. Eventually, Worthington doesn't take in account the relative orientations of the stratal slope and the slope of the water table. We have built in [6] a statistical and geometric two-dimensional model that has the same bias: a figure in elevation can lead to the conclusion that the slope of the water table and the stratal slope have always the same direction; this is seldom the case when considering a three-dimensional situation. However a very important result of our model was the following: we showed a strong correlation between the probability of finding open fissures and the possible existence of interesting karstified networks. This is precisely what Quinif and his colleagues [7] have observed in actual caves. More generally, despite some quantitative disagreements, our model was in good qualitative accordance with the known evolution of flow conduits according to fractures [5]. Our model gave a meaning to the mysterious unit of length appearing in some of Worthington's formulas (It can be the averaged distance between successive fissures).

In what follows, we present a three-dimensional model that has the same basis of our two-dimensional model [6], but that enables to take in account the relative directions between stratal slope and water table. It also enables to study the horizontal repartition of conduits. This is a statistical and geometrical model, designed to avoid taking in account the precise rules governing the flow rates and chemical processes. These rules are very complex and present strong variations with the conduits scale: taking them explicitly in account needs a lot of parameters (as explained for instance in [8]). This would lead to something more difficult to master and with much more sources of uncertainties. Instead of focusing on the evolution of the conduits, we focus on their statistical distribution in an early state (proto-conduits). This probabilistic approach we develop is more universal. As larger conduits develop from proto-conduits, some features of the repartition may still be preserved inside evolved karst and this is why a comparison, at least partial, with actual systems remains very relevant.

2 MATERIAL AND METHODS

Here we present how our model is built, how it works, and how the results can be viewed or used to feed further statistical studies.

2.1 Presentation of the system and the parameters describing it

We model a karst by a quadrangular network where water can flow from a node to another adjacent node along an edge that is permeable only with a certain probability. The three perpendicular families of planes present in our network and containing the edges may correspond to the bedding planes and two families of mutually perpendicular joints. Each edge is the intersection of two fissures. This corresponds either to the assumption that water flows only at intersections of fissures or to the assumption that water flows through fissures in an inhomogeneous fashion as explained in [1]. To take in account the different permeabilities of the three families of fissures, we introduce three different probabilities for an edge to let water flow: p_x, p_y, p_z . For instance the bedding planes may be more permeable than the joints, so the three quantities, each related to one family of edges, can be different. This is also a way to take in account tectonics constraints that can be compressive in one direction and extensive in another.

This 3D model of karst, even oversimplified, already depends upon six parameters: three sizes of the system (or relative coordinates of the resurgence related to the sinkhole) and three probabilities. Yet one assumes that the positions of the inlet and the outlet are fixed; this is the case, for instance, in a situation of allogenic control, with the inlet corresponding to a sinkhole. The Figure 1 exemplifies the coordinates used and shows than an angle φ must be introduced as a seventh parameter. φ precisely allows to take in account the fact that the stratal dip and the water table dip may have the same direction (Figure 2-a) od different directions as depicted in Figures 2-b and 2-c.The water table is assumed to be plane, as justified below. The seven parameters of our model can vary in simple ranges (0 to 1 for probabilities, any number of steps for the position of the outlet. For symmetric systems, as shown in Figure 2-d, φ is periodic in the range 0 to 120°. Simulations ca n be ran for any set of values, which is more universal than using seven static parameters as in other models, [2] for instance.

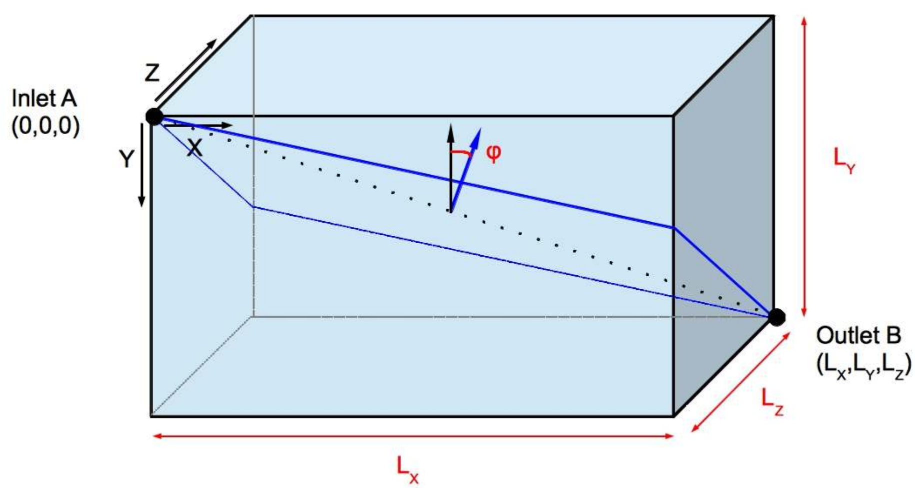


Fig. 1. The model can be embedded in a parallelepiped L_x X L_y X L_z.The water table is in blue

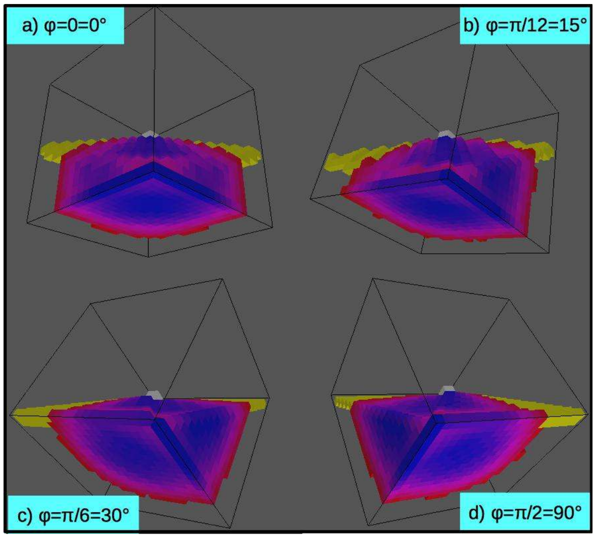


Fig. 2. Four possible configurations of the model

2.2 Rules of calculation

This model corresponds to the early stage of karstification, when proto-conduits are forming. The flow rates in each fissure aren't very important and one assume they don't modify the position of the water table. We focus only on the probability of finding proto-conduits in a location or another of the system. It is explained below how to calculate this probability, that is the most interesting quantity: if a proto-conduit cannot form, it cannot enlarge and this means that in a further evolution more developed conduits cannot exist here. Therefore, this probability of finding proto-conduits is linked to the properties of more developed system corresponding to actual karst systems.

From the inlet (sinkhole, represented by a white cube in the figures), the water circulation is assumed to be driven by the altitude gradient of the water table, as explained for instance in [9]. Water can flow through each different edges of our model only in one sense. The necessity to come out through the outlet (resurgence) introduces another condition on the edges that can be traversed: the total descent below the water table must be exactly compensated by the total ascent toward water table. This is why the system can be embedded in a parallelepiped (Figure 1). Assuming these rules of circulation, we compute for each node the probability P of finding a proto-conduit (able or not to become a conduit after competition and breakthrough) as follows: we start from the inlet with P=1, then this probability is equally balanced over all the neighboring nodes that are available. The process is repeated until all the nodes have been reached. In other words, this is a random walk subjected to the constraints (water table and altitude gradient) explained above.

In order to discuss dimensionless, universal, quantities, we use as “natural” unit of length the mean distance between adjacent fissures. Assuming that this length is the same for the three families is not a limitation: if not the case, it wouldn't change P. It would only change the value of the effective dip and other angles in a way simple to understand. The computation time depends on the number of nodes. But for the same computer, for instance, a 100 X 100 X 100 three-dimensional system takes roughly the same time to be computed than a 1000 X 1000 two-dimensional system. This is why we focused on few situations chosen for illustrating specific features.

2.3 Representation of the calculations

The raw result of a simulation is a set of values of P for each node. This set cannot be directly used. On one hand it is possible to compute statistical quantities, such as the averaged depth under the water table versus the distance from the inlet. On the other hand, it is possible to produce perspective pictures. We used the free software geomview in order to post-process some of the calculations we made and to produce such images. A color code must be used, which enables to put the emphasis on some interesting features: yellow represents the water table, blue stands for high values of P, purple for lower values, and red for the lowest. A white cube indicates the inlet.

3 RESULTS AND DISCUSSIONS

Here we present different results or the model. In each series of results, we focus only on one possible variation of the parameters. We start with isotropic systems, where the permeabilities according to x, y, z are the same. Realistic systems with weak dips are examined, as well as less realistic systems with stronger dips. Then anisotropic systems are examined and the strong influence of the permeabilities on the shape of flow conduits is shown. All these results enable us to develop a general discussion about the validity and the limits of the model.

3.1 Symmetric and isotropic systems

These calculations correspond to important dips (Figure 2) and the same value for p_x ,p_y and p_z .These are extreme conditions because the strong tectonization needed to obtain important dips would certainly fold the bedding planes. Such computations are not the most realistic since our model doesn't take in account any curvature. However they enable to verify the most basic features expected from models of karst. Compared to already existing models [30], the advantage of the present model is still its more realistic and more adaptable geometry. Computations with isotropic systems 100X100X100 and variable values of $p=p_x=p_y=p_z$ show that karstification starts only if p is high enough.

More precisely, the Figure 3-a shows that among seven systems A,B,C,D,E,F,G we computed, with p=0.5 and φ=0, proto-conduits developed only in two of them. In such systems, fluctuations occur more easily at the beginning of the process, near the inlet: later, the number of possibilities for further steps of development is more important and the development becomes more regular. The Figure 3-b shows that among seven systems of size 100 X 100 X 100 we computed, with p=0.47 and φ=0, again only two are subject to the development of proto-conduits. With this lower value of p, the depth dispersion is more important than in Figure 3a

The few number of simulation does not allow to propose a very precise value, but the Figures 3a and b suggest that it is more important (in a range 0.45 to 0.50) than the 0.247 theoretical percolation threshold ([19] for instance), because the conduits don't exactly develop according a fully random walk. They are submitted to additional constraints: they exist only above the water table and are submitted to rules regarding the sense in which water flows.

One could reproach our model for allowing the conduits to develop only following the shortest possible pathways. This is why, in order to examine the influence of this approximation, we computed the same systems using weaker rules: we allowed the water to flow along the edges in both senses (“isotropic diffusion” at place of “oriented diffusion”). The Figure 3c shows that, near the percolation threshold, taking in account the possibility for conduits to use pathways that are not systematically the shortest is essential. One can see that the system B has a very different behavior using one rule or another, and that proto-conduits can develop in the system F using “isotropic” rules.

These results resemble what happens with actual conduits developing in the phreatic zone, for instance those described by Lauritzen in [10] for the caves of Lake Glomdale and detailed in Fig 7 of Ford and Williams' book [5]. This suggests that actual systems with deep phreatic loops and long and sinuous conduits developing far away from the water table and the straight line have formed near the percolation threshold.

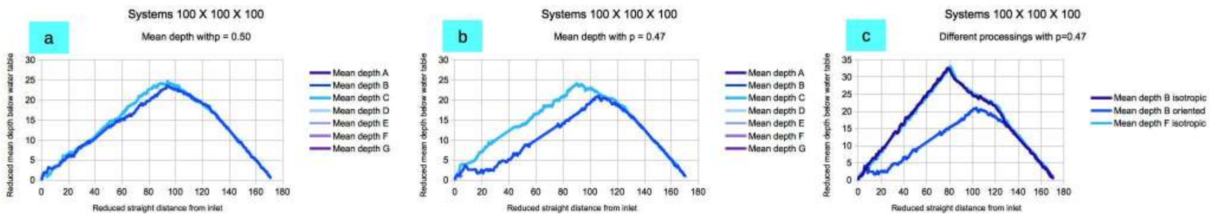


Fig. 3. Mean depth of the protoconduits (if existing) in seven symmetric systems

With a 3D model, the dispersion appears to be weaker than with the rough 2D model previously developed in our previous research (Figure 4 of [6]). The threshold also appears different, lower. Our 2D model is the equivalent of φ=0 for a 3D model. The Figure 4 shows the repartition of P for a system processed with p=0.47, near the percolation threshold. The elevation (Figure 4b) is in agreement with Ford's ideas of [8]: conduits develop close to the water table except when fissures are scarce, then they are strongly dispersed below the water table. The plane view (Figure 4a) suggests the same thing regarding the deviations from the straight line joining inlet and outlet. This can be compared with the cut-offs of the meander of the river Cure described by Haid in [11] and also reported in Fig 7 of Ford and Williams book [5]. Regarding inlet and outlet, they are equivalent to an allogenic situation and, even if not totally sinuous, they exhibit fluctuations from the straight line. The same comparison is possible with the cut-offs of the river Ardèche in the Grotte Saint-Marcel that is described by Mocochain & Bigot in [12].

On the contrary of its very good agreement for low values of p (obviously conduits don't develop at all), our model doesn't give firm conclusions in the case of high values of p. We cannot precise whether conduits develop at the level of the water table or don't develop (as in the state 5 of Ford's model). Eventually, Figure 4a is in qualitative agreement with Annable's model [13], also reported in Fig 7 of Ford and Williams book [5], where conduits don't develop according to a sheer straight line.

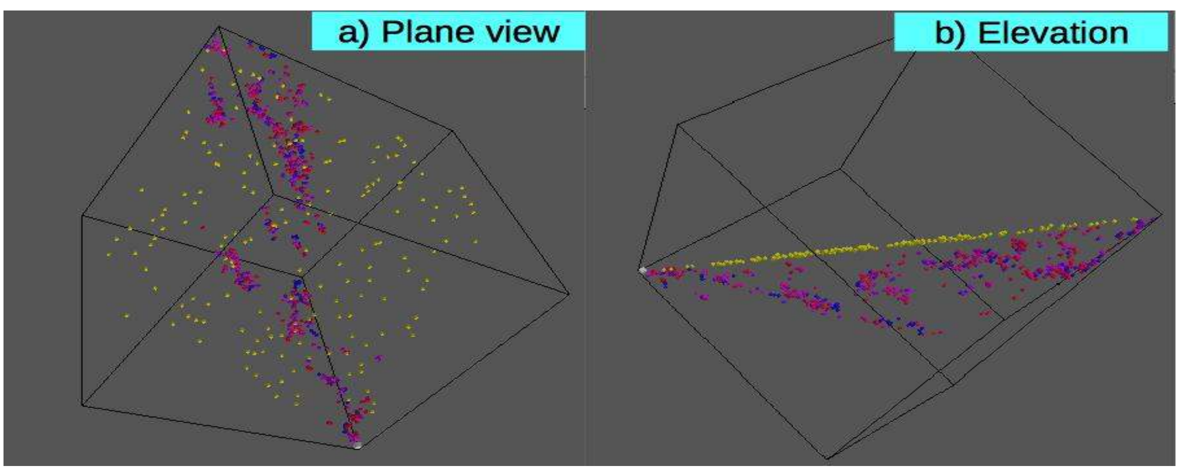
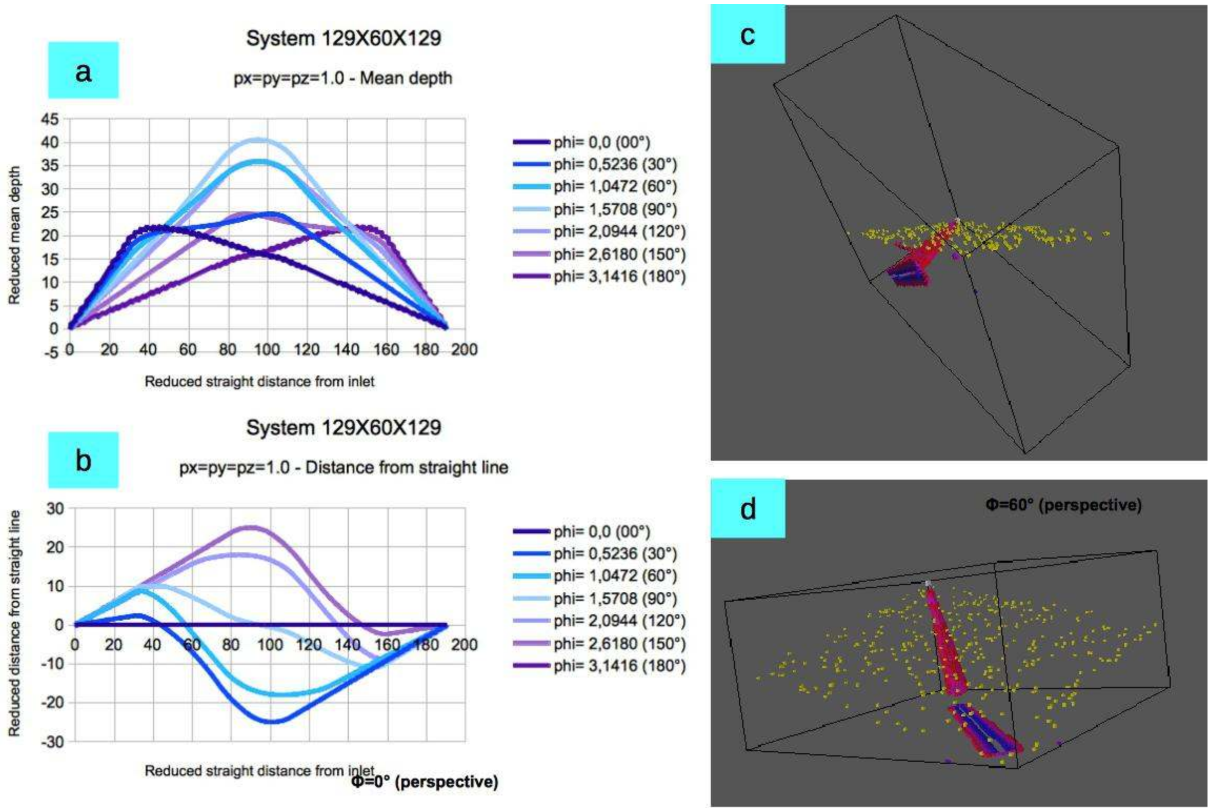


Fig. 4. Plane view (a) and elevation (b) of a system 100 X 100 X 100 computed with isotropic rules and with p=0.47

3.2 Asymmetric isotropic models

We performed some calculations on a system of size 129X60X129 with p_x=p_y=pz=1.0 and different



values of φ. The figure 5 shows the importance of this angle, for the mean depth (figure 5a) and the deviation (

figure 5b) from the straight line are very dependent on it. In other words, for different karst systems having the same size, the same external shape, inlet and outlet at the same relative places, the inner shapes of flow conduits may be very different if the directions of the families of fissures are different. This is very explicit on Figures 5-c and 5-d.Fig. 5. Influence of φ on the mean depth and the deviation from the straight line

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3.3 Asymmetric systems with weak dips

Our model doesn't take in account slope variations among the system. So, like the 2D model, it is more realistic when applied to weakly tectonized systems such as karst plain, where the dips are often weak.

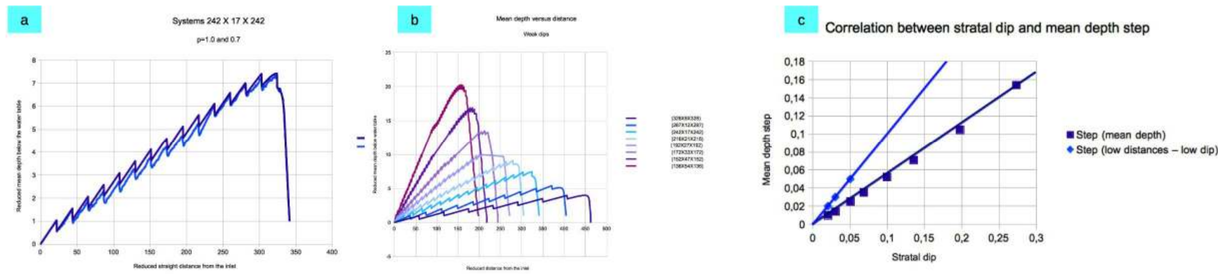

Fig. 6. Study of systems with weak dips

We performed some computations on systems of size 328X9X328, 287X12X287, 242X17X242, 216X21X216, 192X27X192, 172X33X172, 152X47X152, 136X54X136, which all have almost the same calculation time than a 100X100X100 system. We started with $\varphi=\pi=180°$ in order to consider a weak dip on the side of the inlet and to examine the influence of the stratal dip on the mean depth. The Figure 6a suggests that above enough from the percolation threshold, the results are few depending on the value of the probabilities. Choosing probabilities unity for further calculations spared a lot of time: each configuration was computed just once (Otherwise one had to average on several results) The "saw tooth" behavior near the inlet is explained by the fact that conduits cannot exist above the water table. This 3D-model predicts the "quasi-artesian trapping" of some certain authors. The Figure 6b shows the evolution of the mean depth with the straight distance for the different systems. From a qualitative point of view, it appears the more important the stratal dip is, the more the mean depth grows with distance from the inlet. From a quantitative point of view, the results are different from those of our 2D model and more realistic. Especially, and according to Ford in [5], "where strata dip quite steeply (2-5° or more) the bedding planes tend to entrain the ground water to greater depths". This appears very clearly on figure 6c, which shows the correlation between the stratal dip and the slope of the mean depth with dips of 1.2°, 1.7° and 2.9°. At place of Eq. (1) proposed by Worthington in [5], the figure 6c suggests with the same notations the following equation:

$$D=\alpha L^{1}\theta^{1} \qquad (2)$$

α is a dimensionless quantity in the range [0.57,1]. Even if this dimensionless coefficient cannot be compared to Worthington's dimensioned one, the exponents of Eq. (1) and (2) are quite close, especially for L.

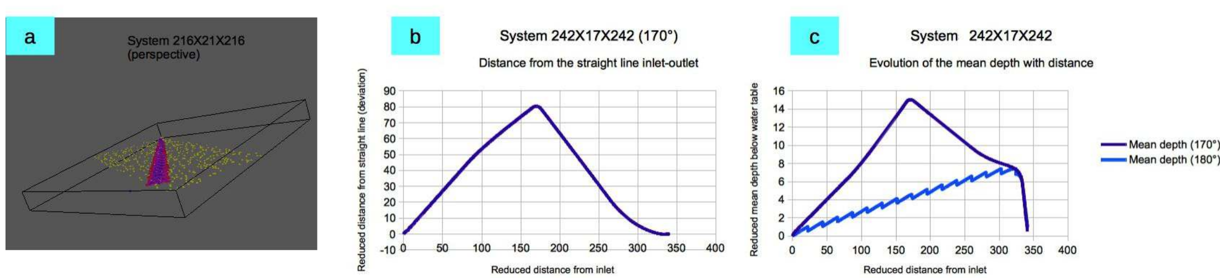

Fig. 7. Influence of an oblique stratal dip on the repartition of the conduits

In order to examine what happens when the stratal dip and the slope of the straight line between inlet and outlet are different, we computed the same systems but with φ=170° at place of φ=180°. Figure 7a shows the repartition of probability in a system 216X21X216 with φ=170°. Obviously, the oblique stratal dip tends to entrain the conduits deeper, and astray from the straight-line inlet-outlet. The case of systems 242X17X242 with φ=180° or φ=170° is numerically investigated on Figures 7b and 7c.

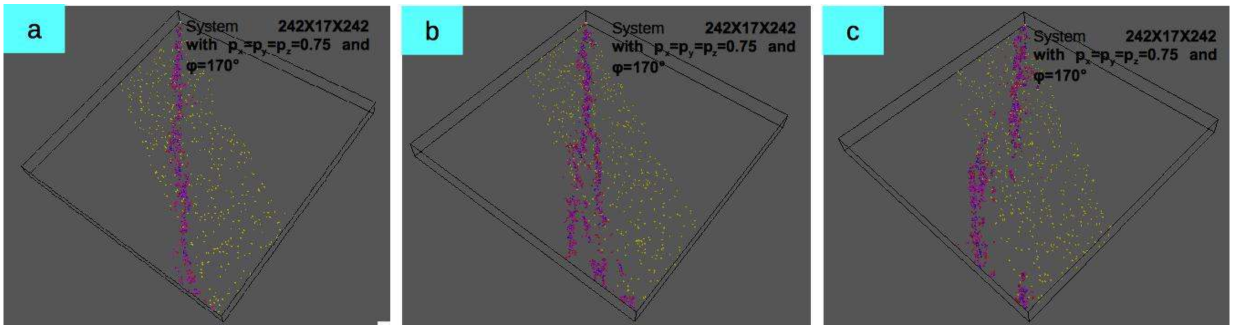

Fig. 8. Perspective view of three different systems 242X17X242 with $p_x=p_y=p_z=0.75$ and φ=170°, near the percolation threshold

Obviously an oblique stratal dip strongly influences the mean depth and the deviation from straight line. Without regarding small scale details, this can be compared to the conduits of the Holloch (Switzerland) described by Bögli in [14] and reported in Fig 7of Ford and Williams book [5]: the stratal dip is oblique regarding the circulation of water. In addition to the results of Figure 7, we also studied the influence of the percolation threshold in the case of weak dips. This threshold is close to the one of two-dimensional systems, because of the "quasi artesian trapping effect" evoked above. We examined the influence of the probability in a homogeneous case $p_x=p_y=p_z$ and Figure 8 shows three systems with $p_x=p_y=p_z=0.75$ (near the percolation threshold): the repartition of the conduits is subjected to strong fluctuations, which can explain the great diversity of actual systems.

3.4 Inhomogeneous and anisotropic case

The present model shows that some karst systems may have the same size, the same external shape with the same inlet and outlet, the same families of fissures, and simultaneously have very different flow conduits repartitions if the probabilities are different. The Figure 9 shows the results of calculations have performed on systems 129X69X129 with φ=π=180° in two different situations: with $p_y=p_z=0.55$ and $p_x=1.0$ (Figure 9a); and with $p_x=p_z=0.55$ and $p_y=1.0$ (Figure 9b). A simple permutation in the probabilities, leading some fissures to be more permeable, or less permeable, leads to spectacular changes in the repartition of P, hence in the shape of the flow conduits.

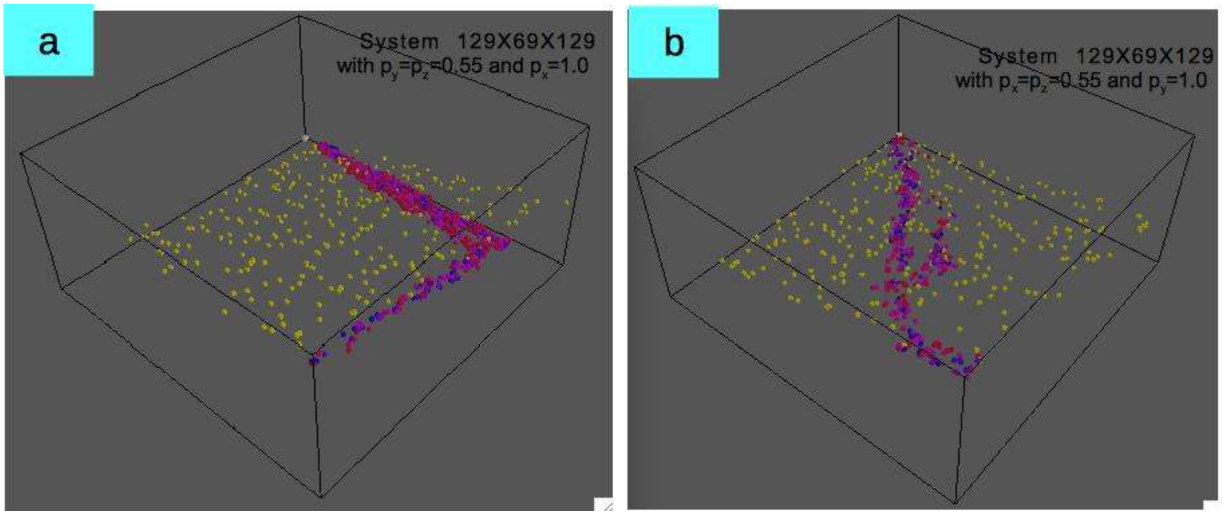

Fig. 9. With different permeabilities, the flow conduits of two systems having the same external characteristics have very different shapes

3.5 **Limits and validation of this model**

We have already discussed the good qualitative accordance between our predictions and some general trends of actual systems. However this is a rough large-scale model: we don't claim we succeeded to make very good and very general quantitative predictions. A full statistical validation would require the processing of a huge number of actual surveys, in order to reconstitute and extract, for instance, the mean depths of the conduits at the time of their formation. As evoked by several authors, for instance Audra and colleagues in [15], a real karst is a complex system that has had a multi-step evolution. Conduits formed in the phreatic zone (and issued from proto-conduits) can be modified in the vadose, unsaturated, zone once the water table has evolved. The position of the outlet is also subjected to evolution. All that means that not all the statistical properties of the proto-conduits are preserved once the system has evolved and is observed at the present time. A reconstitution of conduits formed in the phreatic zone from proto-conduits has to be done; for instance on the basis of their shape. We didn't make such a processing.

A full statistical validation of our model would also require to escape some biases: conduits having evolved from proto-conduits and still in the flooded zone, below the water table, can be explored and surveyed only by cave diving. Such a practice has strong limits, conduits that are too far from entrance or too deep cannot be investigated. Not all the conduits are penetrable by man: some massifs are recognized as karst because of their hydrological features, but the presumed corresponding conduits aren't penetrable and no survey can be done. Eventually, in caves having a complex evolution, with several variations of the water table, the altitude of the conduits at the time of their formation can be very different from their current altitude. A reconstitution may be very difficult.

The only statistical comparison we made is based on Worthington's data [4], with less than fifty caves, and on the study [7] of Quinif and colleagues devoted to a sole area. Regarding individual surveys, the large-scale comparison with the Holloch neglects many details and steps of evolution (some vadose conduits shouldn't be taken in account). In addition we compare a model devoted to an allogenic system to an actual system that has no single inlet. The lower scales comparisons with the river Cure and caves of lake Glomdale don't take in account the whole system but only the surveyed part of it. Eventually, fissures directions scarcely appear on surveys as pointed out by Thérond in [16]. However, the distances between fissures should also be measured and averaged in order to compare several conduit systems from a statistical point of view.

Finally, it appears that a full quantitative validation or our model, as well as any other large-scale model, would require the availability of a large number of data. These data should be precise enough and all in the same numerical format: it appears that developing large-scale models is intimately linked to gathering data. This is still an open problematics. This should encourage cavers, cave divers and other specialists to make more accurate observations and to produce more open surveys with more relevant data.

4 CONCLUSION

It remains difficult to develop “complete” or “standard” models of karst able to render all the features of actual systems. Because of the great variety of karst systems, such a building is very likely impossible. We developed our model not in order to approach this ideal but on the contrary, because it is complementary to other models of classical allogenic karst. On one hand, some models take in account very precise parameters and, at small scale, may be predictive regarding details such the rate of growth of the conduits and the influence of the water table. On the other hand, we propose a rougher model but at a larger scale and using only a small number of very generic parameters. There is a good qualitative accordance between its predictions and some general trends of actual systems. This is a wide-scale rough model: as already pointed out, we don't claim we succeeded to make very good quantitative predictions. The interest of our model is elsewhere: it shows that the most visible parameters of a karst system (size, directions of the families of fissures) are not enough to understand the distribution of flow conduits. Their existence and shape depends on the probabilities for fissures to be permeable, which are uneasy to measure on the field.

Our calculations also illustrate the fact that a model of karst is a complex object in the sense described by Bennett in the pages 227-57 of [17]: it is “easy” to describe, but the calculation time is important. In other words the programs are light but have a huge running time. Eventually, the development of our model establishes a strong link between the evolution of karst conduits regarded as a natural phenomenon and very theoretical notions known in physics as “critical phenomena”. More precisely, the formation of developed systems of flow conduits needs two critical phenomena. The first is known as percolation and is fully taken in account in our model: if the probabilities are too low, below the percolation threshold, the inlet and the outlet are disconnected, it exists no pathway joining them and the karstification can not start. The second critical phenomenon is a dynamical one, very well known in the field of karstology as “breakthrough”. As described for instance by White [20]: a competition exists between the conduits joining the inlet and the outlet. It is a positive feedback retroaction loop; the larger conduits will support higher flow rates and they will enlarge at a faster rate. Our model doesn't take in account this second critical phenomena. This is precisely why, in situations where P is widely spread over the system and not concentrated in specific areas, we are not able to fully discriminate between situations where conduits will differentiate (states 3 or 4 of Ford's model) or not (state 5 of Ford's model). Therefore, the development of a dynamical model, able to take in account at large scale both percolation and dynamical aspects, would be particularly interesting. This could be a strategy to reduce the numerous uncertainties that remain when comparing the results of our model (distribution of proto-conduits) to real karst (distribution of fully evolved conduits). In addition, this way of modeling karst flow conduits would be close to the way of modeling of other systems where, after percolation, the most used pathways are reinforced. For instance, social systems and neural networks related to the emergence of consciousness have this kind of behavior. This establishes links between karst sciences and other topics and this gives to the development of reliable and general global models of percolation coupled to breakthrough a special interest. Such models are more complicated than those usually considered in theoretical physics (described of instance in [18]). The new different approaches that are needed, often more computational, are very stimulating.

REFERENCES

[1] Szymczak P, Ladd AJC. The initial stages of cave formation: Beyond the one-dimensional paradigm. Earth and Planetary Science Letters. 2011;301:424–32.

[2] Perne M, Covington M, Gabrovšek F. Evolution of karst conduit networks in transition from pressurized flow to free-surface flow. Hydrol. Earth Syst. Sci. 2014;18:4617–33.

[3] Rongier G, Collon-Drouaillet P. Conduits karstiques en 3D: une méthode de simulation stochastique. Karstologia. 2014;63:58-60. French with English abstract.

[4] Worthington SHR. Hydraulic and geological factors influencing conduit flow depth. Cave and Karst Science.2004;31(3):123-34.

[5] Ford DC, Williams P. Karst hydrogeology and geomorphology. Chichester: John Wiley & Sons; 2007.

[6] Boudinet P. A statistical model of karst flow conduits. Speleogenesis & Evolution of Karst Aquifers. 2012;12:9-16.

[7] Havron C, Quinif Y, Vandycke S. Tectonique et karstification, le cas de la région de Han sur Lesse (Belgique). Karstologia. 2004;43: 19-26. In French with English abstract.

[8] Ford DC. Perspectives in karst hydrogeology and cavern genesis. Karst Modeling: Special Publication. 2003;5:17-19.

[9] Quinif Y. Dissipation d'énergie et adaptabilité dans les systèmes karstiques. Karstologia. 1998;31:1-11. In French with English abstract.

[10] Lauritzen SE. Hydraulics and dissolution kinetics of a phreatic conduit. Proceedings of the 9th international speleological congress. Barcelona.

[11] Haid A. Yonne. Spelunca. 1996; 62:14. In French without English abstract.

[12] Mocochain L, Bigot JY. Les origines de la Grotte de Saint-Marcel. La Grotte de Saint-Marcel d'Ardèche. Chauzon: Comité Départemental de Spéléologie de l'Ardèche; 2008. In French without English abstract.

[13] Annable WK. Numerical Analysis of conduit evolution in karst aquifers. PhD. Thesis. Waterloo, Ontario: Univ of Waterloo; 2003.

[14] Bögli A. Le Holloch et son Karst. Neuchâtel; Éditions de la Baconnière; 1970. In French without English abstract.

[15] Audra P, Bini A, Gabrovšek F, Häuselmann P, Hobléa F, Jeannin PY, Kunaver J, Monbaron M, Šušteršič F, Tognini P, Trimmel H, Wildberger A. Cave and Karst evolution in the Alps and their relation to paleoclimate and paleotopography. TIME in KARST. Postojna: Karst Waters Institute; 2007.

[16] Thérond R. Le lever géologique des réseaux karstiques. Spelunca. 1998;70:26-36. In French without English abstract.

[17] Bennett CH, Logical depth and physical complexity. The Universal Turing Machine– a Half-Century Survey. Oxford: Oxford University Press;1988.

[18] Ziman, JM. Models of Disorder: The Theoretical Physics of Homogeneously Disordered Systems. Cambridge: Cambridge University Press; 1979.

[19] Pajot S. Percolation et économie. Doctorate Thesis. Nantes: University of Nantes; 2001.

[20] White WB. Karst hydrology: recent developments and open questions. Engineering Geology. 2002;65(2-3): 85-105.

[21] Xu Z, Hu BX, Davis H, Kish S: Numerical study of groundwater flow cycling controlled by seawater/freshwater interaction in a coastal karst aquifer through conduit network using CFPv2. Journal of Contaminant Hydrology. 2015;182: 131-145.

[22] Kovacs A, Perrochet P, Darabos E, Lénart L, Szücs P: Well hydrograph analysis for the characterisation of flow dynamics and conduit network geometry in a karst aquifer, Bükk Mountains, Hungary. Journal of Hydrology. 2015;530: 484-499.

[23] Luhmann AJ, Covington MD, Myre JM, Perne M, Jones SW, Alexander EC, Saar MO: Thermal damping and retardation in karst conduits. Hydrol. Earth Syst. Sci. 2015;19: 137-157.

[24] Borghi A, Renard P, Cornaton F: Can one identify karst conduit networks geometry and properties from hydraulic and tracer test data? Advances in Water Resources. 2016;90: 99-115.

[25] Malard A, Jeannin PY, Vouillamoz J, Weber E: An integrated approach for catchment delineation and conduit-network modeling in karst aquifers: application to a site in the Swiss tabular Jura. Hydrogeology Journal. 2015;23(7): 1341-1357.

[26] Hubinger B, Birk S, Hergarten, S: A New Equation Solver for Modeling Turbulent Flow in Coupled Matrix□Conduit Flow Models. *Groundwater*, 2016.

APPENDIX

Computational and mathematical details, programs and source or result files, can be found on our Internet site: **http://pboudinet.dynalias.com/~speleo/Recherche/Flow_Conduits_3D/liste.php** or directly asked the author.

Geomview can be found on **http://www.geomview.org/**