

On application of two- stage stochastic fully fuzzy linear programming for water resources management optimization

H. A. Khalifa^a and M. Al- Shabi^b

^a Department of Operations Research, Institute of Statistical Studies and Research, Cairo University,
Giza, Egypt

^b Department of Management Information System, College of Business Administration, Taibah
University, Saudi Arabia

Abstract

In this paper, a management of water resources is investigated based on a two- stage stochastic fully fuzzy linear programming. The problem having various scenarios of policy associated with economics penalties of different levels. A proposed approach convert the problem into a triple-objective problem and then a weighting method is utilized for solving it. The advantage of the approach is to generate a set of solutions for water resources planning which help the decision maker to make tradeoffs between the efficiency of economic and the risk violation of the constrains. A case study is given for illustration.

Keywords: Water resources; Policy analysis; Uncertainty; Two- stage; Multi- objective programming; Weighting method

1. Introduction

Water is an essential life- sustaining resource whose existence and availability for human use are often taken for granted. It is often utilized by people who are unaware of where the water originated and what happens to the water boiling use. Beyond meting direct human use, water enables all living species it survives and flourishes and is a renewable natural source (Fu et al. [5]). Water Management is defined as the control and movement of Meta resources that is to minimize the damage to life and property and to maximize efficient beneficial use(Jimenez et al. [11]). One advantage of the good water management of dams and levees is to reduce of the risk of harm which due to flooding (Jimenez et al. [11]). It is clear that the irrigate water management systems make the limited water supplies for agriculture most efficient used.

Corresponding Authors:

E- mail: mshaby@taibahu.edu.sa, malshabi@yahoo.com, hamiden_2008@yahoo.com

^a Present Address: Department of Mathematics, College of Arts and Science, Al- Badaya, Qassim University, Saudi Arabia.

Water Resources Management is the integrating concept for a number of water sub- sectors such as hydropower, water supply and sanitation, irrigation and drainage, and environmental (Gasimov and Yenilmez [6]) .

An integrated water resources perspective ensures that economics, environmental and technical dimensions are taken into account in the management and development of water resources. The water resources management includes (Jairaj and Vedula [10]): The quantitative and qualitative exploration of water resources, water needs an inventory records, measurement and matching of the water resources and water needs (demands) in a special system, and decision support depending with light of the results .

A long with development and regulation, Kirpich [16] investigated another phrase (Holistic water management) which used to outline water management. Grigg [7] introduced a brief of integrated water resources management (IWRM) definition. As known, fuzzy set theory was introduced by Zadeh [33] to deal with fuzziness. Up to now, fuzzy set theory has been applied to broad fields. Fuzzy set theory introduced by Zadeh [33], make a model has to be set up using data which is approximately known. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers). For the fuzzy set theory development, we may referee to the papers of Kaufmann [12], and Dubois and Prade [2], they extended the use of algebraic operations of real numbers to fuzzy numbers by the use a fuzzification principle. Fuzzy linear constraints with fuzzy numbers were studied by Dubois and Prade [2]. Lu et al. [18] introduced the definition of an inexact rough interval fuzzy linear programming method and investigated for generating water allocation to agricultural irrigation system. Shaocheng [22] studied two kinds of linear programming problems with fuzzy numbers called: interval numbers and fuzzy number linear programming, respectively. Tanaka et al. [24] have formulated and proposed a method for solving fuzzy coefficients linear programming. Wang and Qiao [27] put forward a model of linear programming with fuzzy random variable coefficients. Wang and Huang [29] developed an interactive two- stage stochastic fuzzy programming for managing water resources. They proposed an interactive resolution method within an inexact two- stage stochastic programming. A two- stage optimization framework for planning reservoir operations is proposed by Wang and Adams [26], where hydrologic uncertainty and seasonally of reservoir inflows has been modeled as periodic Markov process. Through a two- stage dynamic programming approach, a long- term hydrothermal scheduling of multi- reservoir systems has been examined by Ferrero et al. [4]. Bellman and Zadeh [1] introduced the concept of a maximizing decision making problem. Zhao et al. [34] introduced the complete solution set for the fuzzy linear programming problems using linear and nonlinear

membership functions. For water resources management, Huang and Loucks [9] proposed an inexact two- stage stochastic programming. An interactive fuzzy resolution method for solving linear programming problems with fuzzy parameters is proposed by Jimenez et al. [11]. For water resources management developing, a number of optimization techniques have been developed (Slowinski [23]; Wu et al. [30]; Jairaj and Vedula[10]; Maqsood et al. [19], and Wang et al. [28]. A model for obtaining an optimal multi- period operation within a multi- reservoir system is developed by Eiger and Shamir [3]. Xu et al. [31] investigated and applied an inexact two- stage fuzzy gradient chance- constrained programming method to the water resources management in the Heshui River Basin, Jiangxi Province. To quantify the economic trade- offs when reducing groundwater abstraction to sustainable level, Mortinsen et al.[20] applied a multi- objective multi- temporal deterministic hydro economic optimization approach for this purpose. Fu et al. [5] proposed a two- level a symmetric Nash- Harany leader- follower game model to resolve the conflict arises when different water users compete for a limited water supply. Veeramani et al. [25] studied fuzzy MOLP problem with fuzzy technological coefficients and resources. They solved the problem based on the proposed method introduced by Gasimov and Yenimez[6]. Kiruthiga and Loganathan [15] reduced the Fuzzy MOLP problem to the corresponding ordinary one using the ranking function and hence solved it using the fuzzy programming technique. Hamadameen [8] proposed a technique for solving fuzzy MOLP problem in which the objective functions coefficients are triangular fuzzy numbers. Khalifa [14] studied a water resources management problem as an application of a two- stage fuzzy random programming.

The rest of the paper is organized as follows: Some preliminaries needed in the sake of the paper are given in section2. In section 3, a fully water resources management problem is introduced as specific definition and properties. A solution procedure for solving the problem is considered in section4. In section 5, a numerical example is given for illustration. Finally some concluding remarks are reported in section 7.

2. Preliminaries

In this section, Some basic concepts and related results to fuzzy numbers and some of their arithmetic operations, triangular fuzzy numbers and some of algebraic operations are reviewed

(Kaufmann and Gupta [13], Sakawa [21], Zimmermann [35], and Liang et al. [17]) .

Definition1.(Kaufmann and Gupta [13]). Let R be the set of real numbers, the fuzzy number $\tilde{p}$ is a mapping

$\mu_{\tilde{p}} : R \rightarrow [0, 1]$, with the following properties:

- (i) $\mu_{\tilde{p}}(x)$ is an upper semi- continuous membership function;
- (ii) $\tilde{p}$ is a convex set, i. e., $\mu_{\tilde{p}}(\lambda x^1 + (1 - \lambda)x^2) \geq \min\{\mu_{\tilde{p}}(x^1), \mu_{\tilde{p}}(x^2)\}$ for all $x^1, x^2 \in R, 0 \leq \lambda \leq 1$;
- (iii) $\tilde{p}$ is normal, i. e., $\exists x_0 \in R$ for which $\mu_{\tilde{p}}(x) = 1$;
- (iv) $\text{Supp}(\tilde{p}) = \{x : \mu_{\tilde{p}}(x) > 0\}$ is the support of a fuzzy set $\tilde{p}$.

Definition2. (Sakawa [21]). A triangular fuzzy number(T.F.N.) can be represented completely by a triplet $\tilde{A} = (a_1, a_2, a_3)$, with the following membership is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3, \\ 0, & x > a_3. \end{cases}$$

Also, T.F.N. parametric form for level α can be characterized as:

$$\begin{aligned} \tilde{A}_\alpha &= [(a_2 - a_1)\alpha + a_1, -(a_3 - a_2)\alpha + a_3] \\ &= [\underline{v}(\alpha), \bar{v}(\alpha)]; \forall 0 < \alpha \leq 1. \end{aligned}$$

Definition3. A T.F.N. $\tilde{A} = (a_1, a_2, a_3)$ is called non- negative triangular fuzzy number if $a_1 \geq 0$.

Definition4. (Zimmermann [35]). Let $\tilde{A} = (a_1, a_2, a_3)$, and $\tilde{B} = (b_1, b_2, b_3)$ be two non- negative triangular fuzzy numbers, the formulas for the addition, subtraction, scalar multiplication, and multiplication can be defined:

1. Addition:

$$\begin{aligned} \tilde{A} \oplus \tilde{B} &= (a_1, a_2, a_3) \oplus (b_1, b_2, b_3) \\ &= (a_1 + b_1, a_2 + b_2, a_3 + b_3). \end{aligned}$$

2. Subtraction:

$$\begin{aligned} \tilde{A}(-)\tilde{B} &= (a_1, a_2, a_3)(-)(b_1, b_2, b_3) \\ &= (a_1 - b_3, a_2 - b_2, a_3 - b_1). \end{aligned}$$

3. Multiplication:

$$\tilde{A} \otimes \tilde{B} = \begin{cases} (a_1 b_1, a_2 b_2, a_3 b_3), & a_1 \geq 0 \\ (a_1 b_3, a_2 b_2, a_3 b_1), & a_1 < 0, a_3 \geq 0 \\ (a_1 b_3, a_2 b_2, a_3 b_1), & a_3 < 0 \end{cases}$$

4. Scalar multiplication

$$k\tilde{A} = \begin{cases} (ka_1, ka_2, ka_3), & k > 0, \\ (ka_3, ka_2, ka_1), & k < 0. \end{cases}$$

Remark1. $\tilde{A} \geq \tilde{0}$ if and only if $a_1 \geq 0, a_1 - a_2 \geq 0, a_1 + a_3 \geq 0$.

Definition5.(Kaufmann and Gupta [13]). A trapezoidal fuzzy number(Tr.F.N.) can be represented completely by a quadruplet $\tilde{A} = (a_1, a_2, a_3, a_4)$,with the following membership is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2, \\ \frac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4, \\ 0, & x > a_4 \end{cases}$$

Also, a Tr. F.N. can be characterized by its interval of confidence (or parametric form) at interval of confidence at level α is defined by Liang et al. [17]:

$$\begin{aligned} \tilde{A}_\alpha &= [(a_2 - a_1)\alpha + a_1, -(a_4 - a_3)\alpha + a_4] \\ &= [v(\alpha), v(\alpha)]; \forall 0 < \alpha \leq 1. \end{aligned}$$

Remark2. A Tr. F.N $\tilde{A} = (a_1, a_2, a_3, a_4)$ can also signify a T.F.N. $\tilde{v} = (a_1, a_2, a_3) = ((v)^-, (v)^c, (v)^+)$, if $a_2 = a_3$.

3. Problem formulation and solution concepts

A typical water resources management problem (Huang and Loucks[9]; Wang and Huang [29]as follows:

$$\max f = \sum_{j=1}^n \tilde{B}_j T_j - E \left[\sum_{j=1}^n C_j S_{jQ} \right] \quad (1)$$

Subject to

$$\sum_{j=1}^n (T_j - S_{jQ})(1 + \delta) \leq Q \quad (2)$$

(Water availability constraints)

$$S_{jQ} \leq T_j \leq T_{j\max}; \forall j, \quad (3)$$

(Water- allocation target constraints)

$$S_{jQ} \geq 0; \forall j \quad (4)$$

(Non- negative and technical constraints)

Where,

f : A benefit of system (\$);

B_j : Net benefit to user j per m^3 of water allocated (\$/ m^3) / (First- stage revenue parameters)

T_j : Allocation target for water that is promised to user j (m^3) /(First- Stage decision variables)

$E [.]$: Expected value of a random variable;

C_j : Loss to user j per m^3 of water not delivered, $C_j > NB_j$ (\$/ m^3)/ (Second- Stage cost parameters)

S_{jQ} : Shortage of water to user j when the seasonal flow is Q (m^3)

(Second- Stage decision variables))

Q : Total amount of seasonal flow (m^3) (random variables);

δ : Rate of water loss during transportation;

$T_{j\max}$: Maximum allowable allocation amount for user j (m^3);

m : Total number of water users;

i : Water user, $i = 1, 2, 3$, where $i = 1$ for municipality, $i = 2$ for the industrial user, and $i = 3$ for the agricultural sector

Referring to Huang and Loucks [9] , the above problem can be reformulated as in the following form

$$\max f = \sum_{j=1}^n B_j T_j - \sum_{i=1}^m \sum_{j=1}^n p_i C_j S_{ij} \quad (5)$$

Subject to

$$\sum_{j=1}^n (T_j - S_{ij})(1 + \delta) \leq q_i; \forall i, \quad (\text{Water availability constraints}) \quad (6)$$

$$S_{ij} \leq T_j \leq T_{j_{\max}}; \forall, j \quad (\text{Water- allocation target constraints}) \quad (7)$$

$$S_{ij} \geq 0; \forall i, j \quad (\text{Non- negative and technical constraints}) \quad (8)$$

Where, S_{ij} is the amount by which the water- allocation target T_j is not met when the seasonal flow is q_i with probability p_i .

Consider the fuzzy model for the problem(5)- (8) as

$$\max \tilde{f} = \sum_{j=1}^n \tilde{B}_j \otimes \tilde{T}_j - \sum_{i=1}^m \sum_{j=1}^n p_i (\tilde{C}_j \otimes \tilde{S}_{ij}) \quad (9)$$

Subject to

$$\sum_{j=1}^n (\tilde{T}_j (-) \tilde{S}_{ij}) (1 \oplus \tilde{\delta}) (\leq) \tilde{q}_i; \forall i, \quad (10)$$

$$\tilde{S}_{ij} (\leq) \tilde{T}_j (\leq) \tilde{T}_{j_{\max}}; \forall, j \quad (11)$$

$$\tilde{S}_{ij} \geq \tilde{0}; \forall i, j \quad (12)$$

Where, $\tilde{B}_j, \tilde{C}_j, \tilde{\delta}, \tilde{q}_i, T_{j_{\max}}; \tilde{T}_j$, and $\tilde{S}_{ij}; \forall i, j$ are triangular fuzzy parameters and variables.

Definition6. (Optimal fuzzy solution). The $(\tilde{S}_{ij})^*$ which satisfy the conditions in (10)- (12) is called a fuzzy optimization solution .

4. Solution procedure

In this section a solution procedure for solving the problem (9)- (12) is investigated. The steps of the procedure are given as in the following steps:

Step1: Convert problem (9)- (12) into the corresponding problem using the arithmetic operations of T.F.N.

$$\max \tilde{f} = \sum_{j=1}^n ((B_j T_j)^l, (B_j T_j)^c, (B_j T_j)^u) - \sum_{i=1}^m \sum_{j=1}^n p_i ((C_j S_{ij})^l, (C_j S_{ij})^c, (C_j S_{ij})^u) \quad (13)$$

Subject to

$$\left(\sum_{j=1}^n ((T_j)^l - (S_{ij})^l, (T_j)^c - (S_{ij})^c, (T_j)^u - (S_{ij})^u) (1 + \delta^l, 1 + \delta^c, 1 + \delta^u) \leq \right. \\ \left. ((q_i)^l, (q_i)^c, (q_i)^u) \right) \forall i, \quad (14)$$

$$\left((S_{ij})^l, (S_{ij})^c, (S_{ij})^u\right) \leq \left((T_j)^l, (T_j)^c, (T_j)^u\right) \leq \left((T_{j \max})^l, (T_{j \max})^c, (T_{j \max})^u\right); \forall, j \quad (15)$$

$$\left((S_{ij})^l, (S_{ij})^c, (S_{ij})^u\right) \geq \tilde{0}; \forall i, j \quad (16)$$

Problem (13)- (16) can be rewritten as

$$\max \tilde{f} = \sum_{j=1}^n \left((B_j T_j)^l, (B_j T_j)^c, (B_j T_j)^u \right) - \sum_{i=1}^m \sum_{j=1}^n p_i \left((C_j S_{ij})^l, (C_j S_{ij})^c, (C_j S_{ij})^u \right) \quad (17)$$

Subject to

$$M = \left\{ \begin{array}{l} \sum_{j=1}^n \left((T_j)^l - (S_{ij})^l \right) (1 + \delta^l) \leq (q_i)^l; \forall i \\ \sum_{j=1}^n \left((T_j)^c - (S_{ij})^c \right) (1 + \delta^c) \leq (q_i)^c; \forall i \\ \sum_{j=1}^n \left((T_j)^u - (S_{ij})^u \right) (1 + \delta^u) \leq (q_i)^u; \forall i \\ (S_{ij})^l \leq (T_j)^l \leq (T_{j \max})^l; \forall, j \\ (S_{ij})^c \leq (T_j)^c \leq (T_{j \max})^c; \forall, j \\ (S_{ij})^u \leq (T_j)^u \leq (T_{j \max})^u; \forall, j \\ (S_{ij})^l \geq 0, (S_{ij})^c - (S_{ij})^l \geq 0, (S_{ij})^u - (S_{ij})^c \geq 0; \forall i, j \end{array} \right. \quad (18)$$

Step2: Referring to the problem (17) - (18), it can be converted into

$$\begin{aligned} \max f_1 &= \left(\sum_{j=1}^n (B_j T_j)^c - \sum_{i=1}^m \sum_{j=1}^n p_i (C_j S_{ij})^c \right) \\ \min f_2 &= \left(\sum_{j=1}^n \left((B_j T_j)^u - (B_j T_j)^l \right) - \sum_{i=1}^m \sum_{j=1}^n p_i \left((C_j S_{ij})^u - (C_j S_{ij})^l \right) \right) \\ \max f_3 &= \left(\sum_{j=1}^n \left((B_j T_j)^u + (B_j T_j)^l \right) - \sum_{i=1}^m \sum_{j=1}^n p_i \left((C_j S_{ij})^u + (C_j S_{ij})^l \right) \right) \\ \text{Subject to: } & \left((S_{ij})^c, (S_{ij})^l, (S_{ij})^u \right) \in M \end{aligned} \quad (19)$$

Step3: Solve problem(19) by using the weighting method(Zadeh 1963)[32]

$$\begin{aligned} \max & (0.5 f_1 - 0.2 f_2 + 0.3 f_3) \\ \text{Subject to} & \left((S_{ij})^c, (S_{ij})^l, (S_{ij})^u \right) \in M; \forall i, j \end{aligned} \quad (20)$$

5. Numerical example

Consider the problem introduced by Wang and Huang [29] involving triangular fuzzy numbers as

Table1. Economic data (\$/ m³) and seasonal flows (in 10⁶ m³) under different probability levels

Activity	User		
	Municipal($i = 1$)	Industrial($i = 2$)	Agricultural($i = 3$)
Maximum allowable allocation ($\tilde{T}_{\max}$)	(7,8,9)	(7,8,10)	(7,8,9)
Target of water allocation($\tilde{T}_j$)	(1,2,3)	(2,3,5)	(2,4,5)
Net benefit when water demand is satisfied($\tilde{B}_j$)	(95,100,110)	(45,50,70)	(28,30,33)
Reduction of the net benefit when demand is not delivered ($\tilde{C}_j$)	(220,250,285)	(55,60,90)	(45,50,75)
Continue			
Flow level	Probability		Seasonal flow(%)
Low($i = 1$)	0.3		(2,3,4)
Medium($i = 2$)	0.5		(7,9,13)
High ($i = 3$)	0.2		(14,16,20)
Water loss($\tilde{\delta}$)			(0.15,0.20,0.40)

Putting the above values in the MOLP problem (20) as

$$\max \left[\begin{array}{l} 484 - 3.75(S_{11})^c - 62.5(S_{21})^c - 25(S_{31})^c - 9(S_{12})^c - 15(S_{22})^c - 6(S_{32})^c \\ - 7.5(S_{13})^c - 12.5(S_{23})^c - 5(S_{33})^c - 8.55(S_{11})^u - 33(S_{11})^l - 14.25(S_{21})^u \\ - 55(S_{21})^l - 5.7(S_{31})^u - 22(S_{31})^l - 2.7(S_{12})^u - 8.25(S_{12})^l - 4.5(S_{22})^u \\ - 13.75(S_{22})^l - 1.8(S_{32})^u - 5.5(S_{32})^l - 2.25(S_{13})^u - 6.75(S_{13})^l \\ - 3.75(S_{23})^u - 11.25(S_{23})^l - 1.5(S_{33})^u - 4.4(S_{33})^l \end{array} \right]$$

Subject to

$$\begin{aligned}
(S_{11})^l + (S_{12})^l + (S_{13})^l &\geq 4.9826, \\
(S_{21})^l + (S_{22})^l + (S_{23})^l &\geq 4.9391, \\
(S_{31})^l + (S_{32})^l + (S_{33})^l &\geq 4.8783, \\
(S_{11})^c + (S_{12})^c + (S_{13})^c &\geq 8.975, \\
(S_{21})^c + (S_{22})^c + (S_{23})^c &\geq 8.925, \\
(S_{31})^c + (S_{32})^c + (S_{33})^c &\geq 8.8667, \\
(S_{11})^u + (S_{12})^u + (S_{13})^u &\geq 12.97, \\
(S_{21})^u + (S_{22})^u + (S_{23})^u &\geq 12.907, \\
(S_{31})^u + (S_{32})^u + (S_{33})^u &\geq 12.857, \\
0 \leq (S_{11})^u - (S_{11})^c &\leq 1; \\
0 \leq (S_{12})^u - (S_{12})^c &\leq 1; \\
0 \leq (S_{13})^u - (S_{13})^c &\leq 1, \\
0 \leq (S_{21})^u - (S_{21})^c &\leq 1; \\
0 \leq (S_{22})^u - (S_{22})^c &\leq 1; \\
0 \leq (S_{23})^u - (S_{23})^c &\leq 1, \\
0 \leq (S_{31})^u - (S_{31})^c &\leq 1; \\
0 \leq (S_{32})^u - (S_{32})^c &\leq 1; \\
0 \leq (S_{33})^u - (S_{33})^c &\leq 1; \\
0 \leq (S_{11})^l, (S_{21})^l, (S_{31})^l, (S_{12})^l, (S_{22})^l, (S_{32})^l, (S_{13})^l, (S_{23})^l, (S_{33})^l &\leq 1.
\end{aligned}$$

Table2. Optimal fuzzy solution

Variable	Optimal fuzzy value	Optimum fuzzy value $\tilde{f}^* = (0, 0, 502.2849)$
$\tilde{S}_{11}$	(0,1,2)	
$\tilde{S}_{12}$	(0,1,2)	
$\tilde{S}_{13}$	(6.97, 7.97, 9.97)	
$\tilde{S}_{21}$	(0,0,1)	
$\tilde{S}_{22}$	(0,1,2)	
$\tilde{S}_{23}$	(7.907, 8.907, 9.097)	
$\tilde{S}_{31}$	(0,0,1)	
$\tilde{S}_{32}$	(0,1,2)	
$\tilde{S}_{33}$	(7.857, 8.857, 9.957)	

6. Conclusions

The studying of fully fuzzy linear programming for water resources management problem due to its close connection with human life, which is considered to be great importance. The advantages of two- stage stochastic fully fuzzy linear programming framework are:

- Handle dual uncertainties as triangular fuzzy numbers,
- Provides the decision makers (DMs) with the information about the risk, and
- Enables the DM to quantify the relationship between the value of the objective function and the violating risk.

A proposed approach has been used to convert the problem under study into a multi- objective problem. The advantage of the approach is significant for being used in interactive methods for making any comment by related managers and achieving the solutions logically. Finally, fully fuzzy linear programming for water resources management taking into account the minimal S_{ij} by which the water- allocation target T_j is not met when the seasonal flow is q_i with probability p_i is recommended.

Acknowledgements

The authors are very grateful to the anonymous reviewers for his/ her insightful and constructive comments and suggestions that have led to an improved version of this paper.

References

- [1]. Bellman, R. E., and Zadeh, L. A. (1970). Decision making in a fuzzy environment. *Management Science*, 17, 141-164.
- [2]. Dubois, D., and Prade, H. (1980). *Possibility Theory: An Approach to Computerized Processing of Uncertainty*, Plenum, New York.
- [3]. Eiger, G., and Shamir, U. (1991). Operation of reservoir by stochastic programming. *Engineering Optimization*, (17): 293- 312.
- [4]. Ferrero, R. W., Rivera, J. F., and Shahidehpour, S. M. (1998). A dynamic programming two- stage algorithm for long- term hydrothermal scheduling of multireservoir systems. *IEEE Transactions on Power Systems*, (13): 1534- 1540.
- [5]. Fu, J., Zhong, P., Zhu, F., Ghen, J., Wu, Y., and Xu, B. (2018). Water resources allocation in transboundary river based on asymmetric nash- harsanyi leader- follower game model. *Water*, (10): 270- 287.
- [6]. Gasinov, N.R., and Yenilmez, K. (2002). Solving fuzzy linear programming with linear membership functions. *Turkish Journal of Mathematics*, (26): 375- 396.
- [7]. Grigg, S. N. (2008). Integrated water resources management: Balancing views and improving practice. *Journal of Water International*, 33 (3): 279- 292.

- [8].Hamadameen, O.A.(2017). A noval technique for solving multi- objective linear programming problems. Aro- The Scientific Journal of Koya University, 5(2): 1-8.
- [9].Huang, H. G., and Loucks, D. P. (2000). An inexact two- stage stochastic programming model for water resources management under uncertainty. Civil Engineering and Environmental Systems, (17): 95- 118.
- [10]. Jairaj, P. G., and Vedula, S. (2000). Multi-reservoir system optimization using fuzzy mathematical programming. Water Resources Management, (14): 457- 472.
- [11]. Jimenez, M., Arenas, M., Bilbao, A., and Rodriguez, M. V. (2007). Linear programming with fuzzy parameters: An interactive method resolution. European Journal of Operational Research, (177): 1599- 1609.
- [12]. Kaufmann, A. (1975). Introduction to the Theory of Fuzz Subsets, Vol. I. Academic Press, New York.
- [13]. Kaufmann, A., and Gupta, M. M. (1988). Fuzzy Mathematical Models in Engineering and Management Science, Elsevier Science Publishing Company INC., New York.
- [14]. Khalifa, H.A.(2015). On two- stage fuzzy random programming for water resources management. African Journal of Mathematics and Computer Science Research, 8(3): 31- 36.
- [15]. Kiruthiga, M., and Loganathan, C. (2015). Fuzzy multi- objective linear programming problem using membership function. International Journal of Science, Engineering, and Technology Research, 4(7): 2681- 2686.
- [16]. Kirpich, P. Z. (1993). Holistic approach to irrigation management in developing countries. Journal of Irrigation and Drainage, ASCE, 119(2): 323- 333.
- [17]. Liang,T. F., Chiu,C. S., and Cheng, H.W. (2015). Using possibilistic linear programming for fuzzy transportation planning decisions. Hsiuping Journal, (11): 93- 112.
- [18]. Lu, H., Huang, G., and He, L. (2011). An inexact rough- interval fuzzy linear programming method for generating conjunctive water- allocation strategies to agricultural irrigation systems. Applied Mathematical Modelling, (35): 4330- 4340.
- [19]. Maqsood, I., Huang, G. H., and Yeomans, H. G. (2005). An interval- parameter fuzzy two- stage stochastic program for water resources management under uncertainty. European Journal of Operational Research, (167): 208- 225.
- [20]. Mortinsen, G., Liu, S., Mo, X., Bauer- Gottwen, P. (2018). Optimizing water resources allocation in the Haihe River basin underground water sustainability constraints. Geophysical Research Abstract, 20, EGU2018- 12870-1.
- [21]. Sakawa, M. (1993). Fuzzy Sets and Interactive Multi-objective Optimization. New York, USA: Plenum Press.

- [22]. Shaocheng, T. (1994). Interval number and fuzzy number linear programming. *Fuzzy Sets and Systems*, (66): 301- 306.
- [23]. Slowinski, R. (1986). A multicriteria fuzzy linear programming method for water supply system development planning. *Fuzzy Sets and Systems*, (19): 217- 237.
- [24]. Tanaka, H., Tchihashi, H., and Asal, K. (1984). A formulation of fuzzy linear programming problem based on comparison of fuzzy number. *Control of Cybernetics*, 13(3): 184- 194.
- [25]. Veeramani, C., and Duraisamy, C., and Naagoorgani, A. (2011). Solving fuzzy multi-objective linear programming problems with linear membership functions. *Australian Journal of Basic and Applied Sciences*, 5(8): 1171- 1178.
- [26]. Wang, D., and Adams, B.J. (1986). Optimization of real- time reservoir operations with Markov decision processes. *Water Resources Research*, (22): 345- 352.
- [27]. Wang, G., and Qiao, Z. (1993). Convergence of sequences of fuzzy random variables and its application. *Fuzzy Sets and Systems*, (59): 295- 311.
- [28]. Wang, L.Z., Fang, L., and Hipel, K.W. (2003). Water resources allocations: Cooperative game theoretic approach. programming for resources management. *Journal of Environmental Informatics*, (2): 11- 22.
- [29]. Wang, S., and Huang, H. G. (2011). Interactive two- stage stochastic fuzzy programming for resources management. *Journal of Environmental Management*, (92): 1986- 1995.
- [30]. Wu, S. M., Huang, G. H., and Guo, H. C. (1997). An interactive inexact- fuzzy approach for multi-objective planning of water resource systems. *Water Science and Technology*, (36): 235- 242.
- [31]. Xu, J., Huang, G., Li, Z., and Chen, J. (2017). A two- stage fuzzy chance- constrained water management model. *Environmental Science and Pollution Research*, 24(3): 12437- 12454.
- [32]. Zadeh, L. A. (1963). Optimality and non- scalar valued performance criteria. *IEEE Transactions on Automatic Control*, AC-8, 59- 60.
- [33]. Zadeh, L. A. (1965). Fuzzy sets. *Information Control*, (8) 338- 353.
- [34]. Zhao, R., Govind, R., and Fan, G. (1992). The complete decision set of the generalized symmetrical fuzzy linear programming problem. *Fuzzy Sets and Systems*, (51): 53-65.
- [35]. Zimmermann, H.J. (1990). *Fuzzy sets Theory and Applications*, Kluwer- Nijhoff. Boston, MA.