

Game Analysis Involving Tax Evasion Behavior of Firms, Tax Audit by Government, and Optimal Air Pollution Tax Rates

This paper uses two-stage game theory to analyze the relationship between tax evaders and government regarding air pollution emissions. The production function of a firm in a perfectly competitive market was first considered. Then, the backward induction method and the Cramer rule method to determine the optimal subgame perfect equilibrium in the two-stage game and investigate the relationship between firms' tax evasion behavior and tax variables. This study discovered that the stronger the spillover effect on firms engaging in air pollution control, the higher the tax rate levied by government should be. When firms are in a perfectly competitive market and the financial policy instruments (i.e., air pollution tax and subsidy rate) are known, the conditions for economic stability can be established.

1. Introduction

Tax erosion occurs for various reasons. Some articles have argued that it is caused by high tax rates imposed by governments, which entices taxpayers to report an income that is considerably lower than their actual income (i.e., concealed income); some have remarked that high tax rates prompt taxpayers to shift their operations from on-the-ground markets to underground markets, which increases tax evasion behavior; and others have argued that when concealment costs are lower than the probability of getting caught multiplied by the financial penalty of the concealment, tax evasion behavior will likely ensue, increasing tax erosion. Firm-related tax evasion behavior involves other factors, including the external costs incurred by firms' pollution emission, companies' pollution-related tax evasion, firms' expected utility arising from uncertainty in the probability of being caught for evading taxes, and the effect of other firms' tax evasion behavior on representative firms' decisions to engage in tax evasion behavior. These topics were explored in this study.

2. Base Model

Because tax evaders and a government are in a game-based relationship, this paper employs the W-C-Y model proposed by Wang and Conant (1988) and Yaniv (1995) to investigate how firms balance production output and tax evasion to maximize their expected utility. Additionally, the model is employed to analyze the tax game between firms and government. A firm's actual profit can be represented by the following definitional equation:

$$\pi(Q) = R(Q) - C(Q) \quad (1)$$

where R is the firm's total income, C is the firm's total costs, Q is the production output, and

34 R and C are functions of Q . In addition, $R'(Q) > 0$, $R''(Q) < 0$, $C'(Q) > 0$, and $C''(Q) > 0$,
 35 indicating that the second-order conditions of the profit function are a decreasing concave
 36 function, and that the second-order conditions of the cost function are an increasing convex
 37 function.

38

39 2.1 Model Assumptions

40 This paper assumes that m is the proportional tax levied by a government, that $0 < m < 1$,
 41 and that a firm confronted with proportional tax may evade taxes by reporting a profit lower
 42 than their actual profit.

43 Next, this paper assumes that Z is the profit declared by the firm; $Z \leq \pi$ (i.e., the profit
 44 declared by the firm is less than or equal to its actual profit); the government penalty rate is
 45 τ (where $\tau > 1$); the government performs random profit-checking; and q is the probability
 46 that the firm gets caught (an exogenous variable). In contrast to previous literature, the
 47 concealment cost of a firm is defined as a definite integral with a lower limit of 0:

48 $\int_0^s h(s)ds = H(s)$, where s is the effort made by the firm regardless of whether it is caught for

49 tax evasion ($1 \geq s \geq 0$). In practice, a firm may produce pollution during the production of
 50 their products and/or services, which creates an external cost to the environment. Thus, this
 51 paper adopts the view of Baron (2001) and Wu and Yen (2011) and assumes that the utility
 52 function of a representative firm is affected by both its profit and corporate social
 53 responsibility; the firm emits pollution during product and/or service production; the firm's
 54 expected utility is unaffected by whether its pollution emissions fall within the standard

55 pollution emissions ($\bar{Q}$) specified by the country's environmental protection unit; and

56 $\tilde{Q} = Q - \bar{Q}$ indicates the firm's illegal pollution emissions. Unlike previous articles, this
 57 paper accounts for A^d , the negative psychological effect of h percent of firms evading
 58 taxes ($0 \leq h \leq 1$) on the representative firms in a society; thus, the fewer firms that engage in
 59 tax evasion, the more moral pressure is experienced by representative firms when they evade
 60 taxes. This paper expands the concept of firm's size of tax evasion, which was proposed by
 61 Wu and Yen (2011), by representing it in the form of "actual profits" and "reported profits."
 62 In addition, this paper adds representative-tax-evader concealment cost functions for further
 63 analyses.

64 2.2 Taxpayers' Optimal Solutions in Response to Taxes

65 The first stage involves government formulating the most appropriate policy instruments
 66 (e.g., tax rates, penalties, and maximum pollution emission threshold for product and service
 67 production processes). The second stage entails the taxpayers' optimal solutions given the

68 policy instruments employed by government. A representative firm's expected utility in
69 production output and tax evasion can be represented as follows:

70 i) A representative firm's after-tax profit when it successfully evades taxes can be
71 represented by the following formula:

$$72 \quad \pi^{UA} = (1 - m) \times \pi + m \times (\pi - Z) - H(s) \quad (2)$$

73 ii) A representative firm's after-tax profit when it is caught for tax evasion can be
74 represented by the following formula:

$$75 \quad \pi^A = (1 - m) \times \pi - \tau \times m \times (\pi - Z) - H(s) \quad (3)$$

76 This paper references the studies conducted by Slemrod and Yitzhaki (2000) and Yitzhaki
77 (1987) and proposes the probability (q) that a representative firm is caught for tax evasion to
78 express an incremental function representing concealed profit.

79 The expected utility function for a representative firm caught or not caught evading taxes
80 can thus be expressed as follows:

$$81 \quad EU(Q, Z) = (1 - q) \times U(\pi^{UA}) + q \times U(\pi^A) - A^d \times h - b \times V(E(Q) - \bar{Q}) \quad (4)$$

82 where $V(E(Q) - \bar{Q})$ is the negative effect when a firm's pollution emission exceeds $\bar{Q}$, and
83 b is the firm's level of concern for the external cost created as a result of the pollution
84 emissions generated during product and service production. The decision to be made by a
85 representative firm concerning its production output and tax evasion behavior can be
86 expressed as follows:

$$87 \quad \text{Max } U(Q, Z) = (1 - q) \times U(\pi^{UA}) + q \times U(\pi^A) - A^d \times h - b \times V(E(Q) - \bar{Q}) \quad (5)$$

$$88 \quad \text{s.t. } \pi^{UA} = (1 - m) \times \pi + m \times (\pi - Z) - H(s)$$

$$89 \quad \pi^A = (1 - m) \times \pi - \tau \times m \times (\pi - Z) - H(s)$$

90 The first-order conditions of this decision equation can be represented as follows:

$$91 \quad U_Z = m \times \left(\tau \times q \times U'(\pi^A) - (1 - q) \times U'(\pi^{UA}) \right) = 0 \quad (6)$$

$$92 \quad U_Q = (R' - C') \times \left[(1 - q) \times U'(\pi^{UA}) + q \times (1 - m) \times U'(\pi^A) - \tau \times m \times q \times U'(\pi^A) \right] - b \times V' \times E' = 0 \quad (7)$$

93 where U_Z and U_Q represent a firm's marginal expected utility in reported profit and in its
94 production output, respectively. Solving (6) and (7) yields a firm's optimal reported profit
95 (Z^*) and production output (Q^*). This paper investigates the second-order conditions of a

96 firm's decision-making problems, where

$$97 \quad U_{ZZ} < 0; \quad U_{QQ} < 0; \quad \text{and} \quad H = U_{ZZ} \times U_{QQ} - (U_{ZQ})^2 > 0.$$

98 The perspective of Wu and Yan (2011) and is adopted, and it is assumed that a
99 representative firm is not concerned with the external cost of its pollution emissions during
100 product and service production; that is, $b = 0$. Substituting this into (7) gives

$$101 \quad U_Q = (R' - C') \times [(1-q) \times U'(\pi^{UA}) + q \times (1-m) \times U'(\pi^A) - \tau \times m \times q \times U'(\pi^A)] = 0 \quad (8)$$

102 Equation (8) reveals that when marginal revenue equals marginal cost, a firm's reported
103 profit and its production output are neutral, a finding that supports the Wang-Conant (W-C)
104 proposition.

105 Applying the first-order conditions to (6) enables obtaining the following equation:

$$106 \quad U_{ZQ} = m \times (R' - C') \times [q \times \tau \times U''(\pi^A) \times \{(1-m) - \tau \times m\} - U''(\pi^{UA}) \times (1-q)] \quad (9)$$

107 Equation (6) shows that $\tau \times q \times U'(\pi^A) = (1-q) \times U'(\pi^{UA})$, which can be substituted into (9)
108 to obtain (10).

$$109 \quad U_{ZQ} = m \times (R' - C') \times q \times \tau \times U'(\pi^A) \times \{r(\pi^{UA}) - r(\pi^A) \times (1-m-\tau \times m)\} \quad (10)$$

$$110 \quad \text{where } r(\pi^A) = \frac{-U''(\pi^A)}{U'(\pi^A)} \quad \text{and} \quad r(\pi^{UA}) = \frac{-U''(\pi^{UA})}{U'(\pi^{UA})}.$$

111 i). Equation (10) demonstrates that when $b = 0$ and a firm is in equilibrium, then
112 $(R' - C') = 0$, which suggests that marginal revenue equals marginal costs and that the firm's
113 reported profit and production output have become neutral, supporting the W-C proposition
114 (1988).

115 ii). Equations (7) reveals that when $b > 0$, $(R' - C') = 0$ is not the optimal production output
116 decision. Under this scenario, $(R' - C') > 0$; hence, (10) shows that when a representative
117 firm's coefficient of absolute risk aversion decreases, or $r(\pi^A) > r(\pi^{UA})$,

118 $\frac{r(\pi^{UA})}{r(\pi^A)} > 1 - m - \tau \times m$ ensures that $U_{ZQ} > 0$, meaning that a firm's increased production output

119 leads to increased reported income and decreased tax evasion. Equation (10) thus carries the
120 following critical economic implication: when a firm emits more pollution than the pollution
121 emissions standard set by government, the psychological anxiety experienced by the firm
122 prompts it to increase its production and thus its reported profit, reducing tax evasion.

123 However, for this to occur, the following premise must be true: "the ratio between the
124 coefficient of absolute risk aversion when a firm is not caught for tax evasion and that of
125 absolute risk aversion ratio when a firm is caught for tax evasion" must be greater than "the
126 tax to be paid by the firm for each dollar earned minus the tax to be paid for each dollar
127 earned then multiplied by the tax evasion penalty."

2.3 Comparative Static Analyses of Firm Production Output and Reported Profits

This paper uses the implicit function theorem to determine the relationships of governmental policy instrument variables ($\Omega \in \{\tau, q, m\}$) with optimal production output and reported profit, where τ , q , m denote the penalty, probability that a firm is caught for tax evasion, and tax rate, respectively. Thus,

$$\frac{\partial Z^*}{\partial \Omega} = \frac{U_{z\Omega} \times U_{q\Omega} - U_{z\Omega} \times U_{q\Omega}}{H} \quad (11)$$

$$\frac{\partial Q^*}{\partial \Omega} = \frac{U_{z\Omega} \times U_{q\Omega} - U_{zz} \times U_{q\Omega}}{H} \quad (12)$$

Given that $H = U_{zz} \times U_{qq} - (U_{zq})^2 > 0$, whether (11) and (12) are greater or less than zero is determined by the sign of the numerator H .

i) When $b = 0$, (8) implies that $U_{z\Omega} = U_{q\Omega} = U_{q\Omega} = 0$. Accordingly, (11) and (12) can be expressed as follows:

$$\frac{\partial Z^*}{\partial \Omega} = \frac{-U_{z\Omega} \times U_{q\Omega}}{H} \quad (13)$$

$$\frac{\partial Q^*}{\partial \Omega} = 0 \quad (14)$$

Equation (14) shows that when $b = 0$, changes in policy instruments (e.g., tax rates, penalties, and probabilities that firms are caught for tax evasion) will not change a firm's production output model. Given that $U_{qq} < 0$ and that $H > 0$, whether (13) is greater or less than 0 is determined by whether $U_{z\Omega}$ is greater or less than 0.

Substituting the penalty τ , the probability that a firm is caught for tax evasion q , and the tax rate m into (6) yields the following equations:

$$U_{z\tau} = m \times [q \times U'(\pi^A) - m \times (\pi - Z) \times \tau \times q \times U''(\pi^A)] \quad (15)$$

Equation (15) reveals that $U_{z\tau} > 0$, indicating that a firm's reported profit and tax evasion penalty are in a complementary relationship.

$$U_{zq} = m \times [\tau \times U'(\pi^A) + U'(\pi^{UA})] > 0 \quad (16)$$

Equation (16) shows that a firm's reported profit (Z) and the probability that a firm is caught for tax evasion are also in a complementary relationship. Equation (6) shows that

154 $\tau \times q \times U'(\pi^A) = (1-q) \times U'(\pi^{UA})$. Substituting the coefficient of absolute risk aversion

155 $[r(\pi^i) = -U''(\pi^i)/U'(\pi^i), i = A, UA]$ into (17) yields (18):

$$156 \quad U_{Zm} = \tau \times q \times U'(\pi^A) - (1-q) \times U'(\pi^{UA}) - (\pi + \tau \times \pi) \times m \times \tau \times q \times U''(\pi^A) + \quad (17)$$

$$Z \times m \times (1-q) \times U''(\pi^{UA})$$

$$157 \quad U_{Zm} = m \times (1-q) \times U'(\pi^{UA}) \times [(\pi + \tau \times \pi) \times r(\pi^A) - Z \times r(\pi^{UA})] \quad (18)$$

158 Because $r(\pi^A) > r(\pi^{UA})$ and $(\pi + \tau \times \pi) > Z$, $U_{Zm} > 0$. Equations (15), (16), and (18)

159 reveal that the relationship between changes in policy instrument variables and a

160 representative firm's optimal reported profit is denoted by $U_{Z\Omega} > 0$; that is,

$$161 \quad \frac{\partial Z^*}{\partial \Omega} = \frac{-U_{Z\Omega} \times U_{\Omega\Omega}}{H} > 0$$

$$162 \quad \text{Where } \begin{bmatrix} U_{ZZ} & U_{ZQ} \\ U_{QZ} & U_{QQ} \end{bmatrix} \begin{bmatrix} dZ \\ dQ \end{bmatrix} = \begin{bmatrix} -U_{Z\Omega} \\ -U_{Q\Omega} \end{bmatrix}, \quad H = \begin{bmatrix} U_{ZZ} & U_{ZQ} \\ U_{QZ} & U_{QQ} \end{bmatrix} > 0.$$

163 This indicates that when the government increases penalties, tax rates, or the probability
164 of firms being caught for tax evasion, firms increase their reported profits.

165 ii) If $b > 0$, a firm will be psychologically anxious about emitting more pollution than the
166 pollution emissions standard set by the government; (10) shows that when

$$167 \quad \frac{r(\pi^{UA})}{r(\pi^A)} > 1 - m - \tau \times m, \quad U_{ZQ} > 0. \text{ In addition, (7) shows that } U_{Q\Omega} \neq 0. \text{ The calculations}$$

168 for the two variables are described next.

169 First, the effect of penalties on production output can be obtained by substituting τ into (7)

170 as a first-order condition:

$$171 \quad U_{Q\tau} = -(1-m) \times (R' - C') \times q \times m \times (\pi - Z) - m \times (R' - C') \times q \times U'(\pi^A)$$

$$+ (R' - C') \times \tau \times m^2 \times q \times U''(\pi^A) \times (\pi - Z)$$

$$172 \quad \begin{cases} \text{if } (R' - C') < 0, \text{ then } U_{Q\tau} > 0 \\ \text{if } (R' - C') > 0, \text{ then } U_{Q\tau} < 0 \end{cases} \quad (19)$$

173 Equation (19) demonstrates that if the tax evaded by a firm is the tax base for calculation
174 of the subsequent penalty and $(R' - C') > 0$, an increase in penalty reduces a firm's production

175 output ($U_{Q\tau} < 0$). However, if $(R' - C') < 0$, then an increase in penalty increases a firm's

176 output. In practice, $(R' - C') < 0$ rarely occurs in the long term.

177 Second, the effect of the probability of the government performing tax checking

178 (q) on production output (U_{Qq}) is calculated by substituting q into (7) as a first

179 -order condition:

$$180 \quad U_{Qq} = -(R' - C') \times U'(\pi^{UA}) + (1 - m) \times U'(\pi^A) \times (R' - C') - \tau \times m \times U'(\pi^A) \times (R' - C') \\ = -(R' - C') \times U'(\pi^{UA}) + U'(\pi^A) \times (R' - C') \times (1 - m - \tau \times m) \quad (20)$$

$$181 \quad \begin{cases} \text{If } (R' - C') > 0 \text{ and } m \geq \frac{1}{1 + \tau}, \text{ then } U_{Qq} < 0 \\ \text{If } (R' - C') > 0 \text{ and } m < \frac{1}{1 + \tau}, \text{ then } U_{Qq} \text{ may be positive or negative} \end{cases}$$

182 Equation (20) reveals that the tax evaded by a firm is the tax base for calculation of the

183 subsequent penalty; when $(R' - C') > 0$ and $m \geq \frac{1}{1 + \tau}$, an increase in the probability of the

184 government performing tax audit decreases a firm's output capacity ($U_{Qq} < 0$). However,

185 when $m < \frac{1}{1 + \tau}$, an increase in the probability of the government performing tax audit

186 may increase or decrease a firm's output capacity.

187 Third, the effect of tax rate on production output (U_{Qm}) is calculated by substituting m

188 into (7) as a first-order condition:

$$189 \quad U_{Qm} = -Z \times (1 - q) \times (R' - C') - (R' - C') \times q \times U'(\pi^A) - \\ (\pi + \tau \times \pi - \tau \times Z) \times q \times (1 - m) \times U''(\pi^A) \times (R' - C') - (R' - C') \times \tau \times q \times U'(\pi^A) \\ + \tau \times q \times m \times U''(\pi^A) \times (R' - C') \times (\pi + \tau \times \pi - \tau \times Z) \quad (21)$$

190 Equation (21) can be further modified into the following:

$$191 \quad U_{Qm} = -Z \times (1 - q) \times (R' - C') - (R' - C') \times q \times U'(\pi^A) - (R' - C') \times \tau \times q \times U'(\pi^A) \\ - (\pi + \tau \times \pi - \tau \times Z) \times q \times U''(\pi^A) \times (R' - C') \times (1 - m - \tau \times m) \quad (22)$$

192 Equation (22) shows that when $(R' - C') > 0$ and $m \geq \frac{1}{1 + \tau}$, then $U_{Qm} < 0$; conversely,

193 when $(R' - C') > 0$, and $m < \frac{1}{1 + \tau}$, then U_{Qm} may be positive or negative.

194 Equation (22) thus reveals that the condition $(R' - C') > 0$ alone cannot guarantee

195 whether an increase in tax rate has a positive or negative effect on production output.

This paper uses the implicit function theorem, (6), and (7) to identify the relationships of minimal changes in governmental policy instrument variables with a firm's optimal reported profit and production output, as represented by the following:

$$\frac{\partial Z^*}{\partial \Omega} = \frac{U_{zQ} \times U_{Q\Omega} - U_{z\Omega} \times U_{QQ}}{H} \quad (23)$$

$$\frac{\partial Q^*}{\partial \Omega} = \frac{U_{zQ} \times U_{QZ} - U_{zz} \times U_{Q\Omega}}{H} \quad (24)$$

Given that $H > 0$, whether (23) and (24) are greater than, equal to, or less than zero is determined by the sign of the numerator H .

The relationship between a firm's pollution emissions and its social responsibility is next explored.

iii) Relationship between a firm's social responsibility and its reported profit:

When $b = 0$, $U_{zQ} = U_{QZ} = U_{Q\Omega} = 0$, indicating that marginal revenue equals marginal cost and that a firm's reported profit and its production output have become neutral, supporting the W-C proposition; that is,

$$\frac{\partial Z^*}{\partial \Omega} = \frac{-U_{z\Omega} \times U_{QQ}}{H} \quad (25)$$

$$\frac{\partial Q^*}{\partial \Omega} = 0 \quad (26)$$

Equation (10) shows that when $b > 0$, the representative firm's coefficient of absolute risk aversion is decreased, or $r(\pi^A) > r(\pi^{UA})$. Therefore, $\frac{r(\pi^{UA})}{r(\pi^A)} > 1 - m - \tau \times m$ must be true to ensure that $U_{zQ} > 0$. This signifies that an increase in a firm's production output will increase its reported profit and reduce its tax evasion behavior. In addition, (19)–(22) show that when $b > 0$, increases in penalties, the probability of tax audit by government, and the tax rate do not guarantee that firms' tax evasion behavior will be effectively deterred.

iv) When a firm has a higher social responsibility (b), the amount of profit reported by the firm will be higher only if $\frac{r(\pi^{UA})}{r(\pi^A)} < 1 - m - \tau \times m$; this subsequently lowers the firm's tax evasion behavior, as shown in (27).

Given that $U_Z = m \times \left(\tau \times q \times U'(\pi^A) - (1-q) \times U'(\pi^{UA}) \right) = 0$ (6), $U_{zb} = 0$, and given that

$$U_Q = (R' - C') \times \left[(1-q) \times U'(\pi^{UA}) + q \times (1-m) \times U'(\pi^A) - \tau \times m \times q \times U'(\pi^A) \right] - b \times V' \times E' = 0$$

$U_{Qb} = -V' \times E' < 0$, the following equation can be derived by using $U_{QQ} < 0, U_{ZZ} < 0$,

224 and $H > 0$:

$$225 \quad \frac{\partial Z^*}{\partial b} = \frac{U_{ZQ} \times U_{Qb} - U_{Zb} \times U_{QQ}}{H},$$

$$226 \quad \begin{cases} \text{when } \frac{r(\pi^{UA})}{r(\pi^A)} > 1 - m - \tau \times m, U_{ZQ} > 0 \text{ and } \frac{\partial Z^*}{\partial b} < 0 \\ \text{when } \frac{r(\pi^{UA})}{r(\pi^A)} < 1 - m - \tau \times m, U_{ZQ} < 0 \text{ and } \frac{\partial Z^*}{\partial b} > 0 \end{cases} \quad (27)$$

227 Equation (27) reveals that when $\frac{r(\pi^{UA})}{r(\pi^A)} < 1 - m - \tau \times m$, an increase in a firm's social
 228 responsibility will increase its reported profit, reducing its tax evasion behavior. However,
 229 when $\frac{r(\pi^{UA})}{r(\pi^A)} > 1 - m - \tau \times m$, an increase in a firm's social responsibility will lower its
 230 reported profit, increasing its tax evasion behavior.

231 v) Regardless of the tax rate, an increase in a firm's social responsibility will diminish
 232 its output capacity:

$$233 \quad \frac{\partial Q^*}{\partial b} = \frac{U_{Zb} \times U_{QZ} - U_{ZZ} \times U_{Qb}}{H} = \frac{-U_{ZZ} \times U_{Qb}}{H} \quad (28)$$

234 Given that $U_{Qb} < 0$, $U_{Zb} = 0$, $U_{ZZ} < 0$, and $H > 0$, $\frac{\partial Q^*}{\partial b} < 0$.

235 Equation (28) demonstrates that when a firm has a higher social responsibility ($b > 0$), an
 236 increase in the firm's production output will result in an increase in the negative effect. Thus,
 237 when a firm has a higher social responsibility, it will lower its output capacity to reduce the
 238 negative effect.

239 vi) The effect of increased pollution emission standards ($\bar{Q}$) on a firm's production output
 240 and reported profit is explored. Equation (6) shows that $U_{Z\bar{Q}} = 0$. When

$$241 \quad \frac{r(\pi^{UA})}{r(\pi^A)} > 1 - m - \tau \times m, U_{ZQ} > 0, H > 0, \frac{\partial Z^*}{\partial \bar{Q}} > 0, \text{ and } U_{Q\bar{Q}} = V'' \times E' \times b > 0 \text{ (as shown}$$

242 in (7)), (29) is used to determine that $\frac{\partial Z^*}{\partial \bar{Q}} > 0$. Accordingly, an increase in pollution

243 emission standards will facilitate raising a firm's reported profit:

$$244 \quad \begin{bmatrix} U_{ZZ} & U_{ZO} \\ U_{QZ} & U_{QQ} \end{bmatrix} \begin{bmatrix} dZ \\ dQ \end{bmatrix} = \begin{bmatrix} -U_{Z\bar{Q}} \\ -U_{Q\bar{Q}} \end{bmatrix}$$

$$\frac{\partial Z^*}{\partial \bar{Q}} = \frac{U_{zQ} \times U_{Q\bar{Q}} - U_{z\bar{Q}} \times U_{QQ}}{H} > 0 \quad (29)$$

vii) Similarly, given that $U_{zQ} = 0$, $U_{ZZ} < 0$, $U_{Q\bar{Q}} > 0$, and $H > 0$, the following equation can be derived:

$$\frac{\partial Q^*}{\partial \bar{Q}} = \frac{U_{QZ} \times U_{z\bar{Q}} - U_{Q\bar{Q}} \times U_{ZZ}}{H} > 0 \quad (30)$$

which reveals that $\frac{\partial Q^*}{\partial \bar{Q}} > 0$ and that an increase in pollution emission standards set

by government can help increase a firm's output capacity and thereby its production output.

3. Subgame Perfect Equilibrium Between Taxpayers and Government

This paper explores the production function of representative firms (an exogenous variable) and their tax evasion behavior in addition to the definitional equations of and causal relationships between firms' tax evasion behavior and the tax rates and subsidy rates offered by the government to domestic firms engaging in air pollution control to reduce external costs caused by production. This paper adopts the perspective of Baron (2001) and Wu and Yan (2011) (i.e., that pollution may be emitted during a firm's product and/or service production) and the model introduced by Hsu and Tsui (2005) (i.e., the optimal air pollution tax and subsidy rate model to be used by government while considering the spillover effect) to analyze the two-stage game describing the interaction between tax evaders and government. In Stage 1 of the game, firms identify their optimal production output and air pollution emissions, whereas in Stage 2, the government substitutes the spillover effect and firms' production output and air pollution emissions into the social welfare function to determine the optimal tax subsidy rates to be offered to firms engaging in air pollution control. In the two-stage game settings, when all market participants play the tax game while knowing all information, backward induction can be used to solve the subgame perfect equilibrium between firms and government. Calculations can then be made to identify the optimal production output and air pollution emissions (i.e., Stage 1 solutions) and determine the optimal tax and subsidy rates to be offered by the government to firms engaging in air pollution control (i.e., Stage 2).

3.1 Model Assumptions

Assuming that a firm's revenue function is $R(Q_i)$; m is the air pollution tax levied by government when a firm emits air pollution that exceeds the emission standards; D_0 is the air pollution allowed to be emitted during the production process, as stipulated by the environmental protection agency; $C_i^{max}(Q_i, \bar{D}_i)$ is the production cost for air pollution emitted by the i^{th} firm when the firm does not engage in air pollution control; $C_i(Q_i, \bar{D}_i)$ is the i^{th} firm's production cost if it engages in air pollution control; and $\bar{D}_i$ is the air pollution emitted by the i^{th} firm when the firm does engage in air pollution control. $\frac{\partial C_i}{\partial Q_i} > 0$ and $\frac{\partial C_i}{\partial \bar{D}_i} < 0$, where $\bar{D}_i < \bar{D}_i$.

$C_i(Q_i, \bar{D}_i) - C_i^{max}(Q_i, \bar{D}_i)$ denotes the cost to a firm engaging in air pollution prevention and control. $g_i(\bar{D}_j)$ is defined as the spillover effect of the j^{th} firm's air pollution emissions on the i^{th} firm, where the spillover effect causes the i^{th} firm to invest in air pollution control (where the cost is $g_i(\bar{D}_j) \times [C_i(Q_i, \bar{D}_i) - C_i^{max}(Q_i, \bar{D}_i)]$).

In particular, $g_i(\bar{D}_j) = 1$ indicates that the j^{th} firm has no spillover effect on the i^{th} firm. Note that $0 < g_i(\bar{D}_j) \leq 1$ and $g_i(\bar{D}_j) > 0$; the smaller $g_i(\bar{D}_j)$ is, the stronger the spillover effect becomes. R_Q is the marginal revenue of the i^{th} firm for

producing one additional unit of product or service; C_Q^{max} and C_Q are the marginal cost of the i^{th} firm for producing one additional unit of product or service if they do not and do engage in air pollution control, respectively; a_u is the subsidy rate offered by the government for a firm that engages in air pollution control; v_i is the tax

erosion rate; and $M_h(\bar{D}_i - D_0)$ is a firm's expected penalty when its air pollution emissions exceed the standards (Harford, 1978).

The model introduced in this paper assumes that the i^{th} firm's air pollution concealment cost is minimal (close to zero) and that its profit equals its revenue plus the government's air

pollution control subsidy minus its production costs, air pollution control costs, air pollution tax levied by the government, and expected penalty for air pollution emissions that exceed the standards. Accordingly, the objective function that maximizes profit for the firm is as follows:

$$\underset{Q_i, D_i}{\text{Max}} \pi_i = R(Q_i) - C_i(Q_i, D_i) + a_u \times g_i(D_j) \times \left[C_i(Q_i, D_i) - C_i^{\text{max}}(Q_i, D_i) \right] - M_h(D_i - D_h) - m \times D_i \times (1 - v_i) \quad (31)$$

3.2 Optimal Decisions for Representative Firms

Based on the aforementioned assumptions, given that the financial policy instruments (i.e., air pollution tax and subsidy rate) are known, and assuming that the i^{th} firm is in a perfectly competitive market, the first-order derivative function describing production output and air pollution emission quantity can be solved using (31):

$$\pi_{Q_i} \equiv \frac{\partial \pi}{\partial Q} = R_Q - C_Q + a_u \times g_i(D_j) \times \left[C_Q - C_Q^{\text{max}} \right] = 0 \quad (32)$$

$$\pi_{D_i} \equiv \frac{\partial \pi}{\partial D} = -C_D + a_u \times g_i(D_j) \times C_D - M_h' - m \times (1 - v_i) = 0 \quad (33)$$

Through using conventional model settings, the second-order conditional derivative functions for profit maximization can be formulated:

$$\frac{\partial^2 \pi}{\partial Q_i^2} = R_{QQ} - C_{QQ} + a_u \times g_i(D_j) \times \left[C_{QQ} - C_{QQ}^{\text{max}} \right] < 0 \quad \text{and}$$

$$\frac{\partial^2 \pi}{\partial D_i^2} = C_{DD} \times \left[a_u \times g_i(D_j) - 1 \right] - M_h'' < 0$$

Through taking the total derivative of (32) and (33) simultaneously, the following equation can be derived:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} dQ_i \\ dD_i \end{bmatrix} = \begin{bmatrix} -g_i(D_j) \times [C_{QQ} - C_{QQ}^{\text{max}}] \times da_u - a_u \times [C_{QQ} - C_{QQ}^{\text{max}}] \times g_i' dD_j \\ d(m \times (1 - v)) - g_i(D_j) \times C_D \times da_u - a_u \times C_D \times g_i' dD_j \end{bmatrix} \quad (34)$$

where $Z_{11} \equiv \frac{\partial^2 \pi}{\partial Q_i^2} = R_{QQ} - C_{QQ} + a_u \times g_i(D_j) \times [C_{QQ} - C_{QQ}^{\text{max}}] < 0$, $Z_{12} \equiv [a_u \times g_i(D_j) - 1] \times C_{QD} \geq 0$,

$$322 \quad Z_{21} \equiv \left[a_u \times g_i(\overset{\otimes}{D}_j) - 1 \right] \times C_{\overset{\otimes}{D}Q} \geq 0, \text{ and } Z_{22} \equiv \frac{\partial^2 \pi}{\partial \overset{\otimes}{D}_i^2} = C_{\overset{\otimes}{D}\overset{\otimes}{D}} \times \left[a_u \times g_i(\overset{\otimes}{D}_j) - 1 \right] - M_h'' < 0.$$

$$323 \quad \Gamma = Z_{11} \times Z_{22} - Z_{12} \times Z_{21}$$

324 Comparative static derivative functions can be derived from (34):

$$325 \quad \frac{\partial Q_i}{\partial m} = \frac{-(1-v_i) \times Z_{12}}{\Gamma} < 0 \quad (35)$$

$$326 \quad \frac{\partial Q_i}{\partial v_i} = \frac{m \times Z_{12}}{\Gamma} > 0 \quad (36)$$

$$327 \quad \frac{\partial Q_i}{\partial a_u} = \frac{-Z_{22} \times g_i(\overset{\otimes}{D}_j) \times [C_Q - C_Q^{\max}] + Z_{12} \times g_i(\overset{\otimes}{D}_j) \times C_{\overset{\otimes}{D}}}{\Gamma} \quad (37)$$

$$328 \quad \frac{\partial Q_i}{\partial \overset{\otimes}{D}_j} = \frac{a_u \times g' \times [Z_{12} \times C_{\overset{\otimes}{D}} - Z_{22} \times (C_Q - C_Q^{\max})]}{\Gamma} \quad (38)$$

$$329 \quad \frac{\partial \overset{\otimes}{D}_i}{\partial m} = \frac{(1-v_i) \times Z_{11}}{\Gamma} < 0 \quad (39)$$

$$330 \quad \frac{\partial \overset{\otimes}{D}_i}{\partial v_i} = \frac{-m \times Z_{11}}{\Gamma} > 0 \quad (40)$$

$$331 \quad \frac{\partial \overset{\otimes}{D}_i}{\partial a_u} = \frac{g_i(\overset{\otimes}{D}_j) \times [Z_{21} \times (C_Q - C_Q^{\max}) - Z_{11} \times C_{\overset{\otimes}{D}}]}{\Gamma} \quad (41)$$

$$332 \quad \frac{\partial \overset{\otimes}{D}_i}{\partial \overset{\otimes}{D}_j} = \frac{a_u \times g' \times [Z_{21} \times (C_Q - C_Q^{\max}) - Z_{11} \times C_{\overset{\otimes}{D}}]}{\Gamma} \quad (42)$$

333 Assuming that the conditions for economic stability are established

334 ($\Gamma = Z_{11} \times Z_{22} - Z_{12} \times Z_{21} > 0$), the following economic implications can be inferred:

335 i) Equation (35) reveals that the air pollution tax levied by government decreases a
336 firm's output capacity;

- ii) Equation (39) shows that the air pollution tax levied by government reduces a firm's air pollution emissions;
- iii) Equation (36) indicates that an increase in a firm's tax erosion rate increases its output capacity;
- iv) Equation (40) demonstrates that raising a firm's tax erosion rate increases its air pollution emissions; and
- v) All remaining factors may produce dissimilar results according to the signs of the relevant variables, thus indicating no constancy in the results.

Proposition 1: Assuming that a firm is in a perfectly competitive market, the conditions for economic stability will be present when the financial policy instruments (e.g., air pollution tax and subsidy) are known. In addition, an increase in a firm's tax erosion rate will increase its output capacity [Equation (36)]; an increase in a firm's tax erosion rate will cause its air pollution emissions to rise [Equation (40)], and the air pollution tax levied by government decreases a firm's output capacity [Equation (35)].

3.3 Optimal Decisions for Government

In this section, the backward induction method is adopted to investigate how the government can formulate the optimal tax policy for controlling firms' air pollution emissions. This entails the government setting the optimal air pollution tax rate and subsidy rate to maximize the benefits to society. These benefits include (i) consumer surplus, (ii) producer surplus, (iii) air pollution tax levied, and (iv) penalty income from firms whose air pollution emissions exceed the standards. However, the system also involves the negative items of social welfare, including (v) government expenditure because of subsidies granted to firms that engage in air pollution control, and (vi) the external cost to the society due to because of air pollution emissions.

To simplify the calculation process, this paper defines the number of firms as N . Accordingly, the objective function for the maximization of the benefit to society can be expressed as follows:

$$\begin{aligned} \underset{a_u, m}{Max} S_w = & \sum_{i=1}^N [U(Q_i) - P \times Q_i] + \sum_{i=1}^N [R(Q_i) - C_i(Q_i, D_i^{\otimes})] + \sum_{i=1}^N m \times (1 - v_i) \times D_i^{\otimes} + \sum_{i=1}^N M_h(D_i^{\otimes} - D_0) \\ & - a_u \times \sum_{i=1, i \neq j}^N g_i(D_j^{\otimes}) \times \left[C_i(Q_i, D_i^{\otimes}) - C_i^{max}(Q_i, D_i^{\exists}) \right] - \sum_{i=1}^N I(D_i^{\otimes}) \end{aligned} \quad (43)$$

$$s.t. \quad \sum_{i=1}^N m \times (1 - v_i) \times D_i^{\otimes} + \sum_{i=1}^N M_h(D_i^{\otimes} - D_0) = a_u \times \sum_{i=1, i \neq j}^N g_i(D_j^{\otimes}) \times \left[C_i(Q_i, D_i^{\otimes}) - C_i^{max}(Q_i, D_i^{\exists}) \right] \quad (44)$$

- 370 i) where $\sum_{i=1}^N [U(Q_i) - P \times Q_i]$ is the consumer surplus;
- 371 ii) $\sum_{i=1}^N [R(Q_i) - C(Q_i, \bar{D}_i)]$ is the producer surplus;
- 372 iii) $\sum_{i=1}^N m \times (1 - v_i) \times \bar{D}_i$ is the air pollution tax levied;
- 373 iv) $\sum_{i=1}^N M_h(\bar{D}_i - D_0)$ is the penalty income from firms whose air pollution emissions
- 374 exceed the standards;
- 375 v) $a_u \times \sum_{i=1, i \neq j}^N g_i(\bar{D}_j) \times [C_i(Q_i, \bar{D}_i) - C_i^{\max}(Q_i, \bar{D}_i)]$ is the government expenditure due to
- 376 subsidies granted to firms that engage in air pollution control; and
- 377 vi) $\sum_{i=1}^N I(\bar{D}_i)$ is the external cost to society due to air pollution emissions.

378 If a government uses the air pollution tax exclusively to pay for the subsidy granted to

379 firms that engage in air pollution, the objective function for the maximization of the benefit

380 to society can be obtained by substituting (44) into (43), producing the following equation:

$$381 \quad \underset{a_u, m}{Max} S_W = \sum_{i=1}^N [U(Q_i) - P \times Q_i] + \sum_{i=1}^N [R(Q_i) - C(Q_i, \bar{D}_i)] - \sum_{i=1}^N I(\bar{D}_i) \quad (45)$$

382 4. Solving the Subgame Perfect Equilibrium Between Firms 383 and Government by Using Backward Induction 384

385 4.1 Government's Optimal Subsidy Rate

386 By taking a first-order derivative of (45) with respect to the subsidy rate and making it

387 equal to zero, the following can be obtained:

$$388 \quad [U_{Q_i} - P] + [R_{Q_i} - C_{Q_i}] \times \sum_{i=1}^N \left(\frac{\partial Q_i}{\partial a_u} \right) = (C_{\bar{D}_i} + I') \times \sum_{i=1}^N \left(\frac{\partial \bar{D}_i}{\partial a_u} \right) \quad (46)$$

389 By substituting $R_{Q_i} - C_{Q_i} = -a_u \times g_i(\bar{D}_j) \times [C_{Q_i} - C_{Q_i}^{\max}]$ from (32) and

390 $C_{\bar{D}_i} = \frac{M_h' + m \times (1 - v_i)}{a_u \times g_i(\bar{D}_j) - 1}$ from (33) into (46), the following is obtained:

$$\left[-a_u \times g_i(\bar{D}_j) \times (C_Q - C_Q^{\max}) \right] \times \sum_{i=1}^N \left(\frac{\partial Q_i}{\partial a_u} \right) = \left[\frac{M'_h + m \times (1 - v_i)}{a_u \times g_i(\bar{D}_j) - 1} + I' \right] \times \sum_{i=1}^N \left(\frac{\partial \bar{D}_i}{\partial a_u} \right) \quad (47)$$

Assuming that the “rate of subsidy offered by the government to a firm engaging in air pollution control” and “firm’s output capacity” are neutral, the optimal subsidy rate (an endogenous variable) to be offered by the government to firms that engage in air pollution control can be obtained using (47), as given by

$$a_u^* = \frac{I' - (m \times (1 - v_i) + M'_h)}{I' \times g_i(\bar{D}_j)} \quad (48)$$

4.2 Government’s Optimal Air Pollution Tax Rate

To identify the optimal tax rate, a first-order derivative of (45) with respect to tax rate was taken and then equaled to zero, from which the following is obtained:

$$\left[(U_Q - P) + (R_Q - C_Q) \right] \times \sum_{i=1}^N \left(\frac{\partial Q_i}{\partial m} \right) = (C_D + I') \times \sum_{i=1}^N \left(\frac{\partial \bar{D}_i}{\partial m} \right) \quad (49)$$

Next, $R_Q - C_Q = -a_u \times g_i(\bar{D}_j) \times \left[C_Q - C_Q^{\max} \right]$ (i.e., a firm’s optimal first-order condition)

from (32) and $C_D = \frac{M'_h + m \times (1 - v_i)}{a_u \times g_i(\bar{D}_j) - 1}$ from (33) are substituted into (49):

$$\begin{aligned} \left[-a_u \times g_i(\bar{D}_j) \times (C_Q - C_Q^{\max}) \right] \times \sum_{i=1}^N \left(\frac{\partial Q_i}{\partial m} \right) &= \left[\frac{M'_h + m \times (1 - v_i)}{a_u \times g_i(\bar{D}_j) - 1} + I' \right] \times \sum_{i=1}^N \left(\frac{\partial \bar{D}_i}{\partial m} \right) \\ \Rightarrow \left[-a_u \times g_i(\bar{D}_j) \times (C_Q - C_Q^{\max}) \right] \times \frac{\sum_{i=1}^N \left(\frac{\partial Q_i}{\partial m} \right)}{\sum_{i=1}^N \left(\frac{\partial \bar{D}_i}{\partial m} \right)} &= \left[\frac{M'_h + m \times (1 - v_i)}{a_u \times g_i(\bar{D}_j) - 1} + I' \right] \end{aligned} \quad (50)$$

Assuming that “rate of subsidy offered by the government to a firm engaging in air pollution control” and “firm’s output capacity” are neutral, (50) can be simplified to the following:

$$\frac{M_h' + m \times (1 - v_i)}{a_u \times g_i(D_j) - 1} = -I'$$

Accordingly, the optimal tax rate (an endogenous variable) to be levied by the government can be obtained from the following:

$$m^* = \frac{I' \times \left[1 - a_u \times g_i(D_j) \right] - M_h'}{(1 - v_i)} \quad (51)$$

By considering the spillover effect, the optimal air pollution tax rate and optimal subsidy rate models can be inferred and used to verify the following proposition:

Proposition 2: Assuming that the government's subsidy rate (a_u) for firms that engage in air pollution control is an exogenous variable, the stronger the spillover effect (i.e., the lower $g_i(D_j)$ is), the greater the tax rate should be. In addition, when representative firms' tax erosion rate (v_i) increases, this is a signal to government that it should increase its tax rate to encourage the firms to research and develop air pollution control technologies, which will subsequently increase the spillover effect and enhance the benefit to society (S_w).

5. Conclusion

This study investigated game analyses for tax evaders and tax-levying governments. In the two-stage game settings, when all market participants play the tax game while knowing all information, backward induction can be used to solve the subgame perfect equilibrium between firms and government. The calculation process is divided into two stages; the identification by firms of the optimal production output and air pollution emissions, and the substitution of firms' production output and air pollution emissions into the social welfare function by government to determine the optimal tax and subsidy rates to be offered to firms engaging in air pollution control. Assuming that firms are in a perfectly competitive market and that financial policy instruments (i.e., air pollution tax and subsidy rate) are known, conditions for economic stability can be established. The present study also demonstrated that an increase in a firm's tax erosion rate increases its output capacity and air pollution emissions, whereas an increase in the air pollution tax levied by government decreases a firm's output capacity.

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