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STABILITY ANALYSIS OF ZIKA – MALARIA CO-INFECTION MODEL FOR MALARIA ENDEMIC REGION.

ABSTRACT

In this article, a nonlinear dynamical framework is proposed to contemplate the transmission flow of Zika - Malaria co- infection and impacts of one on each other in the Malaria endemic region because of the likenesses in the side effects of the illness. Strength investigation is done independently on Zika-only model and furthermore on Malaria only model. The disease equilibria purposes of both sub-models were locally and globally asymptotically stable when their basic reproduction number is less than one. The endemic equalibria additionally exist and are locally and comprehensively stable when R_0 more than one.

The disease free equilibrium of the co-infection demonstrate is likewise locally asymptotically stable when R_0 less than one. Be that as it may, the ailment free equilibrium of the co-disease show is not all inclusive stable. The endemic balance of the co-infection show is all around stable utilizing Lyapunov technique. Sensitivity examination is completed on every parameter to decide their impacts on the essential propagation number. Numerical reproduction on the model recommended that the most ideal approach to control the co- infection of Malaria – Zika is to enhance the recovery rates of the two illnesses.

KEYWORDS-Coinfection, Reproduction number, local stability, global stability, sensitivity analysis etc.

1. INTRODUCTION

Zika disease is caused by a virus which is transmitted mostly by female *Aedes aegypti* mosquito [1], which is also responsible for the transmission of chikungunya and dengue fever. The disease was initially discovered in a monkey in the Zika forest of Uganda in 1947[2], however much attention was not given to it until recently when it started causing harm to expectant mothers in the Americas. Mothers in this part of the world were producing microcephaly babies (children with abnormal small head and other neurological disorders), and this caught the attention of the

32 World Health Organization(WHO).The alarming rate of the incidence in Brazil and other parts of
33 the world forced the world health organization to declare the virus as the international emergence
34 concerned. The incidence of Zika virus disease is spreading and this is partly due to the fact that
35 there is neither cure nor vaccine.

36 Only female aedes aegypti mosquito bites an infected human. It has to bite during the viremic
37 stage that is in the first several days of infection, when the virus is still circulating in the human's
38 blood. The mosquito takes up the virus, stores it in her gut and salivary glands, and then
39 transmits it when she bites another uninfected human [3]. As part of the mode of transmission,
40 there has been an evidence that zika virus can be contracted through casual sex with an infected
41 person according to Centre for Disease and preventive Control (CDC).

42 In the light of this, it is very important to know the key parameters in the transmission of the
43 disease and come out with the measures to combat the menace.

44
45 In another advancement Malaria is an old sickness that was first concentrated by Ross in the late
46 1800's [4]. Malaria is a vector borne ailment which is caused by a female Anopheles mosquito.
47 There are four types of parasites specifically: plasmodium falciparum, plasmodium vivax,
48 plasmodium ovale and plasmodium malariae contaminate human.

49
50 Among the parasites, plasmodium falciparum is considered as savage [5]. Plasmodium
51 falciparum is known to be the sort of species that is accepted to cause human diseases in Africa
52 [6]. The disease keeps on being a noteworthy issue in sub-sub-Sahara Africa, Asia, Central and
53 South America and the Middle East. Right around 40% of the total populace settles in endemic
54 ranges [7]. The disease is caused by plasmodium parasite and is transmitted between people
55 through the bite of the female anopheles mosquito. It murders around 700,000 to 2.7million
56 individuals yearly, 75% of whom are African youngsters [8]. The parasites forms into the
57 collection of human between 10-18 days, at that point is passed on when the mosquito infuses
58 salivation while sustaining. In the human body intestinal sickness parasites relocate to the liver,
59 where they develop and increase. Over the long haul the parasites discover its way in the
60 circulation system and keep on developing in the red platelets. At this stage the contaminated
61 people start to indicate side effects like fever, chills, sweating, migraines and other influenza like
62 conditions. The contamination can now and again create considerably more serious responses

including kidney disappointment and passing, particularly if not treated for a timeframe (Mosquito World, 2017).

2. DIFFERENCE BETWEEN ZIKA VIRUS AND MALARIA

Zika infection and Malaria are both mosquito borne sicknesses, and since the ascent of Zika first in Brazil and now reaching out to different parts of the world, a ton of inquiries are being gotten some information about the contrasts between Zika infection and Malaria [9]. The real contrasts is the way we battle the two illnesses since they are spread by two unique sorts of mosquitoes species to be specific female Anopheles for intestinal sickness and female Aedes aegypti for zika infection. Female anopheles mosquito bites during the evening and at first light and can give you the malaria. This can be forestalled by resting in the mosquito bug spray net. Despite what might be expected, the aedes aegypti mosquito bites amid the day time which makes aversion more unpredictable to manage. Notwithstanding, to keep away from been bitten by the female aedes aegypti mosquito having the infection, the therapeutic specialists recommend that utilizing creepy crawly repellant and defensive attire can likewise help in a way [10]. Since zika infection can be transmitted through sex there is the requirement for us to go without or always utilize condom.

Various authors have contributed immersely towards numerous publications on co-infection on disease modeling: a deterministic mathematical model for hepatitis B and tuberculosis by Bowong et al[11]. [12] also formulated and analyzed a deterministic model for hepatitis C virus and HIV/AIDS in 2013 in order to assess their impact on the dynamics of each disease in the presence of treatment. Mushayabasa et al. designed a mathematical model to explore the co-infection of gonorrhea and HIV in the presence of antiretroviral therapy and gonorrhea treatment in 2010.

To the best of my knowledge, this work has not been done by any researcher or been published. It is in this regard that a mathematical model for the Co-infection Malaria-Zika is proposed to access the effect of Zika in the Malaria endemic region. This research has become very necessary for the researcher and the public health officials because the symptoms of both diseases sometimes confuse the medical practitioners and the victims as well.

94 The model in this paper is a system of ordinary nonlinear differential equations and is described,
95 formulated and analyzed below.

96 The paper has been organized as follows, Section 3 deals with the description of the model,
97 Section deals with the analysis of Zika model only, Malaria only, Co-infection Malaria - Zika
98 model Section 4 discusses the sensitivity analysis, Section 5 presents numerical simulation and
99 section 7 is the conclusion.

100

101 **3.0 MODEL FORMULATION AND DESCRIPTION**

102 The model which is about to be described is similar to that of Okosun and Smith [13].

103 This new model subdivides human population into six compartments namely: susceptible
104 humans S_h , infectious malaria only I_m , infectious zika only I_z , both malaria and zika infectious
105 I_{mz} , recovered from malaria only R_m and recovered from zika only R_z and the female mosquito
106 population is also partition into three compartments, that is, susceptible female mosquitoes S_m ,
107 infectious female Anopheles mosquito I_v and infectious female aedes aegypti mosquito I_a .

108 People are recruited into the human population through the rate Π_h and exit through per capital
109 natural death rate μ_h in the six human compartments or through malaria induce death rate (η)
110 in I_m compartment. When an individual in the S_h compartment gets bitten by a mosquito in I_v
111 compartment and malaria infection occurs, then that person moves to I_m compartment and
112 moves to I_z compartment when bitten by a mosquito in I_a compartment and zika infection
113 occurs. An individual in I_m compartment either moves to R_m compartment after recovered from
114 only malaria infection or moves to I_{mz} compartment when bitten by mosquito in I_a compartment
115 and zika infection occurs. Also, an infectious zika person (I_z) moves to I_{mz} compartment
116 when bitten by mosquito in I_v compartment and malaria infection occurs or to R_z compartment
117 when the zika infection subsides and becomes a zika carrier (mild zika infection). Those in
118 R_m compartment moves to S_h compartment for reinfection after they have lost their immunity to
119 malaria and individuals in R_z compartment moves to S_h compartment after they have become
120 zika free.

121 The mosquitoes in S_h compartment moves to I_v compartment when they bit humans in I_m or
122 I_{mz} compartment and to I_a compartment when they bit humans in I_z or I_{mz} compartment.

123 Mosquitoes in either I_a or I_v compartment remain in the infectious state for life or kill by
 124 natural or insecticides. Mosquitoes are recruited into the mosquito population through the rate
 125 Π_m and exit through per capital natural death rate (μ_m) in the three mosquito compartments.

126 Total population of humans and mosquitoes are

$$N_h = S_h + I_m + I_z + I_{mz} + R_m + R_z \dots \dots \dots (1)$$

127 and

$$N_m = S_m + I_v + I_a \dots \dots \dots (2)$$

128
 129
 130

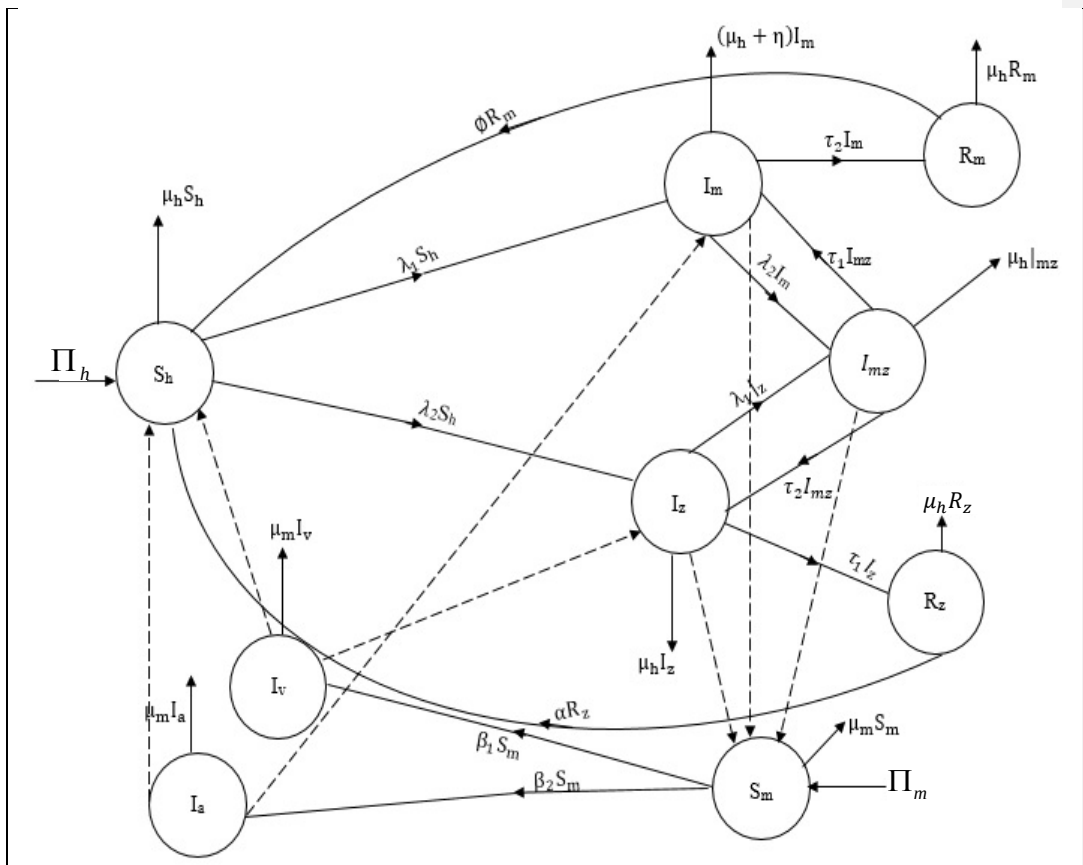


Figure 1.1: Zika –Malaria co-Infection flow chart

131

132

133 Table 1. State Variables for model 3

State Variables	Description
S_h	Number of susceptible individuals
I_m	Individuals who are infected with malaria
I_z	Individuals who are infected with zika virus
I_{mz}	Individuals who are infected with both malaria and zika virus co-infection
R_m	Individuals who recover from Malaria only
R_z	Individuals who recover from Zika only
S_m	Number of susceptible mosquitoes
I_v	Number of female Anopheles mosquitoes
I_a	Number of female aedes aegypti mosquitoes
N_h	Total human population
N_m	Total mosquito population

134

135 Table 2. Parameters for model 3

Parameter	Description
σ_a	The probability of female aedes aegypti mosquito getting infected with zika virus
β_v	Probability of female Anopheles mosquito getting infected with malaria parasite
β_h	Probability of human getting infected with malaria
σ_z	Probability of human getting infected by zika virus
τ_1	Recovery rate for zika virus only.
τ_2	Recovery rate of malaria only
μ_h	Natural death rate for human
μ_m	Natural death rate for mosquitoes

η	Malaria induced death rate
Π_h	Human beings are recruited into their population
Π_m	Mosquitoes are recruited at the rate

136
137 Putting all the assumptions together, the co-infection model for Malaria-Zika is given as (3)
138 below:

$$\left. \begin{aligned}
 \frac{dS_h}{dt} &= \Pi_h + \phi R_m + \alpha R_z - \mu_h S_h - \lambda_1 S_h - \lambda_2 S_h \\
 \frac{dI_m}{dt} &= \lambda_1 S_h - \lambda_2 I_m - (\mu_h + \eta + \tau_2) I_m + \tau_1 I_{mz} \\
 \frac{dI_z}{dt} &= \lambda_2 S_h - \lambda_1 I_z - (\mu_h + \tau_1) I_z + \tau_2 I_{mz} \\
 \frac{dI_{mz}}{dt} &= \lambda_2 I_m + \lambda_1 I_z - (\mu_h + \tau_1 + \tau_2) I_{mz} \\
 \frac{dR_m}{dt} &= \tau_2 I_m - (\phi + \mu_h) R_m \\
 \frac{dR_z}{dt} &= \tau_1 I_z - (\mu_h + \alpha) R_z \\
 \frac{dS_m}{dt} &= \Pi_m - \mu_m S_m - \beta_1 S_m - \beta_2 S_m \\
 \frac{dI_v}{dt} &= \beta_1 S_m - \mu_m I_v \\
 \frac{dI_a}{dt} &= \beta_2 S_m - \mu_m I_a
 \end{aligned} \right\} \dots\dots\dots(3)$$

140 With the force of infection

$$141 \quad \lambda_1 = \frac{\beta_h I_v}{N_h}, \lambda_2 = \frac{\sigma_z I_a}{N_h}, \beta_1 = \frac{\beta_v (I_m + I_{mz})}{N_h}, \beta_2 = \frac{\sigma_a (I_z + I_{mz})}{N_h} \dots\dots\dots(4)$$

142 In this area, we present the basic results concerning the model (3).The following theorem
143 portrays the region within which the model will be examined in the subsequent sections.

144 **Theorem 1.** If the initial state variables are non-negative i.e.

145 $(S_h(0) \geq 0, I_m(0) \geq 0, I_z(0) \geq 0, I_{mz}(0) \geq 0, R_m(0) \geq 0, R_z(0) \geq 0, S_m(0) \geq 0, I_v(0) \geq 0, I_a(0) \geq 0,$
146 then they remain so i.e.

147 $(S_h(t) \geq 0, I_m(t) \geq 0, I_z(t) \geq 0, I_{mz}(t) \geq 0, R_m(t) \geq 0, R_z(t) \geq 0, S_m(t) \geq 0, I_v(t) \geq 0, I_a(t) \geq 0,)$ for
 148 all future times. Moreover

149 $\limsup_{t \rightarrow \infty} N_h(t) \leq \frac{\Pi_h}{\mu_h}$ and $\limsup_{t \rightarrow \infty} N_m(t) \leq \frac{\Pi_m}{\mu_m}$. Again, if $N_h(0) \leq \frac{\Pi_h}{\mu_h}$ and $\left(N_m(0) \leq \frac{\Pi_m}{\mu_m}\right)$ then

150 $N_h(t) \leq \frac{\Pi_h}{\mu_h}$ and $\left(N_m(t) \leq \frac{\Pi_m}{\mu_m}\right)$.

151 More importantly, the region $\Omega = \Omega_h \times \Omega_m$ with

152 $\Omega_h = \left\{ (S_h, I_m, I_z, I_{mz}, R_m, R_z) \in R_+^6 \mid N_h = S_h + I_m + I_z + I_{mz} + R_z + R_z \leq \frac{\Pi_h}{\mu_h} \right\}$ and

153 $\Omega_m = \left\{ (S_m, I_v, I_a) \in R_+^3 \mid N_m = S_m + I_v + I_a \leq \frac{\Pi_m}{\mu_h} \right\}$ is positively invariant

154 The theorem 1 above indicates that the model (3) is biologically and epidemiologically well
 155 posed in the region and thus, the dynamics of the model can be sufficiently studied in Ω . [14]

156 **3.1** Zika only model is given by equation (5) below

157
$$\left. \begin{aligned} \frac{dS_h}{dt} &= \Pi_h + \alpha R_z - \mu_h S_h - \lambda_2 S_h \\ \frac{dI_z}{dt} &= \lambda_2 S_h - (\mu_h + \tau_1) I_z \\ \frac{dR_z}{dt} &= \tau_1 I_z - (\mu_h + \alpha) R_z \\ \frac{dS_m}{dt} &= \Pi_m - \mu_m S_m - \beta_2 S_m \\ \frac{dI_a}{dt} &= \beta_2 S_m - \mu_m I_a \end{aligned} \right\} \dots\dots\dots(5).$$

158 With the force of infection

159 $\lambda_2 = \frac{\sigma_z I_a}{N_h} \text{ and } \beta_2 = \frac{\sigma_a I_z}{N_h} \dots\dots\dots(6)$

160 **3.1.1 Disease free equilibrium (DFE) of Zika only:** Disease free equilibrium (DFE) is the
 161 steady state solution where there is no infection or disease in the population. The disease free is

162 obtained when the infectious state variables are set to zero. For example in Zika,
 163 $I_z = R_z = I_a = 0$.

164 The (DFE) of Zika only model (5) is given as

165
$$E_{zv} = (S_h^*, I_z^*, R_z^*, S_m^*, I_a^*) = \left(\frac{\Pi_h}{\mu_h}, 0, 0, \frac{\Pi_m}{\mu_m}, 0 \right) \dots \dots \dots (7)$$

166 **3.1.2 Basic reproduction number for Zika only**

167 Using the next generation matrix method the basic reproduction number can be defined as the
 168 average number of secondary infectious cases produced by single infective individual in a
 169 population where everyone is susceptible. Applying the next generation matrix method of
 170 [15] the basic reproduction number of the Zika only model is the spectral radius of the matrix
 171 $F_z V_z^{-1}$. Where F_z and V_z are the transmission and transition matrices respectively given by

172
$$F_z = \begin{vmatrix} 0 & \frac{\sigma_z \Pi_h}{\mu_h} \\ \frac{\sigma_a \Pi_m}{\mu_m} & 0 \end{vmatrix} \text{ and } V_z = \begin{vmatrix} (\mu_h + \tau_1) & 0 \\ 0 & \mu_m \end{vmatrix}$$

173 The basic reproduction number of Zika only is denoted by (R_{zv}) and is given as

174
$$R_{zv} = \sqrt{\frac{\sigma_a \sigma_z \Pi_m \Pi_h}{\mu_m^2 \mu_h (\mu_m + \tau_1)}} \dots \dots \dots (8)$$

175 **3.1.3 Local Stability of Disease Free Equilibrium Point for Zika virus**

176 **Theorem2:**

177 The disease free equilibrium point is locally asymptotically stable if $\Re_{zv} < 1$ and unstable if
 178 $\Re_{zv} > 1$.

179
$$J(E_{zv}) = \begin{vmatrix} -\mu_h & 0 & \alpha & 0 & \sigma_z \\ 0 & -(\mu_h + \tau_1) & 0 & 0 & \sigma_z \\ 0 & \tau_1 & -(\mu_h + \alpha) & 0 & 0 \\ 0 & -\frac{\sigma_a \Pi_m \mu_h}{\Pi_h \mu_m} & 0 & -\mu_m & 0 \\ 0 & \frac{\sigma_a \Pi_m \mu_h}{\Pi_h \mu_m} & 0 & 0 & -\mu_m \end{vmatrix} \dots \dots \dots (9)$$

181 The diagonal entries $-\mu_h$, $-\mu_m$ and $-(\mu_h + \alpha)$, are the three eigenvalues of the Jacobian matrix
 182 [16]. Therefore excluding their columns and corresponding rows leads to

$$183 \quad J(E_{zv}) = \begin{vmatrix} -(\mu_h + \tau_1) & \sigma_z \\ \frac{\sigma_a \Pi_m \mu_h}{\Pi_h \mu_m} & -\mu_m \end{vmatrix} \dots\dots\dots (10)$$

184 The eigenvalues of the Jacobian (10) is the characteristic equation of $|J - \lambda I| = 0$

$$185 \quad \begin{vmatrix} -(\mu_h + \tau_1 + \lambda) & \sigma_z \\ \frac{\sigma_a \Pi_m \mu_h}{\Pi_h \mu_m} & -(\mu_m + \lambda) \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 + (\mu_m + \mu_h + \tau_1)\lambda + \mu_m(\mu_h + \tau_1)[1 - R_{zv}^2] = 0 \dots\dots\dots (11)$$

186 If $R_{zv} < 1$, then all the coefficients of equation (11) are positive. Therefore using Routh-Hurwitz
 187 criterion, it can be easily proved that all the roots of the characteristic polynomial (11) have
 188 negative real parts. Hence the disease free equilibrium of Zika only model is locally
 189 asymptotically stable $R_{zv} < 1$,

190 3.1.4 Global stability of disease free equilibrium of Zika Virus

191 **Theorem 3:** The DFE(E_{zv}) of system of equation (5) is globally asymptotically stable if $R_{zv} < 1$
 192 and unstable if $R_{zv} > 1$

193 Proof: We start the proof by dividing (5) into two sub-models, that is, the infectious class and un-
 194 infectious class which are denoted as χ and γ respectively. The variables χ and γ are defined
 195 as, $\chi = (I_z, I_a)$ and $\gamma = (S_h, R_z, S_m)$ respectively. Therefore the model (5) can be written as

$$196 \quad \left. \begin{aligned} \frac{d\chi}{dt} &= R(\chi, \gamma) \\ \frac{d\gamma}{dt} &= S(\chi, \gamma) \end{aligned} \right\} \dots\dots\dots (12)$$

197 The two valued functions $R(\chi, \gamma)$ (dimension 2) and $S(\chi, \gamma)$ (dimension 3) are given by

$$\begin{aligned}
R(\chi, \gamma) &= \left(\frac{\lambda_a I_z}{N_h} S_h - (\mu_h + \tau_1) I_z, \frac{\sigma_a I_z}{N_h} S_m - \mu_m I_a \right)^T \\
S(\chi, \gamma) &= \left(\Pi_h + \alpha R_z - \mu_h S_h - \frac{\lambda I_a}{N_h} S_h, \tau_1 I_z - (\alpha + \mu_h) R_z, \Pi_m - \mu_m S_m - S_m \frac{\sigma_a I_z}{N_h} \right)^T
\end{aligned} \quad \dots\dots\dots(13)$$

Where T is transpose. Now consider the reduced system: $\frac{d\gamma}{dt} = S(\chi, 0)$

$$\begin{aligned}
\frac{dS_h}{dt} &= \Pi_h + \alpha R_z - \mu_h S_h \\
\frac{dR_z}{dt} &= \tau_1 I_m - (\mu_h + \alpha) R_z \\
\frac{dS_m}{dt} &= \beta_2 S_m - \mu_m I_a
\end{aligned} \quad \dots\dots\dots(14)$$

$\gamma^* = (S_h^*, R_z^*, S_m^*) = \left(\frac{\Pi_h}{\mu_h}, 0, \frac{\Pi_m}{\mu_m} \right)$ is a globally asymptotically stable equilibrium point for the

reduced system $\frac{d\gamma}{dt} = S(\chi, 0)$. To see this, solve the second equation in (14) to obtain

$R_z(t) = R_z(0) e^{-(\mu_h + \alpha)t}$; it turns to zero as $t \rightarrow \infty$. In the same way, the last equation in equation

(14) gives $S_m(t) = \frac{\Pi_m}{\mu_m} + \left\{ S_m(0) - \frac{\Pi_m}{\mu_m} \right\} e^{\mu_m t}$ which turns to $\frac{\Pi_m}{\mu_m}$ as $t \rightarrow \infty$ finally, the first

equation of (14) gives

$$S_h(t) = \frac{\Pi_h}{\mu_h} - \left\{ \frac{\Pi_h}{\mu_h} + \left[S_h(0) - \frac{\Pi_h}{\mu_h} \right] \right\} e^{\mu_h t} \rightarrow \frac{\Pi_h}{\mu_h} \text{ as } t \rightarrow \infty. \text{ This asymptotic dynamics is}$$

independent of initial conditions in Ω . Hence the convergence of the solution (14) is global in Ω .

Clearly $R(\chi, \gamma)$ satisfies the following two conditions given as H_2 in [17] namely

1) $R(\chi, 0) = 0$ and

2) $R(\chi, \gamma) = A\chi - \bar{R}(\chi, \gamma), \bar{R}(\chi, \gamma) \geq 0$ on Ω where

$$A = D_\chi R(\chi^*, 0) = \begin{pmatrix} -(\mu_t + \tau_1) & \sigma_z \\ \frac{\sigma_a S_m}{N_h} & -\mu_m \end{pmatrix} \text{ and}$$

$$R(\chi, \gamma) = \begin{pmatrix} \bar{R}_1(\chi, \gamma) \\ \bar{R}_2(\chi, \gamma) \end{pmatrix} = \begin{pmatrix} \sigma_z I_a \left[1 - \frac{S_h}{N_h} \right] \\ \sigma_a I_z \left[1 - \frac{S_m}{N_h} \right] \end{pmatrix}$$

Since the terms in $\bar{R}(\chi, \gamma)$ are nonnegative and the conditions H_2 given in [17] also satisfied, we can conclude that the disease free equilibrium of Zika model only is globally stable.

3.1.5 Existence of endemic equilibrium point of Zika only model: endemic equilibrium point is the steady state solution where the disease (Zika virus) is present in the population. Therefore the endemic equilibrium point of the Zika virus only model is given by

$$E_{zv}^{**} = (S_h^{**}, I_z^{**}, R_z^{**}, S_m^{**}, I_a^{**}) \dots \dots \dots (15)$$

Where

$$\left. \begin{aligned} S_h^{**} &= \frac{N_h \sigma_a \sigma_z \Pi_h \Pi_m \mu_m (\mu_h + \alpha)^2 - R_{zv}^2 N_h \mu_m^2 (\mu_h + \alpha) \Pi_h^2 \alpha \tau_1 + \Pi_h^2 \mu_m^2 \alpha \tau_1 (R_{zv}^2 - 1) (\mu_h + \alpha)}{\sigma_a \sigma_z \Pi_h \Pi_m \mu_m (\mu_h + \alpha)^2 (R_{zv}^2 - 1) + \sigma_a \sigma_z \Pi_h \mu_h \mu_m N_h (\mu_h + \alpha) - R_{zv}^2 \mu_m^2 \mu_h^2 \tau_1 \alpha} \\ I_z^{**} &= \frac{(R_{zv}^2 - 1) (\mu_h + \alpha) \Pi_h \mu_m \mu_h}{\sigma_a \sigma_z \Pi_h \mu_h (\mu_h + \alpha) - R_{zv}^2 \mu_m \mu_h \tau_1 \alpha} \\ R_{zv}^2 &= \frac{(R_{zv}^2 - 1) (\mu_h + \alpha) \Pi_h^2 \mu_m^2 \tau_1}{\sigma_a \sigma_z \Pi_h \mu_h (\mu_h + \alpha)^2 - R_{zv}^2 \mu_m^2 \Pi_h (\mu_h + \alpha) \tau_2 \alpha} \\ S_m^{**} &= \frac{\sigma_a \sigma_z \Pi_m^2 (\mu_h + \alpha) - R_{zv}^2 \Pi_h \Pi_m \mu_m^2 \tau_2 \alpha - (R_{zv}^2 - 1) (\mu_h + \alpha) \sigma_a \Pi_h \Pi_m \mu_m \mu_m^2}{(\mu_h + \alpha) (\sigma_a \sigma_z \Pi_h (\mu_h + \alpha) - R_{zv}^2 \mu_m \mu_h \tau_2 \alpha)} \\ I_a^{**} &= \frac{\sigma_a \mu_h \mu_m \Pi_h \Pi_m (R_{zv}^2 - 1) (\mu_h + \alpha)}{\sigma_a \sigma_z \Pi_m \mu_h \mu_m (\mu_h + \alpha) - R_{zv}^2 \mu_m^2 \mu_h \tau_1 \alpha} \end{aligned} \right\} \dots \dots \dots (16)$$

Theorem 4:

There exists a unique endemic equilibrium point for Zika only model (5) when $R_{zv} > 1$.

3.1.6 Global Stability of Endemic Equilibrium of Zika only model:

Theorem 5:

If $R_{zv} > 1$ then the unique endemic equilibrium E_{zv}^{**} of model (5) is globally stable in the interior of Ω .

228 Proof :

229 This can be determined by constructing Logarithmic Lyapunov function $L(t)$ such that

230 $S_h \leq S_h^*, I_z \leq I_z^*, R_z \leq R_z^*, S_m \leq S_m^*$ and $I_a \leq I_a^*$ and $L(t)$ is given as

231

$$L(t) = \frac{1}{(\mu_h + \lambda_2)} \left(S_h^* S_h - S_h - S_h^* \ln \frac{S_h}{S_h^*} \right) + \frac{1}{(\mu_h + \lambda_2)} \left(I_z^* I_z - I_z - I_z^* \ln \frac{I_z}{I_z^*} \right) +$$

232

$$\frac{1}{(\mu_h + \alpha)} \left(R_z^* R_z - R_z - R_z^* \ln \frac{R_z}{R_z^*} \right) + \frac{1}{(\mu_h + \beta_2)} \left(S_m^* S_m - S_m - S_m^* \ln \frac{S_m}{S_m^*} \right) + \frac{1}{\mu_h} \left(I_a^* I_a - I_a - I_a^* \ln \frac{I_a}{I_a^*} \right) \dots (17)$$

233

234 we take the derivative of $L(t)$ with respect to the state variables which is given as

$$\frac{dL}{dS_h} = \frac{1}{(\mu_h + \lambda_2)} \left(S_h^* - 1 - \frac{1}{S_h} \right) = \frac{1}{(\mu_h + \lambda_2)} \left(\frac{S_h^* S_h - S_h - 1}{S_h} \right)$$

$$\frac{dL}{dI_z} = \frac{1}{(\mu_h + \tau_1)} \left(I_z^* - 1 - \frac{1}{I_z} \right) = \frac{1}{(\mu_h + \tau_1)} \left(\frac{I_z^* I_z - I_z - 1}{I_z} \right)$$

$$\frac{dL}{dR_z} = \frac{1}{(\mu_h + \alpha)} \left(R_z^* - 1 - \frac{1}{R_z} \right) = \frac{1}{(\mu_h + \alpha)} \left(\frac{R_z^* R_z - R_z - 1}{R_z} \right)$$

$$\frac{dL}{dS_m} = \frac{1}{(\mu_h + \beta_2)} \left(S_m^* - 1 - \frac{1}{S_m} \right) = \frac{1}{(\mu_h + \beta_2)} \left(\frac{S_m^* S_m - S_m - 1}{S_m} \right)$$

235

$$\frac{dL}{dI_a} = \frac{1}{\mu_h} \left(I_a^* - 1 - \frac{1}{I_a} \right) = \frac{1}{\mu_h} \left(\frac{I_a^* I_a - I_a - 1}{I_a} \right)$$

236 We then find the product of the derivative of $L(t)$ with respect to the state variables. Thus

237

$$\frac{dL}{dt} = \frac{\partial L}{\partial S_h} \frac{dS_h}{dt} + \frac{\partial L}{\partial I_z} \frac{dI_z}{dt} + \frac{\partial L}{\partial R_z} \frac{dR_z}{dt} + \frac{\partial L}{\partial S_m} \frac{dS_m}{dt} + \frac{\partial L}{\partial I_a} \frac{dI_a}{dt}$$

$$\begin{aligned}
& \frac{dL}{dt} = \frac{1}{(\mu_h + \lambda_2)} \left(\frac{S_h^* S_h - S_h - 1}{S_h} \right) [-(\mu_h + \lambda_2) S_h] \\
& + \frac{1}{(\mu_h + \tau_1)} \left(\frac{I_z^* I_z - I_z - 1}{I_z} \right) [-(\mu_h + \tau_1) I_z] \\
& + \frac{1}{(\mu_h + \alpha)} \left(\frac{R_z^* R_z - R_z - 1}{R_z} \right) [-(\mu_h + \alpha) R_z] \\
& + \frac{1}{(\mu_h + \beta_2)} \left(\frac{S_m^* S_m - S_m - 1}{S_m} \right) [-(\mu_h + \beta_2) S_m] + \frac{1}{\mu_h} \left(\frac{I_a^* I_a - I_a - 1}{I_a} \right) [-\mu_h I_a] \\
& \Rightarrow \frac{dL}{dt} = - (S_h^* S_h - S_h - 1) - (I_z^* I_z - I_z - 1) - (R_z^* R_z - R_z - 1) - (S_m^* S_m - S_m - 1) - (I_a^* I_a - I_a - 1) \\
& \Rightarrow \frac{dL}{dt} = -S_h^* S_h + S_h + 1 - I_z^* I_z + I_z + 1 - R_z^* R_z + R_z + 1 - S_m^* S_m + S_m + 1 - I_a^* I_a + I_a + 1 \\
& \Rightarrow \frac{dL}{dt} = 1 + 1 + 1 + 1 + 1 + S_h (1 - S_h^*) + I_z (1 - I_z^*) + R_z (1 - R_z^*) + S_m (1 - S_m^*) + I_a (1 - I_a^*) \\
& \Rightarrow \frac{dL}{dt} = 5 + S_h (1 - S_h^*) + I_z (1 - I_z^*) + R_z (1 - R_z^*) + S_m (1 - S_m^*) + I_a (1 - I_a^*) \leq 0
\end{aligned}$$

Theorem 6

If a function is positive definite on the entire state space, and has the additional property that $|V(x)| \rightarrow \infty$ as $\|x\| \rightarrow \infty$, and if its derivative V is negative definite on the entire state space, then the equilibrium point at the origin is globally asymptotically stable.

Since $V \leq 0$ and by the LaSalle's Extension (LaSalle's, 1976), therefore it implies that the endemic equilibrium is globally asymptotically stable in the invariant feasible region.

3.2 Malaria only model is given by equation (18) below

$$\begin{aligned}
& \left. \begin{aligned}
\frac{dS_h}{dt} &= \Pi_h + \phi R_m - \mu_h S_h - \lambda_1 S_h \\
\frac{dI_m}{dt} &= \lambda_1 S_h - (\mu_h + \eta + \tau_2) I_m \\
\frac{dR_m}{dt} &= \tau_2 I_m - (\phi + \mu_h) R_m \\
\frac{dS_m}{dt} &= \Pi_m - \mu_m S_m - \beta_1 S_m \\
\frac{dI_v}{dt} &= \beta_1 S_m - \mu_m I_v
\end{aligned} \right\} \dots\dots\dots(18)
\end{aligned}$$

$$\lambda_1 = \frac{\beta_h I_v}{N_h} \text{ and } \beta_1 = \frac{\beta_v I_m}{N_h}$$

The (DFE) of the Malaria only model is given as

$$E_{ma} = (S_h^*, I_m^*, R_m^*, S_m^*, I_v^*) = \left(\frac{\Pi_h}{\mu_h}, 0, 0, \frac{\Pi_m}{\mu_m}, 0 \right) \dots\dots\dots(19)$$

Again, using the next generation matrix method as applied in zika only model, the basic reproduction number of the malaria only model is the spectral radius of the matrix $F_m V_m^{-1}$. Where F_m and V_m are the transmission and transition matrices respectively given by

$$F = \begin{vmatrix} 0 & \beta_h \\ \frac{\beta_v \Pi_m \mu_h}{\Pi_h \mu_m} & 0 \end{vmatrix} \text{ and } V = \begin{vmatrix} (\mu_h + \eta + \tau_2) & 0 \\ 0 & \mu_m \end{vmatrix}$$

The basic reproduction number of malaria only is denoted by R_{ma} is given as

$$R_{ma} = \sqrt{\frac{\beta_h \beta_v \Pi_m \mu_h}{\Pi_h \mu_m^2 (\mu_m + \eta + \tau_2)}} \dots\dots\dots(20)$$

3.2.1 Local Stability of Disease Free Equilibrium Point for Malaria only

Theorem7: The disease free equilibrium point of Malaria only is locally asymptotically stable if $\Re_{ma} < 1$ and unstable if $\Re_{ma} > 1$.

$$J(E_{ma}) = \begin{vmatrix} -\mu_h & 0 & \phi & 0 & \beta_h \\ 0 & -(\mu_h + \eta + \tau_2) & 0 & 0 & \beta_h \\ 0 & \tau_2 & -(\mu_h + \phi) & 0 & 0 \\ 0 & -\frac{\beta_v \Pi_m \mu_h}{\Pi_h \mu_m} & 0 & -\mu_m & 0 \\ 0 & \frac{\beta_v \Pi_m \mu_h}{\Pi_h \mu_m} & 0 & 0 & -\mu_m \end{vmatrix} \dots\dots\dots (21)$$

The diagonal entries $-\mu_h, -\mu_m$ and $-(\mu_h + \phi)$ are the three of the eigenvalues of the Jacobian matrix[16]. Therefore excluding their columns and corresponding rows leads to

$$J(E_{ma}) = \begin{vmatrix} -(\mu_h + \eta + \tau_2) & \beta_h \\ \frac{\beta_v \Pi_m \mu_h}{\Pi_h \mu_m} & -\mu_m \end{vmatrix} \dots\dots\dots (22)$$

The eigenvalues of the Jacobian (22) is the characteristic equation of $|J - \lambda I| = 0$

$$\begin{vmatrix} -(\mu_h + \eta + \tau_2 + \lambda) & \beta_h \\ \frac{\beta_v \Pi_m \mu_h}{\Pi_h \mu_m} & -(\mu_m + \lambda) \end{vmatrix} = 0$$

$$\lambda^2 + (\mu_h + \eta + \mu_m + \tau_2)\lambda + \mu_m (\mu_h + \eta + \tau_2)[1 - R_{ma}^2] = 0 \dots\dots\dots (23)$$

If $R_{ma} < 1$, then all the coefficients of equation (23) are positive. Therefore using Routh-Hurwitz criterion, it can be easily prove that all the roots of the characteristic polynomial (23) have negative real parts. Hence the disease free equilibrium of Malaria only model is locally asymptotically stable $R_{ma} < 1$,

3.2.2 Global stability on DFE Malaria only(E_{ma}) : Here in this section, we try to prove the global stability of the disease free equilibrium.

Theorem 8:

The DFE(E_{ma}) of system of equation (18) is globally asymptotically stable if $R_{ma} < 1$ and unstable if $R_{ma} > 1$

Proof :

The model (3) is now written as

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$$\left. \begin{aligned} \frac{dW}{dt} &= E(V, W) \\ \frac{dV}{dt} &= F(V, W) \end{aligned} \right\} \dots\dots\dots (24)$$

Where $W = (I_m, I_v)$ and $V = (S_h, R_m, S_m)$

The two valued functions $E(V, W)$ (dimension 2) and $F(V, W)$ (dimension 3) are given by

$$\left. \begin{aligned} E(V, W) &= \left(\frac{\beta_h I_v}{N_h} S_h - (\eta + \mu_h + \tau_2) I_m, \frac{\beta_v I_m}{N_h} S_m - \mu_m I_v \right)^T \\ F(V, W) &= \left(\Pi_h + \phi R_m - \mu_h S_h - \frac{\beta_h I_m}{N_h} S_h, \tau_2 I_m - (\phi + \mu_h) R_m, \Pi_m - \mu_m S_m - S_m \beta_1 \right)^T \end{aligned} \right\} \dots\dots\dots (25)$$

Where T is transpose. The reduced form of the system: $\frac{dV}{dt} = F(V, 0)$

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$$\left. \begin{aligned} \frac{dS_h}{dt} &= \Pi_h + \phi R_m - \mu_h S_h \\ \frac{dR_m}{dt} &= \tau_2 I_m - (\mu_h + \phi) R_m \\ \frac{dS_m}{dt} &= \beta_1 S_m - \mu_m I_v \end{aligned} \right\} \dots\dots\dots (26)$$

$V^* = (S_h^*, R_m^*, S_m^*) = \left(\frac{\Pi_h}{\mu_h}, 0, \frac{\Pi_m}{\mu_m} \right)$ is a globally asymptotically stable equilibrium point for the

reduced system $\frac{dV}{dt} = F(V, 0)$. Therefore, $R_m(t) = R_m(0) e^{-(\mu_h + \phi)t}$; it turns to zero as $t \rightarrow \infty$. In

$S_h(t) = \frac{\Pi_h}{\mu_h} + \frac{\phi}{\mu_h} \left\{ R_m(0) e^{-(\mu_h + \phi)t} \right\} - \left\{ \frac{\Pi_h}{\mu_h} + \left[S_h(0) - \frac{\Pi_h}{\mu_h} \right] \right\} e^{\mu_h t} \rightarrow \frac{\Pi_h}{\mu_h}$ as $t \rightarrow \infty$. Hence the

convergence of the solution (26) is global in Ω . Clearly $E(V, W)$ satisfies the following two

conditions given as H_2 in [17] namely

1) $E(V, 0) = 0$ and

2) $E(V, W) = AW - \bar{E}(V, W), \bar{E}(V, W) \geq 0$ on Ω where

$$A = D_W E(V^*, 0) = \begin{pmatrix} -(\eta + \mu_t + \tau_2) & \beta_h \\ \frac{\beta_v S_m}{N_h} & -\mu_m \end{pmatrix} \text{ and}$$

$$\bar{E}(V, W) = \begin{pmatrix} \bar{E}_1(V, W) \\ \bar{E}_2(V, W) \end{pmatrix} = \begin{pmatrix} \beta_h I_v \left[1 - \frac{S_h}{N_h} \right] \\ \beta_v I_m \left[1 - \frac{S_m}{N_h} \right] \end{pmatrix}$$

Since the terms in $\bar{E}(V, W)$ are nonnegative and the conditions H_2 given in [17] also satisfied, we can conclude that the disease free equilibrium of malaria is globally asymptotically stable.

We denote the endemic equilibrium point for malaria model only by E_{ma}^{**} and is given by

$$E_{ma}^{**} = (S_h^{**}, I_m^{**}, R_m^{**}, S_m^{**}, I_v^{**}) \dots \dots \dots (27)$$

To evaluate E_{ma}^{**} we solve the model (18) by equating it to zero and solve for the state variables.

$$\left. \begin{aligned} S_h^{**} &= \frac{N_h \beta_h \beta_v \Pi_h \Pi_m \mu_h (\phi + \mu_h)^2 - R_{ma}^2 N_h \Pi_h^2 \phi \tau_2 \mu_m^2 (\phi + \mu_h) + N_h \alpha \tau_2 \mu_m^2 \Pi_h^2 (R_{ma}^2 - 1) (\phi + \mu_h)}{\beta_h \beta_v \Pi_m \Pi_h \mu_h (\phi + \mu_h)^2 (R_{ma}^2 - 1) + \beta_h \beta_v \Pi_m \mu_h \mu_m N_h (\phi + \mu_h) - R_{ma}^2 \Pi_h \mu_m^2 \tau_2 \phi \mu_h N_h} \\ I_m^{**} &= \frac{(R_{ma}^2 - 1) (\phi + \mu_h) \Pi_h^2 \mu_m^2}{\beta_h \beta_v \Pi_m \mu_h (\phi + \mu_h) - R_{ma}^2 \Pi_h \mu_m^2 \tau_2 \phi} \\ R_{ma}^2 &= \frac{(R_{ma}^2 - 1) (\phi + \mu_h) \tau_2 \mu_m^2 \Pi_h^2}{\beta_h \beta_v \Pi_m \mu_h (\phi + \mu_h)^2 - R_{ma}^2 \mu_m^2 \Pi_h (\phi + \mu_h) \tau_2 \phi} \\ S_m^{**} &= \frac{\beta_h \beta_v \Pi_m^2 \mu_h (\phi + \mu_h) - R_{ma}^2 \Pi_h \Pi_m \tau_2 \phi \mu_m^2 - (R_{ma}^2 - 1) (\phi + \mu_h) \beta_v \Pi_h \Pi_m \mu_m^2 \mu_h}{\beta_h \beta_v \Pi_m \mu_h (\phi + \mu_h)^2 - R_{ma}^2 \mu_m^2 \Pi_h (\phi + \mu_h) \tau_2 \phi} \\ I_v^{**} &= \frac{\beta_v \Pi_m \mu_h \mu_m \Pi_h (R_{ma}^2 - 1) (\phi + \mu_h)}{\beta_h \beta_v \Pi_m \mu_h \mu_m (\phi + \mu_h) - R_{ma}^2 \mu_m \Pi_h \mu_m^2 \tau_2 \phi} \end{aligned} \right\} \dots \dots \dots (28)$$

Theorem9:

307 There exists a unique endemic equilibrium point for Malaria only model (18) when $R_{ma} > 1$.

308 3.2.3 Global Stability of Endemic Equilibrium of Malaria model only.

309 Theorem 10

310 If $R_{ma} > 1$ then the unique endemic equilibrium E_{ma}^{**} of Malaria only model (19) is globally
311 stable in the interior of Ω .

312 Proof

313 This can be determined by constructing Logarithmic Lyapunov function $L(t)$ such that

314 $S_h \leq S_h^*, I \leq I_m^*, R \leq R_m^*, S \leq S_m^*$ and $I \leq I_v^*$ and $L(t)$ is given as

$$315 \quad L(t) = \frac{1}{(\mu_h + \lambda_1)} \left(S_h^* S_h - S_h - S_h^* \ln \frac{S_h}{S_h^*} \right) + \frac{1}{(\mu_h + \eta + \tau_2)} \left(I_m^* I_m - I_m - I_m^* \ln \frac{I_m}{I_m^*} \right) + \\ \frac{1}{(\mu_h + \beta_1)} \left(R_m^* R_m - R_m - R_m^* \ln \frac{R_m}{R_m^*} \right) + \frac{1}{(\mu_h + \phi)} \left(S_m^* S_m - S_m - S_m^* \ln \frac{S_m}{S_m^*} \right) + \frac{1}{\mu_h} \left(I_v^* I_v - I_v - I_v^* \ln \frac{I_v}{I_v^*} \right) \dots (29)$$

316

317 we take the derivative of $L(t)$ with respect to the state variables which is given as

$$\begin{aligned} \frac{dL}{dS_h} &= \frac{1}{(\mu_h + \lambda_1)} \left(S_h^* - 1 - \frac{1}{S_h} \right) = \frac{1}{(\mu_h + \lambda_1)} \left(\frac{S_h^* S_h - S_h - 1}{S_h} \right) \\ \frac{dL}{dI_m} &= \frac{1}{(\mu_h + \eta + \tau_2)} \left(I_m^* - 1 - \frac{1}{I_m} \right) = \frac{1}{(\mu_h + \eta + \tau_2)} \left(\frac{I_m^* I_m - I_m - 1}{I_m} \right) \\ 318 \quad \frac{dL}{dR_m} &= \frac{1}{(\mu_h + \phi)} \left(R_m^* - 1 - \frac{1}{R_m} \right) = \frac{1}{(\mu_h + \phi)} \left(\frac{R_m^* R_m - R_m - 1}{R_m} \right) \\ \frac{dL}{dS_m} &= \frac{1}{(\mu_h + \beta_1)} \left(S_m^* - 1 - \frac{1}{S_m} \right) = \frac{1}{(\mu_h + \beta_1)} \left(\frac{S_m^* S_m - S_m - 1}{S_m} \right) \\ \frac{dL}{dI_v} &= \frac{1}{\mu_h} \left(I_v^* - 1 - \frac{1}{I_v} \right) = \frac{1}{\mu_h} \left(\frac{I_v^* I_v - I_v - 1}{I_v} \right) \end{aligned}$$

319

320 We then find the derivative of $L(t)$ with respect to the state variables. Thus

321

$$322 \quad \frac{dL}{dt} = \frac{\partial L}{\partial S_h} \frac{dS_h}{dt} + \frac{\partial L}{\partial I_m} \frac{dI_m}{dt} + \frac{\partial L}{\partial R_m} \frac{dR_m}{dt} + \frac{\partial L}{\partial S_m} \frac{dS_m}{dt} + \frac{\partial L}{\partial I_v} \frac{dI_v}{dt}$$

$$\begin{aligned}
\frac{dL}{dt} &= \frac{1}{(\mu_h + \lambda_1)} \left(\frac{S_h^* S_h - S_h - 1}{S_h} \right) [-(\mu_h + \lambda_1) S_h] \\
&+ \frac{1}{(\mu_h + \eta + \tau_2)} \left(\frac{I_m^* I_m - I_m - 1}{I_m} \right) [-(\mu_h + \eta + \tau_2) I_m] \\
&+ \frac{1}{(\mu_h + \phi)} \left(\frac{R_m^* R_m - R_m - 1}{R_m} \right) [-(\mu_h + \phi) R_m] \\
&+ \frac{1}{(\mu_h + \beta_1)} \left(\frac{S_m^* S_m - S_m - 1}{S_m} \right) [-(\mu_h + \beta_1) S_m] + \frac{1}{\mu_h} \left(\frac{I_v^* I_v - I_v - 1}{I_v} \right) [-\mu_h I_v] \\
\Rightarrow \frac{dL}{dt} &= -(S_h^* S_h - S_h - 1) - (I_m^* I_m - I_m - 1) - (R_m^* R_m - R_m - 1) - (S_m^* S_m - S_m - 1) - (I_v^* I_v - I_v - 1) \\
\Rightarrow \frac{dL}{dt} &= -S_h^* S_h + S_h + 1 - I_m^* I_m + I_m + 1 - R_m^* R_m + R_m + 1 - S_m^* S_m + S_m + 1 - I_v^* I_v + I_v + 1 \\
\Rightarrow \frac{dL}{dt} &= 1 + 1 + 1 + 1 + 1 + S_h (1 - S_h^*) + I_m (1 - I_m^*) + R_m (1 - R_m^*) + S_m (1 - S_m^*) + I_v (1 - I_v^*) \leq 0
\end{aligned}$$

Therefore the endemic equilibrium of malaria only model is globally asymptotically stable by theorem 6.

3.3 Stability of the Disease Free Equilibrium of Co-Infection Model (Zika -Malaria):

Disease free equilibrium (DFE) state of the co-infection model is given as

$$E_{mz}^* = \left(\frac{\Pi_h}{\mu_h}, 0, 0, 0, 0, \frac{\Pi_m}{\mu_m}, 0, 0 \right) \dots \dots \dots (30)$$

The basic reproduction number (R_{mz}) of co-infection model is given by

$$R_{mz} = \max \left\{ \sqrt{\frac{\beta_h \beta_v \Pi_m \mu_h}{(\mu_h + \eta + \tau_2) \Pi_h \mu_m^2}}, \sqrt{\frac{\sigma_a \sigma_z \Pi_m \mu_h}{(\mu_h + \tau_1) \Pi_h \mu_m^2}} \right\}$$

Representing

$$R_{ma} = \sqrt{\frac{\beta_h \beta_v \Pi_m \mu_h}{(\mu_h + \eta + \tau_2) \Pi_h \mu_m^2}} \text{ and } R_{zv} = \sqrt{\frac{\sigma_a \sigma_z \Pi_m \mu_h}{(\mu_h + \tau_1) \Pi_h \mu_m^2}} \dots \dots \dots (31)$$

Theorem 11

339 The disease free equilibrium point is locally asymptotically stable if $R_0 < 1$ ($R_{ma} < 1$ and $R_{zv} < 1$)
 340 and unstable if $R_0 > 1$.

341 Proof

342 The Jacobian matrix of the system (3) at the disease free equilibrium is given by

$$343 \quad J = \begin{vmatrix} -\mu_H & 0 & 0 & 0 & \phi & \alpha & 0 & -\beta_h & -\sigma_z \\ 0 & -(\mu_h + \eta + \tau_2) & 0 & \tau_1 & 0 & 0 & 0 & \beta_h & 0 \\ 0 & 0 & -(\mu_h + \tau_1) & \tau_2 & 0 & 0 & 0 & 0 & \sigma_z \\ 0 & 0 & 0 & -(\mu_h + \tau_1 + \tau_2) & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau_2 & 0 & 0 & -(\mu_h + \phi) & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_1 & 0 & 0 & -(\mu_h + \alpha) & 0 & 0 & 0 \\ 0 & -\frac{\beta_v \Pi_m \mu_h}{\Pi_h \mu_m} & -\frac{\sigma_a \Pi_m \mu_h}{\Pi_h \mu_m} & -\frac{(\beta_v + \sigma_a) \Pi_m \mu_h}{\Pi_h \mu_m} & 0 & 0 & -\mu_m & 0 & 0 \\ 0 & \frac{\beta_v \Pi_m \mu_h}{\Pi_h \mu_m} & 0 & \frac{\beta_v \Pi_m \mu_h}{\Pi_h \mu_m} & 0 & 0 & 0 & -\mu_m & 0 \\ 0 & 0 & \frac{\sigma_a \Pi_m \mu_h}{\Pi_h \mu_m} & \frac{\sigma_a \Pi_m \mu_h}{\Pi_h \mu_m} & 0 & 0 & 0 & 0 & -\mu_m \end{vmatrix} \quad ..(32)$$

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345

346 Since $-\mu_h$, $-\mu_m$, $-(\mu_m + \phi)$ and $-(\mu_h + \alpha)$ are diagonal entries, then they form the first four
 347 eigenvalues of the Jacobian matrix[18]. This leads to

348

$$349 \quad J = \begin{vmatrix} -A_1 & 0 & A_2 & A_3 & 0 \\ 0 & -A_4 & A_5 & 0 & A_6 \\ 0 & 0 & -A_7 & 0 & 0 \\ A_8 & 0 & A_9 & -A_9 & 0 \\ 0 & A_{10} & A_{10} & 0 & -A_9 \end{vmatrix} \dots\dots\dots(33)$$

350 where

$$351 \quad A_1 = \mu_h + \eta + \tau_2, A_2 = \tau_1, A_3 = \beta_h, A_4 = \mu_h + \tau_1, A_5 = \tau_2, A_6 = \sigma_z, A_7 = \mu_h + \tau_1 + \tau_2, \\ A_8 = \frac{\beta_v \Pi_m \mu_h}{\Pi_h \mu_m}, A_9 = \mu_m, A_{10} = \frac{\sigma_a \Pi_m \mu_h}{\Pi_h \mu_m}$$

352 Using elementary row transformation on (33), we have [19]

353

$$J = \begin{bmatrix} -A_1 & 0 & A_2 & A_3 & 0 \\ 0 & -A_4 & A_5 & 0 & A_6 \\ 0 & 0 & -A_7 & 0 & 0 \\ 0 & 0 & 0 & A_7 A_3 A_8 - A_7 A_1 A_9 & 0 \\ 0 & 0 & 0 & 0 & A_7 A_6 A_{10} - A_7 A_4 A_9 \end{bmatrix} \dots\dots\dots(34)$$

Hence

$$|J - \lambda I| = 0 \Rightarrow \begin{vmatrix} -(A_1 + \lambda) & 0 & A_2 & A_3 & 0 \\ 0 & -(A_4 + \lambda) & A_5 & 0 & A_6 \\ 0 & 0 & -(A_7 + \lambda) & 0 & 0 \\ 0 & 0 & 0 & (A_7 A_3 A_8 - A_7 A_1 A_9 + \lambda) & 0 \\ 0 & 0 & 0 & 0 & (A_7 A_6 A_{10} - A_7 A_4 A_9 + \lambda) \end{vmatrix} = 0.$$

$$\Rightarrow (A_1 + \lambda)(A_4 + \lambda)(A_7 + \lambda)(A_7 A_3 A_8 - A_7 A_1 A_9 + \lambda)(A_7 A_6 A_{10} - A_7 A_4 A_9 + \lambda) = 0$$

$$\Rightarrow (A_1 + \lambda) = 0, (A_4 + \lambda) = 0, (A_7 + \lambda) = 0, (A_7 A_3 A_8 - A_7 A_1 A_9 + \lambda) = 0$$

$$\text{or } (A_7 A_6 A_{10} - A_7 A_4 A_9 + \lambda) = 0$$

360

$$A_1 + \lambda_1 = 0 \Rightarrow \lambda_1 = -(\mu_h + \eta + \tau_2)$$

$$A_4 + \lambda_2 = 0 \Rightarrow \lambda_2 = -(\mu_h + \tau_1)$$

$$361 \quad A_7 + \lambda_3 = 0 \Rightarrow \lambda_3 = -(\mu_h + \tau_1 + \tau_2)$$

$$(A_7 A_3 A_8 - A_7 A_1 A_9) + \lambda_4 = 0 \Rightarrow \lambda_4 = \mu_m (\mu_h + \tau_1 + \tau_2) (\mu_h + \eta + \tau_2) (1 - R_{ma}^2)$$

$$(A_7 A_6 A_{10} - A_7 A_4 A_9) + \lambda_5 = 0 \Rightarrow \lambda_5 = (\mu_h + \tau_1 + \tau_2) \left[\mu_m (\mu_h + \tau_1) (1 - R_{zv}^2) \right]$$

362 Finally, since all the eigenvalues of the Jacobian matrix of equation (34) are negative or have
 363 negative real parts when $R_0 < 1$ ($R_{ma} < 1$ and $R_{zv} < 1$), therefore the disease free equilibrium point
 364 is locally asymptotically stable when $R_0 < 1$. We now consider the global stability of disease free
 365 equilibrium.

366 **Theorem 12**

367 The DFE(E_{mz}) of system of equation is globally asymptotically stable if $R_{mz} < 1$ and unstable if

$$368 \quad R_{mz} > 1$$

369 **Proof:**

370 The model (3) is now written as :

371
$$\left. \begin{aligned} \frac{dA}{dt} &= \chi(A, B) \\ \frac{dB}{dt} &= \gamma(A, B) \end{aligned} \right\} \dots\dots\dots(35)$$

372 Where $A = (I_m, I_z, I_{mz}, I_v, I_a)$ and $B = (S_h, R_m, R_z, S_m)$.

373 The two valued functions as $\chi(A, B)$ and $\gamma(A, B)$ are defined as

374
$$\chi(A, B) = \left[\begin{aligned} &\frac{\beta_h I_v}{N_h} S_h - \frac{\sigma I_a}{N_h} I_m - (\mu_h + \eta + \tau_2) I_m + \tau_1 I_{mz}, \frac{\sigma I_a}{N_h} S_h - \frac{\beta_h I_v}{N_h} I_z - (\mu_h + \tau_1) I_z + \tau_2 I_{mz}, \\ &\frac{\sigma I_a}{N_h} I_m + \frac{\beta_h I_v}{N_h} I_z - (\mu_h + \tau_1 + \tau_2) I_{mz}, \frac{\beta_v (I_m + I_{mz})}{N_h} S_m - \mu_m I_v, \frac{\sigma_a (I_z + I_{mz})}{N_h} S_m - \mu_m I_a \end{aligned} \right]^T$$

375
$$\gamma(A, B) = \left[\begin{aligned} &\Pi_h + \alpha R_z + \phi R_m - \mu_h S_h - \lambda_1 S_h - \lambda_2 S_h, \tau_2 I_m - (\phi + \mu_h) R_m \\ &\tau_1 I_z - (\mu_h + \alpha) R_z, \Pi_m - \mu_m S_m - \beta_1 S_m - \beta_2 S_m \end{aligned} \right]^T$$

376
$$\dots\dots\dots(36)$$

376 where T is the transpose . The reduced form of the system: $\frac{dB}{dt} = \gamma(0, B)$ is given as (37)

377
$$\left. \begin{aligned} \frac{dS_h}{dt} &= \Pi_h + \alpha R_z + \phi R_m - \mu_h S_h - \lambda_1 S_h - \lambda_2 S_h \\ \frac{dR_m}{dt} &= \tau_2 I_m - (\phi + \mu_h) R_m \\ \frac{dR_z}{dt} &= \tau_1 I_z - (\mu_h + \alpha) R_z \\ \frac{dS_m}{dt} &= \Pi_m - \mu_m S_m - \beta_1 S_m - \beta_2 S_m \end{aligned} \right\} \dots\dots\dots(37)$$

378

379 $B^* = (S_h^*, R_m^*, R_z^*, S_m^*) = \left(\frac{\Pi_h}{\mu_h}, 0, 0, \frac{\Pi_m}{\mu_m} \right)$ is globally asymptotically stable equilibrium point for

380 the reduced form of the system $\frac{dB}{dt} = \gamma(0, B)$. Therefore,

381
$$S_h(t) = \Pi_h + \phi \left\{ R_m(0) e^{-(\mu_h + \phi)t} \right\} + \alpha \left\{ R_z(0) e^{-(\mu_h + \alpha)t} \right\} - \mu_h \left\{ \frac{\Pi_h}{\mu_h} + \left[S_h(0) - \frac{\Pi_h}{\mu_h} \right] \right\} e^{\mu_h t} \rightarrow \frac{\Pi_h}{\mu_h}$$

382 as $t \rightarrow \infty$. Hence the convergence of the solution (37) is global in Ω . Therefore $\chi(A, B)$ must
 383 satisfy the following two conditions given as H_2 in [17] namely

384 1. $\chi(0, B) = 0$ and

385 2. $\chi(A, B) = TA - \bar{\chi}(A, B), \bar{\chi}(A, B) \geq 0$ on Ω (38)

$$386 \quad T = D_A \chi(0, B^*) = \begin{pmatrix} -\left(\mu_h + \eta + \tau_2 + \frac{\sigma_z I_a}{N_h}\right) & 0 & \tau_1 & \beta_h & \frac{\sigma_z I_m}{N_h} \\ 0 & -\left(\frac{\beta_h I_v}{N_h} + \mu_h + \tau_1\right) & \tau_2 & -\frac{\beta_h I_z}{N_h} & \sigma_z \\ \frac{\sigma_z I_a}{N_h} & \frac{\beta_h I_v}{N_h} & -(\mu_h + \tau_1 + \tau_2) & \frac{\beta_h I_z}{N_h} & \frac{\sigma_z I_m}{N_h} \\ \frac{\beta_v S_m}{N_h} & 0 & \frac{\beta_v S_m}{N_h} & 0 & 0 \\ 0 & \frac{\sigma_a S_m}{N_h} & \frac{\sigma_a S_m}{N_h} & 0 & 0 \end{pmatrix}$$

387

388 At $I_m = I_z = I_{mz} = I_v = I_a = 0$

$$389 \quad T = D_A \chi(0, B^*) = \begin{pmatrix} -(\mu_h + \eta + \tau_2) & 0 & \tau_1 & \beta_h & 0 \\ 0 & -(\mu_h + \tau_1) & \tau_2 & 0 & \sigma_z \\ 0 & 0 & -(\mu_h + \tau_1 + \tau_2) & 0 & 0 \\ \frac{\beta_v S_m}{N_h} & 0 & \frac{\beta_v S_m}{N_h} & 0 & 0 \\ 0 & \frac{\sigma_a S_m}{N_h} & \frac{\sigma_a S_m}{N_h} & 0 & 0 \end{pmatrix}$$

390 And

391

$$\chi(A, B) = \begin{pmatrix} \chi_1(A, B) \\ \chi_2(A, B) \\ \chi_3(A, B) \\ \chi_4(A, B) \\ \chi_5(A, B) \end{pmatrix} = \begin{pmatrix} \beta_h I_v \left(1 - \frac{S_h}{N_h}\right) + \lambda_2 I_m \\ \lambda_2 I_a \left(1 - \frac{S_h}{N_h}\right) + \lambda_1 I_z \\ -(\lambda_1 I_z + \lambda_2 I_m) \\ \frac{\beta_v (I_m + I_{mz})}{N_h} \left(1 - \frac{S_m}{N_m}\right) \\ \frac{\sigma_a (I_z + I_{mz})}{N_h} \left(1 - \frac{S_m}{N_m}\right) \end{pmatrix}$$

Since $\chi_3(A, B) < 0$ and does not completely satisfy the two conditions stated in equation (38), the disease free equilibrium may not globally asymptotically stable.

3.3.1 Existence of endemic equilibrium point of the Coinfection Malaria and Zika virus

Obtaining an explicit expression from Endemic equilibrium point of the co-infection model (Malaria and Zika virus) is not forthcoming due to the complex nature of the equations involved. Therefore we assume that the endemic equilibrium may exist as

$E_{mz}^{**} = (S_h^{**}, I_m^{**}, I_z^{**}, I_{mz}^{**}, R_m^{**}, R_z^{**}, S_m^{**}, I_v^{**}, I_a^{**})$. Let's now determine the global stability of the endemic equilibrium as follows

Theorem 13

If $R_{ma} > 1$ then the unique endemic equilibrium E_{mz}^{**} of System (3) is globally stable in the interior of Ω .

Proof

This can be determined by constructing Logarithmic Lyapunov function $L(t)$ such that

$S_h \leq S_h^{**}, I \leq I_m^{**}, I \leq I_z^{**}, I \leq I_{mz}^{**}, R \leq R_m^{**}, R \leq R_z^{**}, S \leq S_m^{**}, I \leq I_v^{**}$ and $I \leq I_a^{**}$ and $L(t)$ is given as

$$\begin{aligned}
L(t) = & \frac{1}{(\mu_h + \lambda_1 + \lambda_2)} \left(S_h^* S_h - S_h - S_h^* \ln \frac{S_h}{S_h^*} \right) + \frac{1}{(\mu_h + \eta + \tau_2 + \lambda_2)} \left(I_m^* I_m - I_m - I_m^* \ln \frac{I_m}{I_m^*} \right) \\
& + \frac{1}{(\mu_h + \lambda_1 + \tau_1)} \left(I_z^* I_z - I_z - I_z^* \ln \frac{I_z}{I_z^*} \right) + \frac{1}{(\mu_h + \tau_1 + \tau_2)} \left(I_{mz}^* I_{mz} - I_{mz} - I_{mz}^* \ln \frac{I_{mz}}{I_{mz}^*} \right) \\
& + \frac{1}{(\phi + \mu_h)} \left(R_m^* R_m - R_m - R_m^* \ln \frac{R_m}{R_m^*} \right) + \frac{1}{(\mu_h + \alpha)} \left(R_z^* R_z - R_z - R_z^* \ln \frac{R_z}{R_z^*} \right) \\
& + \frac{1}{(\mu_m + \beta_1 + \beta_2)} \left(S_m^* S_m - S_m - S_m^* \ln \frac{S_m}{S_m^*} \right) + \frac{1}{\mu_m} \left(I_v^* I_v - I_v - I_v^* \ln \frac{I_v}{I_v^*} \right) + \frac{1}{\mu_h} \left(I_a^* I_a - I_a - I_a^* \ln \frac{I_a}{I_a^*} \right) \dots \dots \dots (39)
\end{aligned}$$

we take the derivative of $L(t)$ with respect to the state variables which is given as

$$\begin{aligned}
\frac{dL}{dS_h} &= \frac{1}{(\mu_h + \lambda_1 + \lambda_2)} \left(S_h^* - 1 - \frac{1}{S_h} \right) = \frac{1}{(\mu_h + \lambda_1 + \lambda_2)} \left(\frac{S_h^* S_h - S_h - 1}{S_h} \right) \\
\frac{dL}{dI_m} &= \frac{1}{(\mu_h + \lambda_2 + \tau_2 + \eta)} \left(I_m^* - 1 - \frac{1}{I_m} \right) = \frac{1}{(\mu_h + \lambda_2 + \tau_2 + \eta)} \left(\frac{I_m^* I_m - I_m - 1}{I_m} \right) \\
\frac{dL}{dI_z} &= \frac{1}{(\mu_h + \lambda_1 + \tau_1)} \left(I_z^* - 1 - \frac{1}{I_z} \right) = \frac{1}{(\mu_h + \lambda_1 + \tau_1)} \left(\frac{I_z^* I_z - I_z - 1}{I_z} \right) \\
\frac{dL}{dI_{mz}} &= \frac{1}{(\mu_h + \tau_1 + \tau_2)} \left(I_{mz}^* - 1 - \frac{1}{I_{mz}} \right) = \frac{1}{(\mu_h + \tau_1 + \tau_2)} \left(\frac{I_{mz}^* I_{mz} - I_{mz} - 1}{I_{mz}} \right) \\
\frac{dL}{dR_m} &= \frac{1}{(\mu_h + \phi)} \left(R_m^* - 1 - \frac{1}{R_m} \right) = \frac{1}{(\mu_h + \phi)} \left(\frac{R_m^* R_m - R_m - 1}{R_m} \right) \\
\frac{dL}{dR_z} &= \frac{1}{(\mu_h + \alpha)} \left(R_z^* - 1 - \frac{1}{R_z} \right) = \frac{1}{(\mu_h + \alpha)} \left(\frac{R_z^* R_z - R_z - 1}{R_z} \right) \\
\frac{dL}{dS_m} &= \frac{1}{(\mu_h + \beta_1 + \beta_2)} \left(S_m^* - 1 - \frac{1}{S_m} \right) = \frac{1}{(\mu_h + \beta_1 + \beta_2)} \left(\frac{S_m^* S_m - S_m - 1}{S_m} \right) \\
\frac{dL}{dI_v} &= \frac{1}{\mu_m} \left(I_v^* - 1 - \frac{1}{I_v} \right) = \frac{1}{\mu_m} \left(\frac{I_v^* I_v - I_v - 1}{I_v} \right) \\
\frac{dL}{dI_a} &= \frac{1}{\mu_m} \left(I_a^* - 1 - \frac{1}{I_a} \right) = \frac{1}{\mu_m} \left(\frac{I_a^* I_a - I_a - 1}{I_a} \right)
\end{aligned}$$

414 We then find the derivative of $L(t)$ with respect to the state variables. Thus

$$\begin{aligned}
415 \quad \frac{dL}{dt} &= \frac{\partial L}{\partial S_h} \frac{dS_h}{dt} + \frac{\partial L}{\partial I_m} \frac{dI_m}{dt} + \frac{\partial L}{\partial I_z} \frac{dI_z}{dt} + \frac{\partial L}{\partial I_{mz}} \frac{dI_{mz}}{dt} + \frac{\partial L}{\partial R_m} \frac{dR_m}{dt} + \frac{\partial L}{\partial R_z} \frac{dR_z}{dt} + \frac{\partial L}{\partial S_m} \frac{dS_m}{dt} + \frac{\partial L}{\partial I_v} \frac{dI_v}{dt} + \frac{\partial L}{\partial I_a} \frac{dI_a}{dt} \\
\frac{dL}{dt} &= \frac{1}{(\mu_h + \lambda_1 + \lambda_2)} \left(\frac{S_h^* S_h - S_h - 1}{S_h} \right) [-(\mu_h + \lambda_1 + \lambda_2) S_h] \\
&+ \frac{1}{(\mu_h + \lambda_2 + \tau_2 + \eta)} \left(\frac{I_m^* I_m - I_m - 1}{I_m} \right) [-(\mu_h + \lambda_2 + \tau_2 + \eta) I_m] \\
416 \quad &+ \frac{1}{(\mu_h + \lambda_1 + \tau_1)} \left(\frac{I_z^* I_z - I_z - 1}{I_z} \right) [-(\mu_h + \lambda_1 + \tau_1) I_z] \\
&+ \frac{1}{(\mu_h + \tau_1 + \tau_2)} \left(\frac{I_{mz}^* I_{mz} - I_{mz} - 1}{I_{mz}} \right) [-(\mu_h + \tau_1 + \tau_2) I_{mz}] \\
&+ \frac{1}{(\mu_h + \phi)} \left(\frac{R_m^* R_m - R_m - 1}{R_z} \right) [-(\mu_h + \phi)] \\
&+ \frac{1}{(\mu_h + \alpha)} \left(\frac{R_z^* R_z - R_z - 1}{R_z} \right) [-(\mu_h + \alpha)] \\
417 \quad &+ \frac{1}{(\mu_h + \beta_1 + \beta_2)} \left(\frac{S_m^* S_m - S_m - 1}{S_m} \right) [-(\mu_h + \beta_1 + \beta_2)] \\
&+ \frac{1}{\mu_m} \left(\frac{I_v^* I_v - I_v - 1}{I_v} \right) [-\mu_m] \\
&+ \frac{1}{\mu_m} \left(\frac{I_a^* I_a - I_a - 1}{I_a} \right) [-\mu_m]
\end{aligned}$$

$$\begin{aligned}
&\Rightarrow \frac{dL}{dt} = -(S_h^* S_h - S_h - 1) - (I_m^* I_m - I_m - 1) - (I_z^* I_z - I_z - 1) - (I_{mz}^* I_{mz} - I_{mz} - 1) - (R_m^* R_m - R_m - 1) \\
&\quad - (R_z^* R_z - R_z - 1) - (S_m^* S_m - S_m - 1) - (I_v^* I_v - I_v - 1) - (I_a^* I_a - I_a - 1) \\
&\Rightarrow \frac{dL}{dt} = -S_h^* S_h + S_h + 1 - I_m^* I_m + I_m + 1 - I_z^* I_z + I_z + 1 - I_{mz}^* I_{mz} + I_{mz} + 1 - R_m^* R_m + R_m + 1 - R_z^* R_z + R_z + 1 \\
&\quad - S_m^* S_m + S_m + 1 - I_v^* I_v + I_v + 1 - I_a^* I_a + I_a + 1 \\
&\Rightarrow \frac{dL}{dt} = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + S_h(1 - S_h^*) + I_m(1 - I_m^*) + I_z(1 - I_z^*) + I_{mz}(1 - I_{mz}^*) \\
&\quad + R_m(1 - R_m^*) + R_z(1 - R_z^*) + S_m(1 - S_m^*) + I_v(1 - I_v^*) + I_a(1 - I_a^*) \\
&\Rightarrow \frac{dL}{dt} = 9 + S_h(1 - S_h^*) + I_m(1 - I_m^*) + I_{mz}(1 - I_{mz}^*) + I_z(1 - I_z^*) + R_m(1 - R_m^*) + R_z(1 - R_z^*) \\
&\quad + S_m(1 - S_m^*) + I_v(1 - I_v^*) + I_a(1 - I_a^*) \leq 0
\end{aligned}$$

Hence, using theorem 6, the endemic equilibrium of co-infection model is globally asymptotically stable. We now move on to sensitivity analysis

4.0 Sensitivity Analysis of the Co-Infection Model Parameters:

We now perform sensitivity analysis on the parameters of the model to determine which parameter will increase or decrease the basic reproduction number (R_0) when it is increased by a small margin. It is computed using the normalized forward sensitivity index. In terms of differentiable expression, it is defined as follows

$$S = \frac{\partial R_0}{\partial \tau_1} \times \frac{\tau_1}{R_0} \text{ where } \tau_1 \text{ is the parameter under consideration}$$

Positive sensitivity index means, an increase in that parameter will lead to corresponding increase in the basic reproduction number (R_0). However, negative sensitivity index means an increase in negative parameter will lead to a decrease in (R_0). Using the parameters in table 3 below, we compute the sensitivity indices for the parameters in the model(3) and the result is stated in table 4 below.

435 **Table 3.** Description of variables and parameters of Malaria and Zika

parameter	interpretation	values	source
σ_a	The probability of female aedes aegypti mosquito getting infected with zika virus	0.406	Assumed
β_v	Probability of female Anopheles mosquito getting infected with malaria parasite	0.09	Blayneh et al.(2007)
β_h	Probability of human getting infected with malaria	0.034	Assumed
σ_z	Probability of human getting infected by zika virus	0.45	Assumed
τ_1	Recovery rate for zika virus only.	0.0181	Assumed
τ_2	Recovery rate of malaria only	0.00136	Assumed
μ_h	Natural death rate for human	0.00004	Yang(2000)
μ_m	Natural death rate for mosquitoes	0.067	Assumed
η	Malaria induced death rate	0.05	Assumed
Π_h	Human beings are recruited into their population	800	Assumed
Π_m	Mosquitoes are recruited at the rate	1000	Blayneh et al.(2007)
ϕ	Progression from R_m to S_h	0.000137	Assumed
α	Progression from R_z to S_h	0.0055	Assumed

436 **Table 4.** Sensitivity indices of model parameters to R_{ma} and R_{zv}

R_{ma}	Parameter values	Sensitivity index	R_{zv}	Parameter values	Sensitivity index
τ_2	0.00136	-0.0126	μ_m	0.067	-0.8343
Π_h	800	-0.4521	τ_1	0.0181	-0.4497

μ_h	0.00004	0.4816	Π_h	800	-0.4836
μ_m	0.067	-0.3089	μ_h	0.00004	0.4822
β_h	0.034	0.4893	σ_a	0.406	0.4866
η	0.05	-0.4315	Π_m	1000	0.6281
β_v	0.09	0.4816	σ_z	0.45	0.4851
Π_m	1000	0.4839			

437

438 From the table, the most positive sensitivity index for malaria is β_h and most negative is τ_2 . Also,
439 the most positive sensitivity index for zika is Π_m and most negative is τ_1 . Now, 10% and 15%
440 increase in β_h and Π_m parameters respectively will cause an increase in R_{ma} and R_{zv} by 5% and
441 7% respectively.

442 5.0 Numerical simulation

443 Using the following initial values,

444
$$S_h(0) = 19900, I_m(0) = 1500, I_z(0) = 140, I_{zm}(0) = 170, R_m(0) = 100, R_z(0) = 20,$$

$$S_m(0) = 30000, I_v(0) = 33400, I_a(0) = 6200$$

445 and values in table 3 above, the numerical solution of the model 3 is given below in figure 1 and
446 2 respectively.

447

448

449

450

451

452

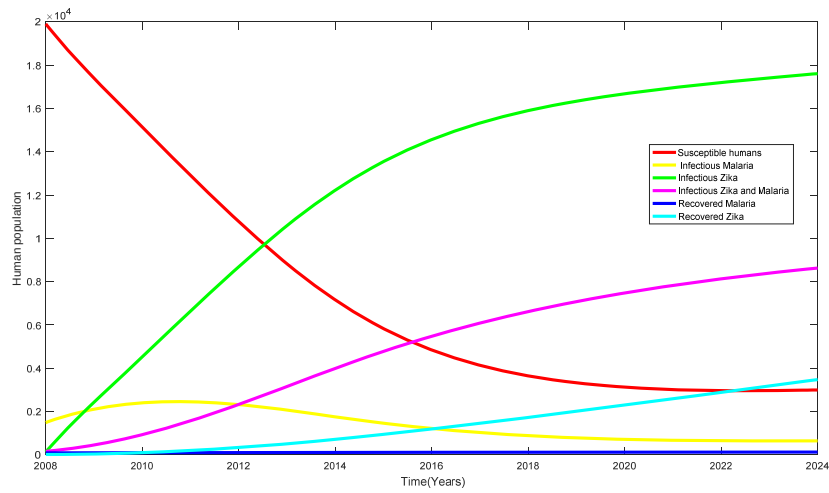


Figure 2; the plot represents the numerical solution of the model with respect to time.

453

454 The figure 1 represents the initial behavior of the model. As time increases from 2008 the
 455 susceptible human drops while those of infectious zika, infectious malaria and recovered zika
 456 increases and malaria infectious increases to its peak at 2011 and start to drop gradually.
 457 However, the recovered malaria trajectory remains low throughout the period.

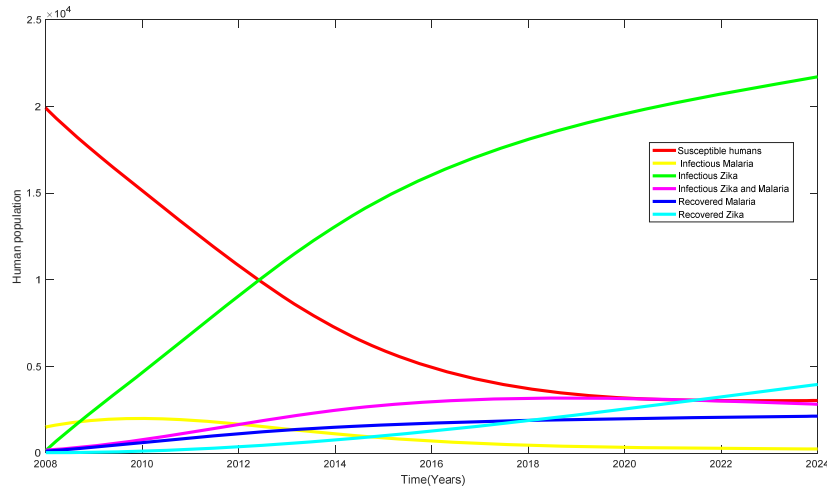


Figure 3; represents the graph of increasing the most negative sensitive index value in malaria only model.

458

459 As the most negative value(μ_m) of the sensitivity index in malaria only model increases, the
 460 recovery rate of malaria increases while the malaria infection and the co-infection trajectories are
 461 decreasing. However, the other trajectories did not show any improvement, from the model we
 462 can say that improving the recovery rate of malaria only may improve the malaria and the co-
 463 infection situation but not zika.

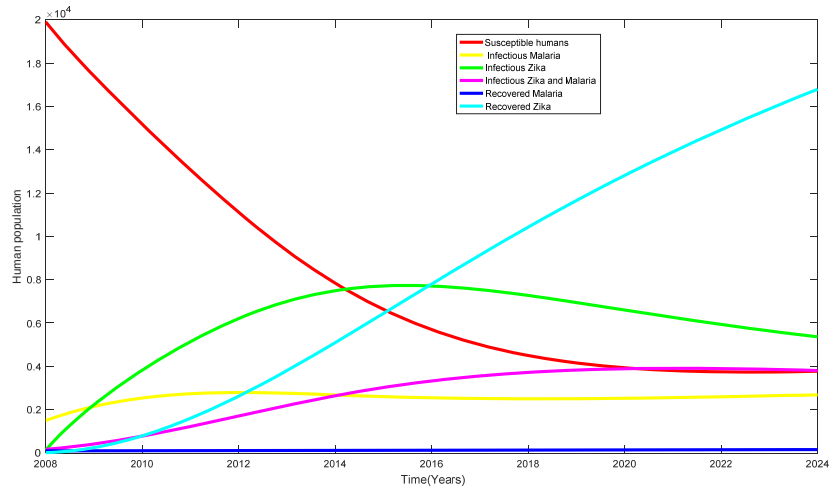


Figure 4 illustrates the plot of increasing the most negative parameter in zika only model.

464

465 Increasing the recovery rate of zika only, did show an improvement in the trajectories of
 466 infectious zika, recovered zika and co-infection. However, there is little improvement in the
 467 trajectory of susceptible humans. Surprisingly, malaria infection increased and the recovery
 468 remain the same.

469

T

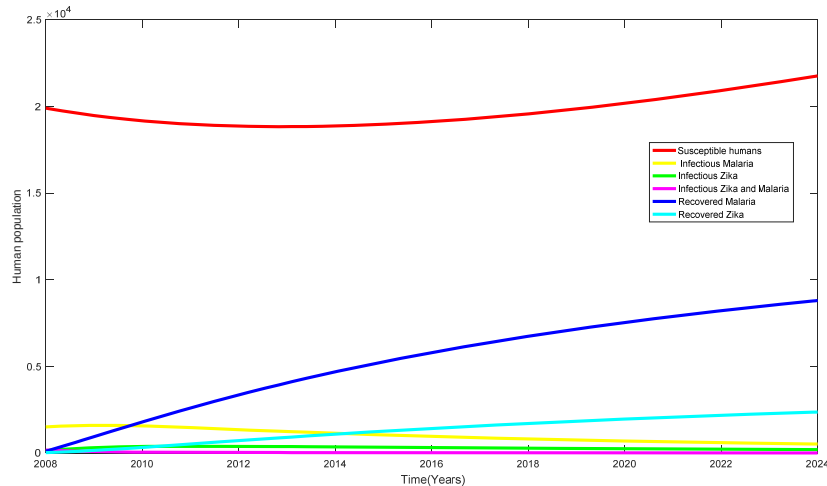


Figure 5 illustrates the plot of increasing the most negative values of both sub-models.

470

471 Improving the recovery rates in both sub-models exhibit great improvement in all the trajectories
 472 except recovered zika which looks almost the same as it was at the beginning of the epidemic.
 473 From the four (4) figures above, the best strategy to control the Malaria- Zika co-infection is to
 474 improve upon the recovery rates of both infections simultaneously.

475 CONCLUSION

476 In this article, we formulated the co-infection model to describe the transmission dynamics of
 477 both Malaria and Zika virus diseases in Malaria endemic region. Stability analysis has been
 478 performed on the model. Sensitivity analysis on the basic reproduction number indicated that
 479 improving the recovery rate of both diseases can reduce the value of the basic reproduction
 480 number considerably. Numerical simulations on the model also confirmed that improving the
 481 recovery rates of both diseases simultaneously is the best to control or eliminate the diseases.
 482 The article is part of on-going thesis, therefore will end here and communicate any future
 483 findings to the research community and policy makers in subsequent articles.

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