

Author's comment on reports by JABS-RJ and LT-MI

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So far, a good number of research works have been done on bilateral sequence spaces for instance, see [1], [5], [2], [11] and [12]. Leon and Montes, in [8] has worked on the complex bilateral sequence space $\ell^2(\mathbb{Z})$ to obtain various results on hypercyclic bilateral weighted shift. In [6], Menet generalized this result to the complex bilateral sequence spaces $\ell^p(\mathbb{Z})$ with $1 \leq p < \infty$ and $c_o(\mathbb{Z})$ and afterwards, to the complex weighted spaces $\ell^p(v, \mathbb{Z})$ and $c_o(v, \mathbb{Z})$. Shkarin, in [10] and [9] used the bilateral sequence spaces $\ell_\infty(\mathbb{Z})$, $\ell_p(\mathbb{Z})$ with $1 \leq p < \infty$ and $c_o(\mathbb{Z})$ to obtain various results associated with weighted bilateral shift on these spaces and also used $\{f_j\}_{j \in \mathbb{Z}}$, a sequence of elements of $\mathcal{B}$ where $\mathcal{B}$ is a Banach space.

In the same way, we have introduced and studied the Banach space X -valued bilateral sequence spaces $c_o(\mathbb{Z}, X, \bar{\lambda}, \bar{p})$, $c(\mathbb{Z}, X, \bar{\lambda}, \bar{p})$ and $\ell(\mathbb{Z}, X, \bar{\lambda}, \bar{p})$ in [15, 14] involving bilateral sequence $\bar{p} = (p_k)_{k=-\infty}^\infty$ and the multiplier $\bar{\lambda} = (\lambda_k)_{k=-\infty}^\infty$. In fact $c_o(\mathbb{Z}, X, \bar{\lambda}, \bar{p})$, $c(\mathbb{Z}, X, \bar{\lambda}, \bar{p})$ and $\ell(\mathbb{Z}, X, \bar{\lambda}, \bar{p})$ will generalize various scalar and vector valued sequence spaces, including the sequence spaces $c_o(X, \bar{\lambda}, \bar{p})$, $c(X, \bar{\lambda}, \bar{p})$ and $\ell(X, \bar{\lambda}, \bar{p})$ studied by Srivastava and Srivastava [13] which itself are the generalization of the well known complex sequence spaces $c_o(\bar{p})$, $c(\bar{p})$ and $\ell(\bar{p})$ studied by Maddox [3, 4] and many others but $\bar{p} = (p_k)_1^\infty$ and $\bar{\lambda} = (\lambda_k)_1^\infty$.



In our papers [19], [18] and [17], we have defined Köthe-Toeplitz duals i.e., α - and β - duals of the sets of X -valued bilateral sequences which will generalize these duals defined by Robinson [7], see also Maddox [4] for the class of usual unilateral vector sequences and investigated the Köthe-Toeplitz duals of $c_o(\mathbb{Z}, X, \bar{\lambda}, \bar{p})$, $c(\mathbb{Z}, X, \bar{\lambda}, \bar{p})$ and $\ell(\mathbb{Z}, X, \bar{\lambda}, \bar{p})$. We have also investigated continuous duals of these bilateral sequence spaces.

Also, we have constructed new sequence spaces $c_o(\mathbb{Z}, X, M, \bar{\lambda}, \bar{p})$ and $\ell(\mathbb{Z}, X, M, \bar{\lambda}, \bar{p})$ by using Orlicz function M in [16]. We also examine some of the properties like containment, linearity and completeness etc. of these newly constructed sequence spaces.

Further we will characterize matrix transformation for these bilateral sequence spaces. We may study these spaces in the theory of weighted bilateral shifts and also in the operator theory in future.

In my opinion this paper contains relevant results and is related to recent researches in functional analysis and its applications.

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