

Numerical Study of Viscous Dissipation and Temperature Dependent Viscosity on MHD Free Convection Flow Over a Sphere with Heat Conduction

ABSTRACT

The effect of viscous dissipation and temperature dependent viscosity on MHD free convection flow over a sphere with heat conduction has been analyzed in the article. The governing equations are transformed into dimensionless non-similar equations by using set of useful transformations and solved numerically by finite difference method along with Newton's linearization approximation. Attention has been focused on the evaluation of shear stress in terms of local skin friction and local Nusselt number, the velocity and the temperature profiles over the whole boundary layer are shown graphically and tabular form for different values of the Prandtl's number P_r , magnetic parameter M , the temperature dependent viscosity ε , viscous dissipation parameter N and the Joule heating parameter J .

Keywords: Free convection, Joule-heating, Magneto hydrodynamics, Variation viscosity and viscous dissipation.

1 .INTRODUCTION

Free convection flow is often encountered in cooling of nuclear reactors or in the study of the structure of stars and planets. Along with the free convection flow the phenomenon of the boundary layer flow of an electrically conducting fluid over a sphere in the presence of a Joule-heating term and magnetic field are also very common because of their applications in nuclear engineering in connection with the cooling of reactors. Numerical study of viscous dissipation and temperature dependent viscosity on magneto hydrodynamic (MHD) free convection flow with heat conduction have been taken on various geometrical shapes such as vertical flat plate, cylinder, sphere etc. studied by many investigators and it has been a very popular research topic for many years. Alam *et al.* [1] investigated the viscous dissipation effects on MHD natural convection flow over a sphere in the presence of heat generation. The effect of viscous dissipation on natural convection flow alone a sphere with heat generation is considered by Akter, S., *et al.* [2]. Miraj, M., *et al.* [3] discussed the conjugate effects of radiation and viscous dissipation on natural convection flow over a sphere with pressure work. Molla, M. M., *et al.* [4] carried out the effects of temperature dependent thermal conductivity on MHD natural convection flow from an isothermal sphere. The effects of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate with heat generation and Joule heating examined by

Islam, S., *et al.* [5]. Nasrin, R., *et al.* [6] performed the combined effects of viscous dissipation and temperature dependent thermal conductivity on magneto hydrodynamic (MHD) free convection flow with conduction and joule heating along a vertical flat plate. Gitima [7] presented the analysis of the effect of variable viscosity and thermal conductivity in micro polar fluid for a porous channel in presence of magnetic field. Cebeci, T., & Bradshaw, P. [8] established the physical and computational aspects of convective heat transfer. Keller H.B. [9] developed the numerical methods in boundary layer theory. Nasrin, R., *et al.* [10] made the MHD free convection flow along a vertical flat plate with thermal conductivity and viscosity depending on temperature. Nabil Eldabe T.M., *et al.* [11] explored the effects of temperature dependent viscosity and viscous dissipation on MHD convection flow from an isothermal horizontal circular cylinder in the presence of stress work and heat generation. Safiqul Islam K. M., *et al.* [12] established the effects of temperature dependent thermal conductivity on natural convection flow along a vertical flat plate with heat generation. Molla, M. M., *et al.* [13] studied the effect of temperature dependent viscosity on MHD natural convection flow from an isothermal sphere. Alim, M. A., *et al.* [14] showed the heat generation effects on MHD natural convection flow along a vertical wavy surface with variable thermal conductivity. Md. Raihanul Haque *et al.* [15] observed the effects of viscous dissipation on natural convection flow over a sphere with temperature dependent thermal conductivity. Charruaudeau, J. [16] examined the influence de gradients de propriétés physiques en convection force application au cas du tube. Nazar *et al.* [17] established the free convection boundary layer on an isothermal sphere in a micro polar fluid. Molla, M.M. *et al.* [18] performed the natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation / absorption. Md., M. Alam *et al.* [19], analyzed the free convection from a vertical permeable circular cone with pressure work and non-uniform surface temperature. Pinakee Dey *et al.* [20] studied the effects an asymptotic method for over-damped forced nonlinear vibration systems with slowly varying coefficients.

In the present work, we have investigated the viscous dissipation and temperature dependent viscosity with MHD and Joule heating effect on the skin friction and the local heat transfer coefficient in the entire region from up stream to down stream of a viscous incompressible and electrically conducting fluid over a sphere. The transformed non similar boundary layer equations governing the flow together with the boundary conditions based on conduction and convection were solved numerically using the implicit finite difference method with Keller box [9] scheme by Cebeci and Bradshaw [8] along with Newton's linearization approximation method. We have studied the effect of the Prandtl's number Pr , the viscous dissipation parameter N , the Joule heating parameter J , Magnetic parameter M and variation viscosity parameter ε on the velocity and temperature fields as well as on the skin friction and local heat transfer coefficient. All the investigations for the fluid with low Prandtl's number appropriate for the liquid metals are carried out.

2. FORMULATION OF THE PROBLEM

We consider a steady two-dimensional natural convection boundary layer flow of an electrically conducting and viscous incompressible fluid over a sphere, H_0 is the magnetic field strength and σ is the electrical conductivity. The surface temperature of the sphere is assumed as T_w and T_∞ being

80 the ambient temperature of the fluid. When $T_w > T_\infty$ an upward flow is established along the surface
 81 due to free convection and the flow is downward for $T_w < T_\infty$.
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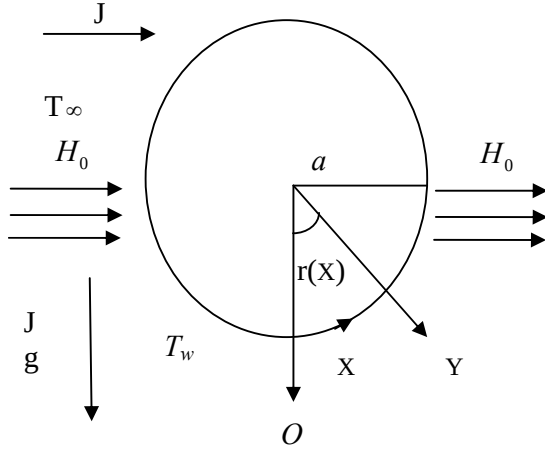


Fig.-1. Physical model and coordinate system

83
 84
 85 The mathematical model for the assumed physical problem is prescribed by the following
 86 conservation equation of mass, momentum and energy. Under these considerations the governing
 87 equations are

$$\frac{\partial}{\partial X}(rU) + \frac{\partial}{\partial Y}(rV) = 0 \quad (1)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{1}{\rho} \frac{\partial}{\partial Y} \left(\mu \frac{\partial U}{\partial Y} \right) + g\beta(T - T_\infty) \sin\left(\frac{X}{a}\right) - \frac{\sigma H_0^2 U}{\rho} \quad (2)$$

$$U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{K}{\rho C_p} \frac{\partial^2 T}{\partial Y^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial U}{\partial Y} \right)^2 + \frac{\sigma H_0^2 U^2}{\rho C_p} \quad (3)$$

88 The boundary conditions for the governing equations are

$$\left. \begin{aligned} U = V = 0, \quad T = T_w \quad \text{on} \quad Y = 0 \\ U \rightarrow 0, T \rightarrow T_\infty \quad \text{at} \quad Y \rightarrow \infty \end{aligned} \right\} \quad (4)$$

$$r = a \sin\left(\frac{X}{a}\right), \text{ where } r = r(X) \quad (5)$$

89 Where a is the radius of the sphere and r is the radial distance from the symmetrical axis to the
 90 surface of the sphere. Here we will consider $\mu = \frac{\mu_\infty}{1 + \alpha(T - T_\infty)}$ is the dependent viscosity where

$$\alpha = \frac{1}{\mu_f} \left(\frac{\delta \mu}{\delta T} \right)_f.$$

92 3. TRANSFORM OF THE GOVERNING EQUATIONS

93 The above equations are made dimensionless by using the following substitutions:

$$x = \frac{X}{a}, y = Gr^{1/4} \frac{Y}{a}, u = \frac{U}{u_0} = \frac{a}{\nu} Gr^{1/2} U, \quad (6)$$

$$v = \frac{a}{\nu} Gr^{1/4} V, \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

94 Where, $u_0 = \frac{\nu}{a} Gr^{1/2}$ is the characteristic velocity of the fluids. Here, we will consider

$$95 \quad \beta = -\frac{1}{\rho} \left(\frac{\delta \rho}{\delta T_f} \right)_p, \quad Gr = \frac{g \beta a^3}{\nu^2} (T_w - T_\infty),$$

$$96 \quad \varepsilon = \frac{1}{\mu_f} \left(\frac{\delta \mu}{\delta T} \right)_f (T - T_\infty), \quad \frac{\mu}{\mu_\infty} = \frac{1}{1 + \varepsilon \theta}$$

97 Using the above transformations into equations (1) to (3), we have

$$\therefore \frac{\delta}{\delta x} (ru) + \frac{\delta}{\delta y} (rv) = 0 \quad (7)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{-\varepsilon}{(1 + \varepsilon \theta)^2} \frac{\partial u}{\partial y} \frac{\partial \theta}{\partial y} + \frac{1}{1 + \varepsilon \theta} \frac{\partial^2 u}{\partial y^2} + \theta \sin x - Mu \quad (8)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + N \left(\frac{\partial u}{\partial y} \right)^2 + J(u^2) \quad (9)$$

98 The boundary conditions associated with (9) to (10) become

$$\left. \begin{aligned} u = v = 0, \quad \theta = 1 \quad \text{at } x = 0, \text{ for any } y \\ u = v = 0, \quad \theta = 1 \quad \text{at } y = 0, x > 0 \\ u \rightarrow 0, \quad \theta \rightarrow 0 \quad \text{as } y \rightarrow \infty, x > 0 \end{aligned} \right\} \quad (10)$$

99 Here, Where, $u_0 = \frac{\nu}{a} Gr^{1/2}$ is the characteristic velocity of the fluids, $M = \frac{\sigma H_0^2 a^2}{\mu_\infty Gr^{1/2}}$ is magnetic

100 parameter $Gr = g \beta (T_w - T_\infty) a^3 / \nu^2$ is the Grashof number and θ is the non-dimensional temperature

101 function, $Pr = \frac{\mu C_p}{k_\infty}$ is the Prandtl's number, and $\varepsilon = \frac{1}{\mu_f} \left(\frac{\delta \mu}{\delta T} \right)_f (T - T_\infty)$ is the variation viscosity

102 parameter. Also $N = \frac{Gr}{a^2 C_p (T_w - T_\infty)}$ is the viscous dissipation parameter, $J = \frac{\sigma H_0^2 a g \beta (T_w - T_\infty)}{\rho C_p}$ is

103 the Joule heating parameter. To solve equations (9) and (10) subject to the boundary conditions (11),

104 we assume the following variables $u = \frac{1}{r} \frac{\partial \psi}{\partial y}$ and $v = -\frac{1}{r} \frac{\partial \psi}{\partial x}$ where $\psi = x r(x) f(x, y)$ and $\psi(x, y)$ is

105 a non-dimensional stream function which is related to the velocity components in the usual way as

$$106 \quad u = \frac{1}{r} \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial x}$$

107

The momentum and energy equations (9) to (10) reduce to

$$\frac{1}{1+\varepsilon\theta} \frac{\delta^3 f}{\delta y^3} + \left(1 + \frac{x}{\sin x} \cos x\right) f \frac{\delta^2 f}{\delta y^2} - \left(\frac{\delta f}{\delta y}\right)^2 - \frac{\varepsilon}{(1+\varepsilon\theta)^2} \frac{\delta\theta}{\delta y} \frac{\delta^2 f}{\delta y^2} \quad (12)$$

$$+ \frac{\theta \sin x}{x} - M \frac{\partial f}{\partial y} = x \left(\frac{\delta f}{\delta y} \frac{\delta^2 f}{\delta y \delta x} - \frac{\delta f}{\delta x} \frac{\delta^2 f}{\delta y^2} \right)$$

$$\frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} + \left(1 + \frac{x}{\sin x} \cos x\right) f \frac{\partial \theta}{\partial y} + Nx^2 \left(\frac{\partial^2 f}{\partial y^2} \right)^2 \quad (13)$$

$$J \left(\frac{\partial f}{\partial y} \right)^2 = x \left(\frac{\partial f}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial f}{\partial x} \frac{\partial \theta}{\partial y} \right)$$

108 The corresponding boundary conditions are

$$\left. \begin{aligned} f &= \frac{\partial f}{\partial y} = 0, \quad \theta = 1 \text{ at } y = 0 \quad \text{for any } y \\ f &= \frac{\partial f}{\partial y} = 0, \quad \theta = 1 \text{ at } y = 0, x > 0 \\ \frac{\partial f}{\partial y} &\rightarrow 0, \theta \rightarrow 0 \text{ as } y \rightarrow \infty, x > 0 \end{aligned} \right\} \quad (14)$$

109 In practical application, the physical quantities of principal interest are the heat transfer and the skin-
110 friction coefficient, which can be written in non- dimensional form as

$$Nu_x = \frac{aGr^{-1/4}}{k(T_w - T_\infty)} q_w \text{ and } Cf_x = \frac{Gr^{-3/4}a^2}{\mu\nu} \tau_w \quad (15)$$

111 Where $q_w = -k \left(\frac{\partial T}{\partial Y} \right)_{Y=0}$ and $\tau_w = \mu \left(\frac{\partial U}{\partial Y} \right)_{Y=0}$, k being the thermal conductivity of the fluid. Using the

112 new variables (6), we have the simplified form of the heat transfer and the skin- friction coefficient as

$$\theta'(x, 0) = Nu_x = - \left(\frac{\partial \theta}{\partial y} \right)_{y=0} \text{ and } f''(x, 0) = Cf_x = x \left(\frac{\partial^2 f}{\partial y^2} \right)_{y=0} \quad (16)$$

113 4. METHOD OF SOLUTION

114 This paper deals with the natural convection flow on variation viscosity and viscous dissipation of
115 viscous incompressible fluid over a heated sphere with Joule heating and magneto hydrodynamics
116 being investigated using the very efficient implicit finite difference method known as the Keller box
117 scheme developed by Keller [9], which has been well documented by Cebeci and Bradshaw [8].

118 To apply the aforementioned method, equations (12) and (13) with their boundary conditions (14) are
119 first converted into the following system of first order equations. For this purpose we introduce new
120 dependent variables $u(x, y)$, $v(x, y)$, $p(x, y)$ and $g(x, y)$ so that the transformed momentum and
121 energy equations can be written as

$$f' = u \quad (17)$$

$$u' = v \quad (18)$$

$$g' = p \quad (19)$$

$$P_1 v' + P_2 f u' - u^2 - P_3 g u' + P_4 g - P_5 u = x \left(u \frac{\partial u}{\partial x} - v \frac{\partial f}{\partial x} \right) \quad (20)$$

$$\frac{1}{\text{Pr}} p' + P_1 f p - P_4 u g + P_6 (u')^2 + P_7 u^2 = x \left(u \frac{\partial g}{\partial x} - p \frac{\partial f}{\partial x} \right) \quad (21)$$

122 where $x = x, \theta = g$ and $P_1 = \frac{1}{1 + \varepsilon \theta}, P_2 = 1 + \frac{x \cos x}{\sin x}, P_3 = \frac{\varepsilon}{(1 + \varepsilon \theta)^2},$

123 $P_4 = \frac{\sin x}{x}, P_5 = M, P_6 = N \varepsilon^2, P_7 = J$

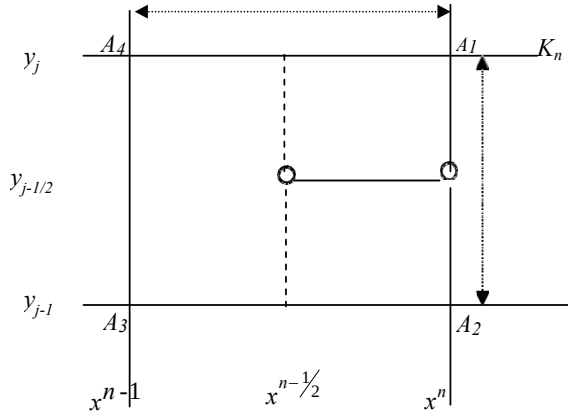


Fig. 2: Net rectangle of difference approximations for the Box scheme.

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125

126 and the boundary conditions (14) are

$$\begin{aligned} f(x, 0) = 0, \quad u(x, 0) = v(x, 0) = 0, \quad g(x, 0) = 1, \quad x > 0 \\ u(0, y) = v(0, y) = 0, \quad g(0, y) = 1 \\ u \rightarrow 0, v \rightarrow 0 \text{ as } y \rightarrow \infty, \quad x > 0 \end{aligned} \quad (22)$$

127

128 Now we consider the net rectangle on the (x, y) plane as shown in the Fig. 2 and denote the net

129 points by

$$x^0 = 0, x^n = x^{n-1} + k_n, n = 1, 2, 3, \dots, N \text{ and } y_0 = 0, y_j = y_{j-1} + h_j, j = 1, 2, 3, \dots, J \quad (23)$$

130 Here n and j are just sequence of numbers on the (x, y) plane, k_n and h_j are the variable mesh

131 widths. Approximate the quantities f, u, v and p , at the points (x^n, y_j) of the net by

132 $f_j^n, u_j^n, v_j^n, p_j^n$ which call net function. It is also employed that the notation P_j^n for the

133 quantities midways between net points shown in Fig. 2 and for any net functions as

$$x^{n-1/2} = \frac{1}{2}(x^n + x^{n-1}) \quad (24)$$

$$y_{j-1/2} = \frac{1}{2}(y_j + y_{j-1/2}) \quad (25)$$

$$g_j^{n-1/2} = \frac{1}{2}(g_j^n + g_j^{n-1}) \quad (26)$$

134 The finite difference approximation according to box method to the three first order ordinary
 135 differential equation (18)-(20) are written for the mid - point $\left(x^n, y_{j-1/2}\right)$ of the segment A_1A_2 shown
 136 in Fig.2.

$$\frac{f_j^n - f_{j-1}^n}{h_j} = u_{j-1/2}^n = \frac{u_{j-1}^n + u_j^n}{2} \quad (27)$$

$$\frac{u_j^n - u_{j-1}^n}{h_j} = v_{j-1/2}^n = \frac{v_j^n + v_{j-1}^n}{2} \quad (28)$$

$$\frac{g_j^n - g_{j-1}^n}{h_j} = p_{j-1/2}^n = \frac{p_j^n + p_{j-1}^n}{2} \quad (29)$$

137 The finite difference approximation to the first order differential equation (20) and (21) are written for
 138 the mid - point $\left(x^{n-1/2}, y_{j-1/2}\right)$ of the rectangle $A_1A_2A_3A_4$. This procedure yields

$$\begin{aligned} & \frac{1}{2} \left(P_1 \frac{v_j^n - v_{j-1}^n}{h_j} \right) + \frac{1}{2} \left(P_1 \frac{v_j^{n-1} - v_{j-1}^{n-1}}{h_j} \right) + (P_2 f v)_{j-1/2}^{n-1/2} - (u^2)_{j-1/2}^{n-1/2} - (P_3 g v)_{j-1/2}^{n-1/2} + (P_4 g)_{j-1/2}^{n-1/2} \\ & - (P_5 u)_{j-1/2}^{n-1/2} = x_{j-1/2}^{n-1/2} \left(u_{j-1/2}^{n-1/2} \frac{u_{j-1/2}^n - u_{j-1/2}^{n-1}}{k_n} - v_{j-1/2}^{n-1/2} \frac{f_{j-1/2}^n - f_{j-1/2}^{n-1}}{k_n} \right) \end{aligned} \quad (30)$$

$$\begin{aligned} & \frac{1}{2Pr} \left(\frac{p_j^n - p_{j-1}^n}{h_j} \right) + \frac{1}{2Pr} \left(\frac{p_j^{n-1} - p_{j-1}^{n-1}}{h_j} \right) + (P_1 f p)_{j-1/2}^{n-1/2} - (P_4 u g)_{j-1/2}^{n-1/2} \\ & + (P_6 v^2)_{j-1/2}^{n-1/2} + (P_7 u^2)_{j-1/2}^{n-1/2} = x_{j-1/2}^{n-1/2} \left(u_{j-1/2}^{n-1/2} \frac{g_{j-1/2}^n - g_{j-1/2}^{n-1}}{k_n} - p_{j-1/2}^{n-1/2} \frac{f_{j-1/2}^n - f_{j-1/2}^{n-1}}{k_n} \right) \end{aligned} \quad (31)$$

139 The above equations are to be linearized by using Newton's Quasi-linearization method. Here
 140 programme has been used Lahey Fortran 90. Then linear algebraic equations can be written in block
 141 matrix which form a coefficient matrix. The whole procedure, namely reduction to first order followed
 142 by central difference approximations, Newton's Quasi-linearization method and the block Thomas
 143 algorithm, is well known as the Keller-box method.

144 5. RESULTS AND DISCUSSION

145 For getting appropriate solutions and proper curves by using suitable data for some test values of
 146 Prandtl's number $Pr = 0.10, 0.72, 1.00, 1.74$ and for a wide range of the values of the viscous
 147 dissipation parameter $N = 0.90, 0.70, 0.50, 0.20$, temperature dependent viscosity parameter $\varepsilon =$
 148 $1.50, 1.20, 1.00, 0.70, 0.40, 0.20$, magnetic parameter $M = 1.50, 1.20, 0.90, 0.60, 0.30$ and the Joule
 149 heating parameter $J = 0.90, 0.70, 0.30, 0.10$. To satisfy the boundary conditions and also use of
 150 worthy values of various parameters to determine the solution is uniformly convergence. we know
 151 the values of the functions $f(x, y)$, $\theta(x, y)$ and their derivatives for the different values of the

Prandtl's number Pr , the temperature dependent parameter ε , the viscous dissipation parameter N , the magnetic parameter M and the Joule heating parameter J . Also we may calculate the numerical values of the local heat transfer coefficient $\theta'(x, 0)$ and the velocity gradient $f''(x, 0)$ on the surface that are important from the physical point of view.

Fig.3 (a) and Fig.3 (b) deal with the effect of the viscous dissipation parameter N ($= 0.20, 0.50, 0.70, 0.90$) for distinct values of the controlling parameters $Pr = 0.72, J = 0.80, M = 0.50$ and $\varepsilon = 2.00$ on the velocity profiles $f'(x, y)$ and the temperature profiles $\theta(x, y)$. From Fig. 3(a), it is revealed that the velocity profile $f'(x, y)$ increases very small with the increase of the viscous dissipation parameter N which indicates that viscous dissipation increases the fluid motion slowly. In Fig.3 (b) it is shown that the temperature profiles $\theta(x, y)$ small increase for increasing values of N with other controlling parameter.

The effects of magnetic parameter or Hartmann number M for $Pr = 0.72, J = 0.70, N = 0.50$ and $\varepsilon = 2.00$ on the velocity profiles and temperature profiles are shown in figures 4(a) to 4(b). Figure 4(a) and figure 4(b) represent respectively the effects of magnetic parameter M on the velocity and temperature profiles. From these figures, it is seen that the velocity profiles decrease with the increasing values of M and the temperature profiles increase with the increasing values of M .

From figure 5(a) shows the effects of the velocity profile for various values of the temperature dependent parameter $\varepsilon = 0.20, 0.40, 0.70, 1.00, 1.20, 1.50$ while the other controlling parameters $Pr = 0.72, J = 0.90, M = 1.00$ and $N = 0.60$. Corresponding distribution of the temperature profile is shown in figure 5(b). From figure 5(a), it is seen that if the temperature dependent viscosity parameter increases, the velocity of the fluid also increases. On the other hand, it is observed that the temperature profile increases within the boundary layer due to increase of temperature dependent viscosity parameter ε which is evident from figure 5(b).

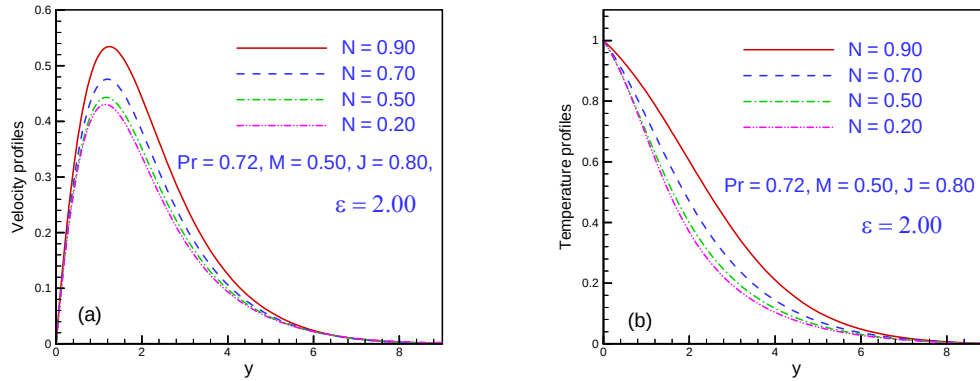
From Fig. 6(a) that an increase in the Joule heating parameter is associated with a considerable increase in velocity profiles but near the surface of the plate the velocity increases and become maximum and then decreases and finally approaches to zero. Fig. 6(b) shows the distribution of the temperature profiles $\theta(x, y)$ against y for the same values of the Joule heating parameter J and each of which attains maximum at the surface.

Fig. 7(a) depicts the velocity profile for different values of the Prandtl's number, $Pr = 0.10, 0.72, 1.00, 1.74$ while the others controlling parameters $N = 0.80, J = 0.50, M = 1.00$ and $\varepsilon = 2.00$. Corresponding distribution of the temperature profile $\theta(x, y)$ in the fluids is shown in Fig. 7(b).

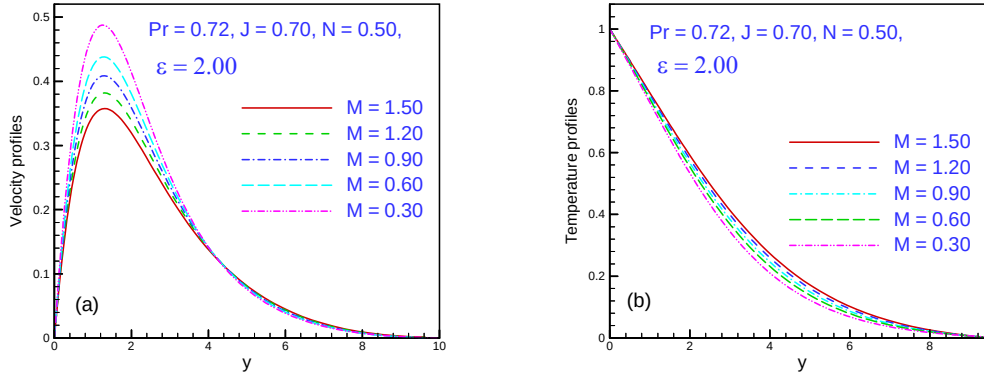
From Fig. 7(a), it is illustrate that if the Prandtl's number increases the velocity of the fluid decreases. On the other hand, from Fig. 7(b) it is seen that the temperature profile decreases within the boundary layer due to increase of the Prandtl number Pr .

Numerical values of the velocity gradient $f''(x, 0)$ and the local heat transfer coefficient $\theta'(x, 0)$ are depicted graphically in Fig.8(a) and Fig.8(b) respectively against the axial distance x for different values of the viscous dissipation parameter N ($= 0.20, 0.50, 0.70, 0.90$) for the fluid having Prandtl's number $Pr = 0.72, J = 0.80, M = 1.00$ and $\varepsilon = 2.00$. From Fig. 8(a), it is performed that the skin-

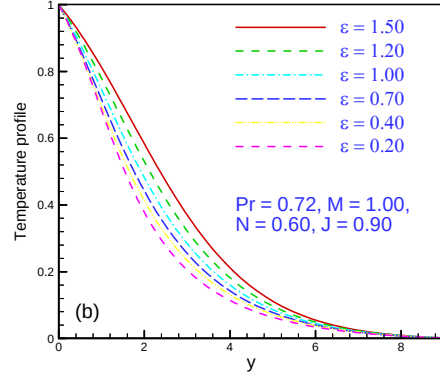
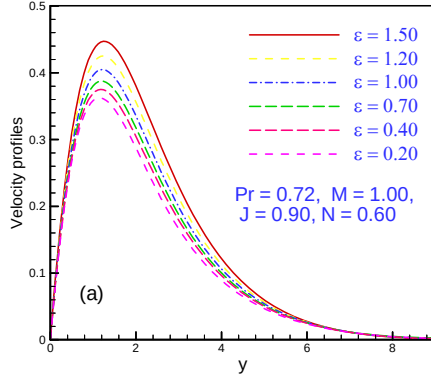
190 friction $f''(x,0)$ increases when the viscous dissipation parameter N increases. It is also observed in
 191 Fig. 8(b), the local heat transfer coefficient $\theta'(x,0)$ decreases while viscous dissipation parameter N
 192 increases.
 193 The effects of magnetic parameter or Hartmann number M for various values of $Pr = 0.72$, $J = 0.70$, N
 194 $= 0.50$ and $\varepsilon = 2.00$ on the local skin friction coefficient $f''(x,0)$ and local heat transfer coefficient
 195 $\theta'(x,0)$ are shown in figure 9(a) and figure 9(b). From figure 9(a) and figure 9(b) it is evident that the
 196 increasing values of magnetic parameter M leads to decrease both the skin friction co-efficient
 197 $f''(x,0)$ and the local heat transfer co-efficient $\theta'(x,0)$.
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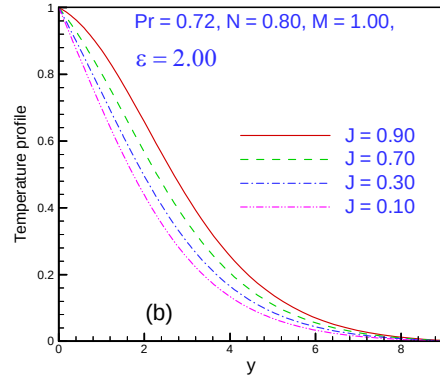
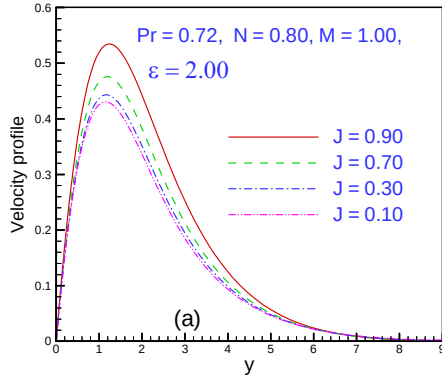
3(a) and 3(b): Variation of dimensionless velocity profiles $f'(x,y)$ and temperature profiles $\theta(x,y)$ against dimensionless distance y for different values of viscous dissipation parameter N with $Pr = 0.72$, $\varepsilon = 2.00$, $M = 1.00$ and $J = 0.80$.



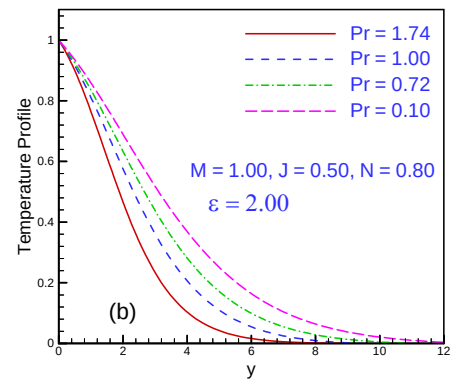
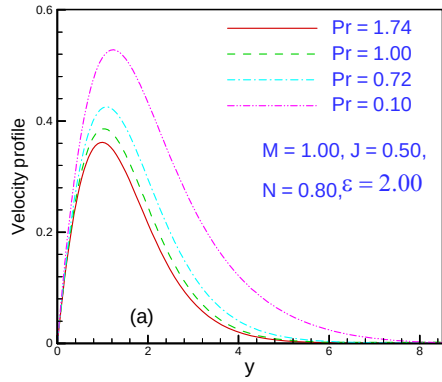
4(a) and 4(b): Variation of dimensionless velocity profiles $f'(x,y)$ and temperature profiles $\theta(x,y)$ against dimensionless distance y for various values of magnetic parameter M with $Pr = 0.72$, $\varepsilon = 2.00$, $J = 0.70$ and $N = 0.50$.



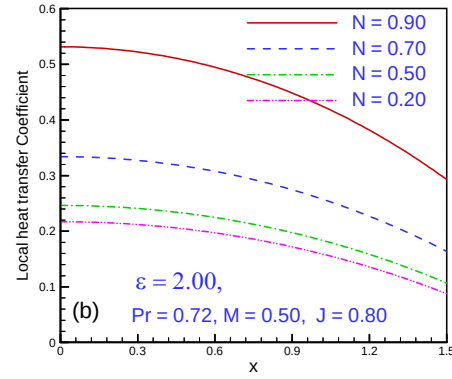
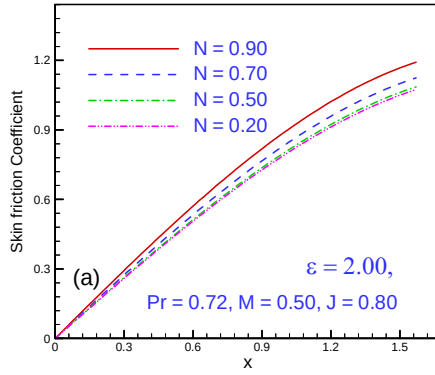
5(a) and 5(b): Variation of dimensionless velocity profiles $f'(x, y)$ and temperature profiles $\theta(x, y)$ against dimensionless distance y for distinct values of variation viscosity parameter ε with $Pr = 0.72$, $N = 0.60$, $M = 1.00$ and $J = 0.90$.



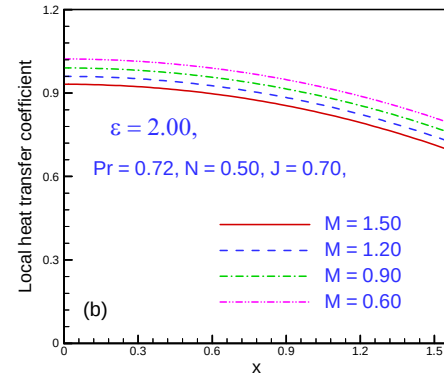
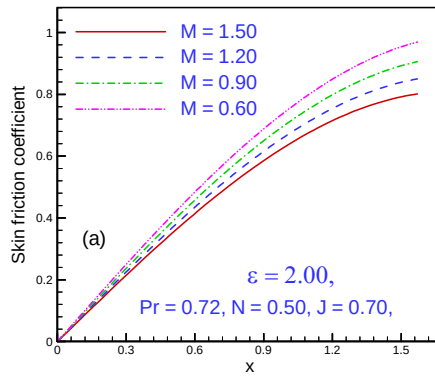
6(a) and 6(b): Variation of dimensionless velocity profiles $f'(x, y)$ and temperature profiles $\theta(x, y)$ against dimensionless distance y for different values of Joule heating parameter J with $N = 0.80$, $M = 1.00$, $\varepsilon = 2.00$ and $J = 0.60$.



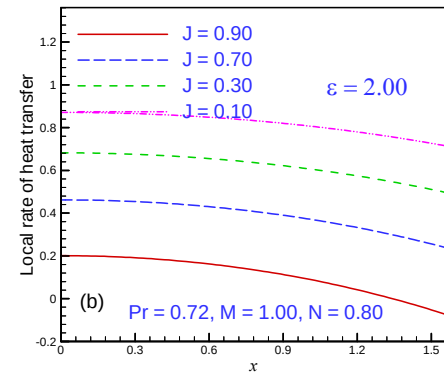
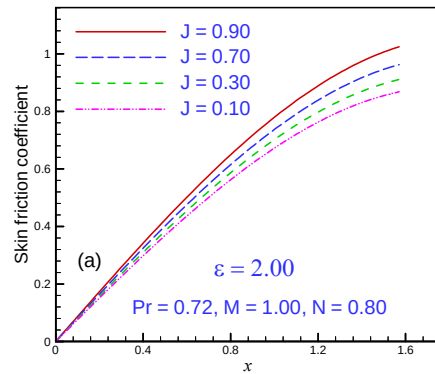
7a) and 7(b): Variation of velocity profiles $f'(x, y)$ and temperature profiles $\theta(x, y)$ against dimensionless distance y for various values of Prandtl's number Pr with $N = 0.80$, $J = 0.50$, $M = 1.00$ and $\varepsilon = 2.00$.



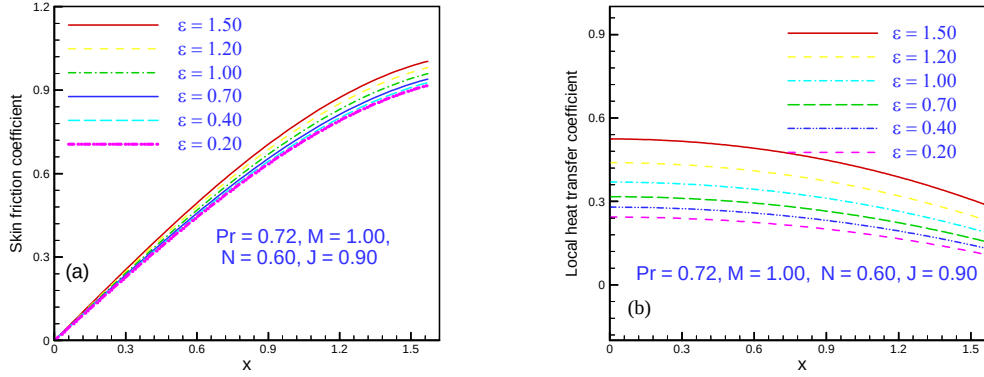
8(a) and 8(b): Variation of dimensionless skin friction coefficient $f''(x,0)$ and local heat transfer coefficient $\theta'(x,0)$ against dimensionless distance x for distinct values of viscous dissipation parameter N with $Pr = 0.72$, $\varepsilon = 2.00$, $J = 0.80$ and $M = 1.00$.



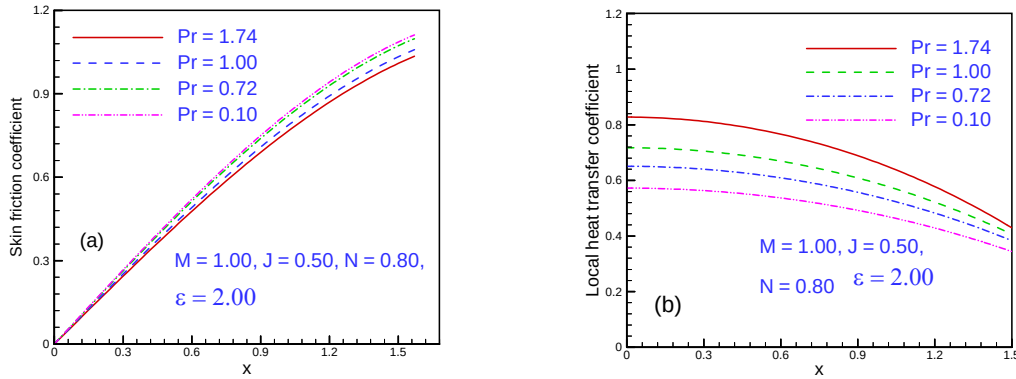
9(a) and 9(b): Variation of dimensionless skin friction coefficient $f''(x,0)$ and local heat transfer coefficient $\theta'(x,0)$ against dimensionless distance x for different values of magnetic parameter M with $Pr = 0.72$, $J = 0.70$, $N = 0.50$ and $\varepsilon = 2.00$.



10(a) and 10(b): Variation of dimensionless skin friction coefficient $f''(x,0)$ and local heat transfer coefficient $\theta'(x,0)$ against dimensionless distance x for distinct values of Joule heating parameter J with $N = 0.80$, $N = 0.80$, $M = 1.00$ and $\varepsilon = 2.00$.



11(a) and 11(b): Variation of dimensionless skin friction coefficient $f''(x,0)$ and local heat transfer coefficient $\theta'(x,0)$ against dimensionless distance x for various values of variation viscosity parameter ε with $N = 0.60$, $J = 0.90$, $M = 1.00$ and $Pr = 0.72$.



12(a) and 12(b): Variation of dimensionless skin friction coefficient $f''(x,0)$ and local heat transfer coefficient $\theta'(x,0)$ against dimensionless distance x for distinct values of Prandtl's number Pr with $N = 0.80$, $J = 0.50$, $M = 1.00$ and $\varepsilon = 2.00$.

The analysis of the effects of Joule heating parameter J ($= 0.10, 0.30, 0.70, 0.90$) on the the surface shear stress in terms of local skin-friction coefficient $f''(x,0)$ and the rate of heat transfer in terms of local Nusselt number $\theta'(x,0)$ against x for $Pr = 0.72$, $N = 0.80$, $M = 1.00$ and $\varepsilon = 2.00$ is shown in Fig.10(a) and Fig.10(b). It is found that the values of the skin-friction $f''(x,0)$ and the local heat transfer coefficient $\theta'(x,0)$ both increases for increasing values of Joule heating parameter J . Here it has been observed that the values of the skin-friction $f''(x,0)$ increases by 76.734% and the local heat transfer coefficient $\theta'(x,0)$ increases by 83.264% while J increased from 0.10 to 0.90.

From Fig. 11(a), it is found that increase in the value of the temperature dependent viscosity parameter ε leads to increase of the value of the shear stress coefficient $f''(x,0)$ which is usually expected. Again from Fig. 11(b) it is illustrate that the increase of the temperature dependent viscosity parameter ε leads to increase of the local heat transfer coefficient $\theta'(x,0)$.

From Fig. 12(a), it is seen that increase in the value of the Prandtl's number Pr ($= 0.10, 0.72, 1.00, 1.74$)

leads to decrease of the value of shear stress $f''(x,0)$. Similar results hold in local heat transfer coefficient $\theta'(x,0)$ shown in Fig. 12(b) for the same values of Prandtl number Pr while $J = 0.50$, $N = 0.80$, $M = 1.00$ and $\varepsilon = 2.00$.

Table-1: Skin friction coefficient and rate of heat transfer against x for different values of magnetic parameter M with other controlling parameters $Pr = 0.72$, $J = 0.70$, $N = 0.50$ and $\varepsilon = 2.00$.

	$M = 0.60$		$M = 0.90$		$M = 1.20$		$M = 1.50$	
x	$f''(x,0)$	$\theta'(x,0)$	$f''(x,0)$	$\theta'(x,0)$	$f''(x,0)$	$\theta'(x,0)$	$f''(x,0)$	$\theta'(x,0)$
0.00000	0.00000	1.02248	0.00000	0.99028	0.00000	0.96003	0.00000	0.93160
0.10472	0.08795	1.02141	0.08340	0.98919	0.07933	0.95893	0.07569	0.93045
0.20944	0.17522	1.01838	0.16613	0.98612	0.15800	0.95582	0.15073	0.92731
0.40143	0.33132	1.00768	0.31395	0.97528	0.29842	0.94485	0.28455	0.91625
0.50615	0.41318	0.99902	0.39132	0.96651	0.37181	0.93597	0.35437	0.90730
0.80285	0.62615	0.96344	0.59178	0.93047	0.56121	0.89952	0.53401	0.87053
1.01229	0.75407	0.92818	0.71112	0.89478	0.67310	0.86343	0.63938	0.83415
1.20428	0.85072	0.88812	0.80021	0.85425	0.75569	0.82249	0.71638	0.79291
1.30900	0.89399	0.86301	0.83950	0.82887	0.79161	0.79685	0.74945	0.76711
1.50098	0.95406	0.81068	0.89264	0.77603	0.83900	0.74355	0.79206	0.71350
1.57080	0.96930	0.78954	0.90550	0.75471	0.84993	0.72206	0.80142	0.69192

The values of the local skin friction coefficient $f''(x,0)$ and local heat transfer coefficient $\theta'(x,0)$ for different values of magnetic parameter or Hartmann number M while $Pr = 0.72$, $N = 0.50$, $J = 0.70$ and $\varepsilon = 2.00$ are given in **Table-1** which is shown in below. Moreover, it is examined that the values of local skin friction coefficient $f''(x,0)$ decrease at various position of x for magnetic parameter $M = 0.60, 0.90, 1.20, 1.50$. The rate of the local skin friction coefficient $f''(x,0)$ decreases by 14.72% as the magnetic parameter M changes from 0.60 to 1.50 and $x = 0.80285$. Furthermore, it is seen that the numerical values of the local heat transfer coefficient $\theta'(x,0)$ decrease as magnetic parameter or Hartmann number M rising. The rate of decrease of the local heat transfer coefficient $\theta'(x,0)$ is 9.64% at position $x = 0.80285$ as the magnetic parameter M changes from 0.60 to 1.50.

Table-2: Comparisons of the present numerical results of Nu_x or the Prandtl numbers $Pr = 0.70, 7.00$ without effect of the viscous dissipation parameter, Joule heating parameter and temperature dependent viscosity parameter with those obtained by Molla *et al.* [18] and Nazar *et al.* [17]

	$Pr = 0.70$			$Pr = 7.00$		
x in degree	Naza <i>et al.</i> [17]	Molla <i>et al.</i> [18]	Present	Naza <i>et al.</i> [17]	Molla <i>et al.</i> [18]	Present
0	0.4576	0.4576	0.4492	0.9595	0.9582	0.9527
10	0.4565	0.4564	0.4485	0.9572	0.9558	0.9487
20	0.4533	0.4532	0.4467	0.9506	0.9492	0.9410
30	0.4480	0.4479	0.4401	0.9397	0.9383	0.9279
40	0.4405	0.4404	0.4340	0.9239	0.9231	0.9140
50	0.4308	0.4307	0.4259	0.9045	0.9034	0.8945
60	0.4189	0.4188	0.4106	0.8801	0.8791	0.8611
70	0.4046	0.4045	0.3982	0.8510	0.8501	0.8395
80	0.3879	0.3877	0.3769	0.8168	0.8161	0.8133
90	0.3684	0.3683	0.3607	0.7774	0.7768	0.7707

The comparison of the local heat transfer coefficient between the present work and the work of Nazar *et al.* (2002) and Molla *et al.* (2004) are presented in **Tables-2** respectively. We observed that the

comparison without the effect of viscous dissipation parameter, temperature dependent viscosity parameter, magnetic parameter and Joule heating parameter in the present problem is similar to the previous work.

5. CONCLUSIONS

From the present investigation, the following conclusions may be drawn:

- Increase in the values of viscous dissipation parameter N leads to an increase in the velocity profile, the temperature profile, the local skin friction coefficient $f''(x,0)$ but the local rate of heat transfer $\theta'(x,0)$ decreases with the increase in viscous dissipation parameter N for $J = 0.80$, $\varepsilon = 2.00$, $M = 1.00$ and $Pr = 0.72$.
- The velocity profile, the skin friction coefficient $f''(x,0)$ and local rate of heat transfer $\theta'(x,0)$ all are decreasing for increasing values of magnetic parameter or Hartmann number M , but the temperature profile increase with the increase of magnetic parameter M .
- The velocity profiles, the temperature profiles, the local skin friction coefficient $f''(x,0)$ and also the local heat transfer coefficient $\theta'(x,0)$ increase significantly when the values of variation viscosity parameter ε increase.
- Significant effects of Joule heating parameter J on velocity and temperature profiles as well as on local skin friction coefficient and the rate of heat transfer have been found in this investigation. An increase in the values of Joule heating parameter J leads to an increase in both velocity and temperature profiles. The local skin friction coefficient $f''(x,0)$ increases at different position of x , also the local rate of heat transfer $\theta'(x,0)$ increases at different position of x for $Pr = 0.72$, $\varepsilon = 2.00$, $M = 1.00$ and $N = 0.80$.
- Increasing values of Prandtl's number Pr leads to decrease the velocity profiles. The temperature profiles, the local skin friction coefficient $f''(x,0)$ and also the local rate of heat transfer $\theta'(x,0)$ increases with the increase of Prandtl's number Pr when $J = 0.50$, $\varepsilon = 2.00$, $M = 1.00$ and $N = 0.80$.

Nomenclature			
a	Radius of the sphere	u, v	dimensionless velocity components
C_p	specific heat	u_0	characteristic velocity
Cf_x	Skin friction coefficient	X, Y	Cartesian coordinates
f	dimensionless stream function	x, y	dimensionless Cartesian coordinates
g	acceleration due to gravity	X/a	angle of the sphere
G_r	Grashof number		

H_0	Magnetic field strength	Greek Symbol	
J	Joule heating parameter		
M	Magnetic parameter	β	co-efficient of thermal expansion
N	viscous dissipation parameter	ε	Temperature dependent viscosity parameter
Nu_x	Local heat transfer coefficient	γ, x	dimensionless similarity variables
		K	thermal conductivity
Pr	Prandtl's number	μ	dynamic viscosity of the fluid
		μ_∞	the ambient dynamic viscosity
r	Radius of the radical distance	ν	kinematic viscosity of the fluid
T	temperature	θ	dimensionless temperature
T_w	temperature at the wall	ρ	density of the fluid
T_∞	Temperature at ambient fluid	σ	electrical conductivity
U, V	velocity components	ψ	stream function

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