

Re: On Fuzzy Soft G-Metric Spaces

June 6, 2018

Abstract

We suggest some improvements of the results contained in [Anonymous, On Fuzzy Soft G-Metric Spaces, Journal of Advances in Mathematics and Computer Science, Under Review]

Key Words and Phrases: Soft set; Fuzzy set; Fuzzy soft set; Fuzzy soft G-metric space; Fixed point

2010 Mathematics Subject Classification: 03E72, 54A40, 54E40, 54E50, 47H10.

Contents

1	Suggestions	1
2	Some Other Comments	2

1 Suggestions

Let X be a nonempty set. It is well known that a map $d : X^2 \mapsto \mathbb{R}^+$ is called a metric if it satisfies the following

- (a) $d(x, y) \geq 0$ and $d(x, y) = 0$ iff $x = y$, for all $x, y \in X$
- (b) $d(x, y) = d(y, x)$ for all $x, y \in X$
- (c) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$

If we replace the set X with the set of all fuzzy points of an absolute fuzzy soft set, and $\mathbb{R}^+$ with the set of all nonnegative fuzzy soft real numbers, then we get a refinement of metric space called fuzzy soft metric space [Thangaraj Beaula and Christinal Gunaseeli, On fuzzy soft metric spaces, Malaya J. Mat. 2(3)(2014) 197–202].

It is well known that if X is a nonempty set and $G : X^3 \mapsto \mathbb{R}^+$ satisfies Definition 1.1 [Binayak S Choudhury, Sanjay Kumar Asha, and Krishnapada Das, Some fixed point theorems in G-metric spaces, Math. Sci. Lett. 1, No. 1, 25-31 (2012)], then G is called a G-metric and (X, G) is called a G-metric space. If we replace the set X with the set of all fuzzy points of an absolute fuzzy soft set, and $\mathbb{R}^+$ with the set of all nonnegative fuzzy

soft real numbers, then we get a refinement of G-metric space called fuzzy soft G-metric space, which appears is a new concept introduced in [Anonymous, On Fuzzy Soft G-Metric Spaces, Journal of Advances in Mathematics and Computer Science, Under Review].

The main work carried out in [Anonymous, On Fuzzy Soft G-Metric Spaces, Journal of Advances in Mathematics and Computer Science, Under Review] is refinement of the topology and analysis on G-metric spaces and its extensions to the setting of fuzzy soft G-metric space. For example compare Proposition 3.5 [Anonymous, On Fuzzy Soft G-Metric Spaces, Journal of Advances in Mathematics and Computer Science, Under Review] with Proposition 2.9 [Aysegul Caksu Gulera, Esra Dalan Yildirim, Oya Bedre Ozbakira, A fixed point theorem on soft G-metric spaces, J. Nonlinear Sci. Appl. 9 (2016), 885–894].

Aside, the author have also tried to obtain some fixed point theorems in the setting of Fuzzy Soft G-Metric Spaces. Apart from improving readability of paper, and using nice notations, the author should note that Corollary 4.11 does not seem to follow from Theorem 4.10, as the right hand side of the inequality in Theorem 4.10 does not have any term containing $\bar{d}$

Apart from the suggestions thus far, I will accept paper for publication if the author can prove the fuzzy soft G-metric space analogue of Theorem 2 [Clement Boateng Ampadu, A New Proof of the Convex Contraction Mapping Theorem in Metric Spaces, International Journal of Mathematical Archive, To Appear], that is, prove the following

Theorem 1.1. *Let $(\hat{E}, \hat{G})$ be a complete fuzzy soft G-metric space, and suppose $T : (\hat{E}, \hat{G}) \mapsto (\hat{E}, \hat{G})$ satisfies for all $f_{e_i} \in FSC(\hat{E})$ for $i = 1, 2, 3$, the following*

$$\hat{G}(T^2 f_{e_1}, T^2 f_{e_2}, T^2 f_{e_3}) \leq \bar{a}G(T f_{e_1}, T f_{e_2}, T f_{e_3}) + \bar{b}G(f_{e_1}, f_{e_2}, f_{e_3})$$

where $\hat{0} \leq \hat{3}\bar{a} + \bar{b} < \hat{1}$. Then, $T^2 f_e = f_e$ for some unique $f_e \in FSC(\hat{E})$, and T^2 is fuzzy soft G-continuous at f_e

2 Some Other Comments

The author should note that the contractive condition is of “second-order”, and this contractive condition generalizes the Banach contractive condition from metric space to fuzzy soft G-metric space. Note that f_e must be a fixed point of T^2 (and not necessarily of T) and T must be fuzzy soft G-(2-continuous) at f_e , that is, T^2 must be fuzzy soft G-continuous at f_e (and T need not be fuzzy soft G-continuous at f_e)

Note that example given in [Clement Boateng Ampadu, A New Proof of the Convex Contraction Mapping Theorem in Metric Spaces, International Journal of Mathematical Archive, To Appear] is very good in the sense that

$$Fix(T) = Fix(T^n) = \{1\}$$

for all $n = 1, 2, 3, \dots$ and T^n is continuous at $x = 1$ for all $n = 1, 2, 3, \dots$

Perhaps the author should construct an example for which $Fix(T) \neq Fix(T^2)$ but $Fix(T^2)$ contains only one point and T^2 is fuzzy soft G-continuous at that point.