

Differential Transform Decomposition Method for the solution of Boundary Layer Equations in a finite domain

Abstract

A new method called differential transform decomposition method (DTDM) for solving differential equations was developed. This method was derived by coupling the scheme of Differential Transform with Adomian polynomials to decompose the non-linear terms in any given differential equation. The method was applied on single and coupled boundary value problems in a finite domain, computational results of the solved problem were in good agreement when compared with the referenced solution for single boundary value problem and when solved with another numerical method for coupled boundary value problem.

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1. Introduction

Fluid dynamics is an important aspect of applies physics and Engineering. The necessity for the developing the understanding of this field is indispensable, for example when one considers the amount of fluid in the surrounding environment, the large quantities of fluid operating in human body on a daily basis. Of particular interest in fluid dynamics is the study of laminar boundary layer flow of an incompressible fluid. Practical examples of boundary layer theory are aerodynamic extraction of plastic sheets, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along liquid film condensation process (Yasir and Naeem, 2010).

The pioneering work in this area was done by sakiadis(1961), over years similarity solutions were proposed, for stretching walls(Crane 1970, Banks 1983, Magyari and Keller 2000), for porous medium(Ingham and Pop, 2002). However, since the closed form solutions for these sets of problems are difficult to obtain, Numerical and approximate analytical solutions for solving such problems have been gaining popularity in the recent years. Among these methods are homotopy perturbation, Variational iteration and Modified Variational methods, Weighted residual method. Details of

these methods could found in Xu (2007), Wazwaz(2000), Noor and Mohyudin(2009), Oderinu and Aregbesola(2012) respectively.

In this letter, Differential transform Decomposition Method(DTDM) which is obtained through Differential Transform Scheme and Adomian polynomial is used to obtain analytical solution of boundary layer equations in both finite and semi infinite domain.

2. Method of Solution

Suppose we have differential equation of the form

$$Lu + Ru + Nu = g(t) \quad (2.1)$$

where L is the operation of the highest ordered derivative or the space variable and R is the remainder of the linear operator, N(u) being the nonlinear term and g(t) the forcing term. In differential Transform method, to solve equation 2.1 we transform as

$$D^T(Lu) + D^T(Ru) + D^T(Nu) = D^T(g(t)) \quad (2.2)$$

before iterating. But with the proposed method equation (2.1) will transformed as

$$D^T(Lu) + D^T(Ru) + A_n = D^T(g(t))$$

where A_n is the Adomian decomposition of $N(u)$, D^T is the corresponding transform of each term and the nonlinear term decomposed by using Adomian polynomials which is given by George Adomian.

Adomian(1984, 1994) gave a polynomial of the form

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} (N \sum_{r=0}^{\infty} \lambda^r u_r) |_{\lambda=0} \quad n = 0, 1, 2, \dots$$

for decomposing the nonlinear term in any differential equation. Of course, we can infer modifications of A_n for products of different dependent fuctions as

$$N(U, V, W) = UVW$$

in the form

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} (N \sum_{r=0}^{\infty} \lambda^r U_r) \cdot (N \sum_{r=0}^{\infty} \lambda^r V_r) \cdot (N \sum_{r=0}^{\infty} \lambda^r W_r) |_{\lambda=0} \quad n = 0, 1, 2, \dots$$

Definition2.1(Abdel Halim, 2008):The transformation of the kth derivative of a function in one variable is as

$$F_k = \frac{1}{k!} \left[\frac{d^k f(t)}{dt^k} \right]_{t=t_0}$$

and the inverse transformation is

$$f(t) = \sum_{k=0}^{\infty} F_k (t - t_0)^k$$

where F_k is the differential transform of $f(t)$. Table 2.1 below shows the transform of functions from the basic definitions of differential transform as provided by (Abdel-Halim,2008).

Table 2.1

Functional form	Transformed form
$f(t) = u(t) \pm v(t)$	$F_k = U_k \pm V_k$
$f(t) = \lambda u(t)$	$F_k = \lambda U_k$
$f(t) = \frac{d^n u(t)}{dt^n}$	$F_k = \frac{(k+1)!}{k!} U(k+n)$
$f(t) = t \frac{du(t)}{dt}$	$U_k = \sum_{r=0}^k \delta(r-1)(k-r+1)U_{k-r+1}$
$f(t) = t \frac{d^2 u(t)}{dt^2}$	$U_k = \sum_{r=0}^k \delta(r-1)(k-r+1)(k-r+2)U_{k-r+2}$
$f(t) = \frac{du(t)}{dt} \frac{du(t)}{dt}$	$U_k = \sum_{r=0}^k (r+1)(k-r+1)U_{r+1}U_{k-r+1}$
$f(t) = \frac{d^2 u(t)}{dt^2} \frac{d^2 u(t)}{dt^2}$	$U_k = \sum_{r=0}^k (r+1)(r+2)(k-r+1)(k-r+2)U_{r+2}U_{k-r+2}$
$f(t) = u(t) \frac{d^2 u(t)}{dt^2}$	$U_k = \sum_{r=0}^k (k-r+1)(k-r+2)U_r U_{k-r+2}$

2.1. Illustration

Consider the third order non-homogeneous nonlinear ordinary differential equation with initial conditions given by

$$f''' + b_1 f f'' + b_2 f'^2 + b_3 f = 0 \quad (2.3)$$

$$f(0) = \alpha, \quad f'(0) = \beta, \quad f''(0) = \gamma \quad (2.4)$$

where f' , f'' and f''' are the derivatives of f with respect to η and b_1, b_2, b_3 are constants. The nonlinear terms in equation (2.3) are

$$A = f'' \quad \text{and} \quad B = f'^2 \quad (2.5)$$

substituting equation (2.5) into equation (2.1), we have

$$f''' + b_1 A + b_2 B + b_3 f = 0 \quad (2.6)$$

Now using the differential transform in table 2.1 we have

$$f_{k+3} = \frac{-1}{(k+1)(k+2)(k+3)} (b_1 A_k + b_2 B_k + b_3 f_k) \quad k = 0, 1, 2, 3, \dots \quad (2.7)$$

from the initial condition (2.4)

$$f(0) = \alpha \quad \text{implies} \quad f_0 = \alpha \quad f'(0) = \beta \quad \text{implies} \quad (k+1)f_{k+1} = \beta \quad f_1 = \beta \quad (2.8)$$

$$f''(0) = \gamma \quad (k+1)(k+2)f_{k+2} = \gamma \quad f_2 = \frac{\gamma}{2} \quad (2.9)$$

where A_k and B_k are the decomposed expressions of $f f''$ and f'^2 respectively using Adomian polynomial. where

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left\{ \left(\sum_{r=0}^n \lambda^r f_r \right) \left(\sum_{r=0}^n \lambda^r (r+1)(r+2)f_{r+2} \right) \right\} \Big|_{\lambda=0} \quad n = 0, 1, 2, \dots \quad (2.10)$$

and

$$B_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left(\left(\sum_{r=0}^n \lambda^r (r+1) f_{r+1} \right) \left(\sum_{r=0}^n \lambda^r (r+1) f_{r+1} \right) \right) \Big|_{\lambda=0} \quad n = 0, 1, 2, \dots \quad (2.11)$$

Solving equations (2.7, 2.8, 2.9, 2.10 and 2.11) simultaneously to obtain the iterative terms $f_3, f_4, f_5, \dots$ and finally the overall solution using the inverse transform formula in definition 2. 1 as

$$f(\eta) = \sum_{i=0}^n f_i \eta^i \quad n \text{ is fixed number} \quad (2.12)$$

3. Numerical Examples

The fluid problem given by Saeed and Moradi(2012) which is govern by the differential equation

$$f^{(iv)} + S(-\eta f''' - 3f'' - \beta f' f'' + f f''') = 0 \quad (3.1)$$

with the boundary conditions

$$f(0) = 0, \quad f''(0) = 0, \quad f(1) = 1, \quad f'(1) = 0 \quad (3.2)$$

which describes two dimensional and one dimensional(Axisymmetric) unsteady flows due to normally expanding or contracting parallel plates. That is

$$\beta = \begin{cases} 0 & \text{Axisymmetric case} \\ 1 & \text{two dimensional} \end{cases} \quad (3.3)$$

To solve equation (3.1) subject to the boundaries 3.2 with the proposed method we transform equation 3.1 as

$$f_{k+4} = \frac{S}{(k+1)(k+2)(k+3)(k+4)} \left(\sum_{r=0}^k \delta_{r-1} (k-r+1)(k-r+2)(k-r+3) f_{k-r+3} + 3(k+1)(k+2) f_{k+2} + \beta A_k - B_k \right) \quad k = 0, 1, 2, \dots \quad (3.4)$$

Where

$$A_k = f' f'' = (k+1) f_{k+1} (k+1)(k+2) f_{k+2}$$

and

$$B_k = f f''' = f_k (k+1)(k+2)(k+3) f_{k+3}$$

A_k and B_k are decomposed by Adomian polynomials as

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left(\left(\sum_{i=0}^n \lambda^i (i+1) f_{i+1} \right) \left(\sum_{i=0}^n \lambda^i (i+1)(i+2) f_{i+2} \right) \right) \Big|_{\lambda=0}$$

and

$$B_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left(\left(\sum_{i=0}^n \lambda^i f_i \right) \left(\sum_{i=0}^n \lambda^i (i+1)(i+2)(i+3) f_{i+3} \right) \right) \Big|_{\lambda=0}$$

from the boundary conditions we have that $f(0) = 0$ implies $f_0 = 0$

$f''(0) = 0$ which implies $f_2 = 0$

we need to set $f'(0) = \alpha$ and $f'''(0) = b$ where α and b are to be determined. This implies that $f_1 = \alpha$ and $f_3 = \frac{b}{6}$

By decomposing the polynomials A_n and B_n we have

$$A_0 = 2f_1f_2 \quad B_0 = 6f_0f_3$$

$$A_1 = \alpha b \quad B_1 = \alpha b$$

$$A_2 = 12\alpha f_4 \quad B_2 = 24\alpha f_4$$

$$A_3 = \frac{1}{2}b^2 + 20\alpha f_5 \quad B_3 = \frac{1}{6}b^2 + 60\alpha f_5$$

$$A_4 = 10f_4b + 30\alpha f_6 \quad B_4 = 5f_4b + 120\alpha f_6$$

$$A_5 = 15f_5b + 48f_4^2 + 42\alpha f_7 \quad B_5 = 11f_5b + 24f_4^2 + 210\alpha f_7$$

Substituting the polynomial A_i and B_i into equation (3.4) and iterating the resulting expression, this gives

$$f_4 = 0$$

$$f_5 = \frac{1}{120}Sb(4 + \beta\alpha - \alpha)$$

$$f_6 = 0$$

$$f_7 = \frac{1}{5040}Sb(24S + 10S\beta\alpha - 18S\alpha + 3\beta b + S\alpha^2\beta^2 - 4S\alpha^2\beta - b + 3S\alpha^2)$$

$$f_8 = 0$$

$$f_9 = \frac{1}{362880}S^2b(-264S\alpha + 84\beta b + 114S\alpha^2 + 16\alpha b - 15S\alpha^3 + 104S\beta\alpha + 18S\alpha^2\beta^2 - 100S\alpha^2\beta - 42\beta\alpha b - 9S\alpha^3\beta^2 + 23S\alpha^3\beta^2\alpha + S\alpha^3\beta^3 + 192S - 52b)$$

Now Applying the inverse transform on $f_0 \dots f_9$ to obtain the solution as

$$f = \sum_{i=0}^9 f_i \eta^i \quad (3.5)$$

To obtain the values of α and b in the resulting equation (3.6), the two boundary conditions left over, that is the boundary conditions at the other end $f(1) = 1$ and $f'(1) = 0$ will be imposed on the resulting equation (3.5). This procedure yield two simultaneous equations in b and α which can be solved to obtain the values of the constants in each case depending on the value of β and S . The obtained values of α and b corresponds to $f'(0)$ and $f'''(0)$ respectively which on substitution into equation (3.5) gives the general solution.

Tables 3.1 and 3.2 shows the results of $f(\eta)$ for both positive and negative S in comparison with that obtained through Runge-Kutta method of order four (Saeed and Moradi, 2012).

Table3.1. Showing the results of equation (3.1) for axisymmetric case $\beta = 0$ in comparison with the results of Runge-Kutta method of order four

S	η	$f(\eta)$ proposed	R-k method Saeed and Moradi (2012)
-1.5	0.2	0.319289	0.319526
	0.4	0.603438	0.603830
	0.6	0.822480	0.822876
	0.8	0.956588	0.956801
-0.5	0.2	0.302580	0.302582
	0.4	0.578079	0.578082
	0.6	0.800777	0.800780
	0.8	0.947700	0.947702
0.5	0.2	0.290330	0.290322
	0.4	0.559266	0.559252
	0.6	0.784318	0.784303
	0.8	0.940711	0.940703
1.5	0.2	0.281110	0.281010
	0.4	0.544952	0.544779
	0.6	0.771556	0.771371
	0.8	0.935142	0.935036

Table3.2. Showing the results of equation (3.1) for axisymmetric case $\beta = 1$ in comparison with the results of Runge-Kutta method of order four

S	η	$f(\eta)$ proposed	R-k method Saeed and Moradi (2012)
-1.5	0.2	0.333846	0.333618
	0.4	0.624728	0.624358
	0.6	0.839687	0.839325
	0.8	0.963169	0.962984
-0.5	0.2	0.305546	0.305545
	0.4	0.582472	0.582470
	0.6	0.804394	0.804382
	0.8	0.949109	0.949108
0.5	0.2	0.288256	0.288260
	0.4	0.556137	0.556143
	0.6	0.781664	0.781671
	0.8	0.939636	0.939640
1.5	0.2	0.276407	0.276432
	0.4	0.537709	0.537752
	0.6	0.765202	0.765249
	0.8	0.932444	0.932471

The above example is for single boundary layer equation in a finite domain, this procedure is also efficient for the solution of coupled boundary value problems.

Now considering the coupled boundary layer equations modelled by Ravikumar et al (2013), which is governed by

$$\begin{aligned} -\frac{du}{dy} &= \frac{1}{Re} \frac{d^2u}{dy^2} + G_r\theta + G_m\phi - \frac{u}{Re\alpha} - MReu \\ -\frac{d\theta}{dy} &= \frac{1}{ReP_r} \frac{d^2\theta}{dy^2} + \frac{E}{Re} \left(\frac{du}{dy}\right)^2 \\ -\frac{d\phi}{dy} &= \frac{1}{ReSc} \frac{d^2\phi}{dy^2} - KRe\phi + S_0 \frac{d^2\theta}{dy^2} \\ u(0) &= 0, \quad \theta(0) = 1, \quad \phi(0) = 1 \\ u(1) &= 0, \quad \theta(1) = m, \quad \phi(1) = n \end{aligned} \quad (3.6)$$

Where P_r is the prandtl number, Sc is the Schmidt number, Re is the Reynolds number, G_r is the Grashof number for heat transfer, G_m is the Grashof number for mass transfer, E is the Eckert number, α is the permeability parameter, M is the Hatmann number, θ is the dimensionless temperature, ϕ is the dimensionless concentration, S_0 is the Soret number, m and n are constants. To solve equation (3.6) using the method discussed above, equation (3.6) will be transformed as

$$\begin{aligned} U_{s+2} &= \frac{Re}{(s+1)(s+2)} \left(\frac{U_s}{Re\alpha} + MReU_s - G_r\theta_s - G_m\phi_s - (s+1)U_{s+1} \right) \\ \theta_{s+2} &= \frac{Re}{(s+1)(s+2)} \left((s+1)\theta_{s+1} + \frac{E}{Re} A_s \right) \\ \phi_{s+2} &= \frac{ReSc}{(s+1)(s+2)} (kRe\phi_s - S_0(s+1)(s+2)\theta_{s+2} - (s+1)\phi_{s+1}) \quad s = 0, 1, 2, \dots \end{aligned} \quad (3.7)$$

In equation (3.7) the only nonlinear term is $\left(\frac{du}{dy}\right)^2$ and this is represented with A_s . The expression for $A_s = ((i+1)U_{i+1})^2$, $i = 0, 1, 2, \dots$ will be generated by Adomian Polynomial as

$$A_n = \frac{1}{n!} \left(\left(\sum_{i=0}^n (i+1)U_{i+1} \right)^2 \right)_{\lambda=0} \quad n = 0, 1, 2, \dots$$

simplifying A_n $n = 0, 1, 2, \dots$ then we have

$$\begin{aligned} A_0 &= U_0^2 \\ A_1 &= 4U_1U_2 \\ A_2 &= 4U_2^2 + 6U_1U_3 \\ A_3 &= 12U_2U_3 + 8U_1U_4 \\ A_4 &= 9U_3^2 + 16U_2U_4 + 10U_1U_5 \\ A_5 &= 24U_3U_4 + 20U_2U_5 + 12U_1U_6 \end{aligned} \quad (3.8)$$

The boundary conditions $u(0) = 0$, $\theta(0) = 1$, $\phi(0) = 1$ implies that $U_0 = 0$, $\theta_0 = 1$, $\phi_0 = 1$. Since we are dealing with three systems of ordinary differential equations

then there is a need for three additional conditions at the initial point, that which are assumed to be $u'(0) = c$, $\theta'(0) = e$, $\phi'(0) = f$, transforming these assumed equations gives $U_1 = c$, $\theta_1 = e$, $\phi_1 = f$ with these constants to be determined later. Solving equations (3.7) and (3.8) simultaneously, then we iteratively obtain U_s , θ_s , ϕ_s $s = 0, 1, 2..$

$$\begin{aligned} U_2 &= \frac{1}{2}R\left(\frac{U_0}{R\alpha} + G\theta_0 - G_r\phi_0 - U_1\right) \\ \theta_2 &= \frac{1}{2}RP_r\left(\theta_1 + \frac{EA_0}{R}\right) \\ \phi_2 &= \frac{1}{2}RS_c\left(kR\phi_0 - S_0RP_r\left(\theta_1 + \frac{EA_0}{R}\right) - \phi_1\right) \end{aligned} \quad (3.9)$$

... So that

$$\begin{aligned} u(y) &= \sum_{i=0}^n U_i y^i \\ \theta(y) &= \sum_{i=0}^n \theta_i y^i \\ \phi(y) &= \sum_{i=0}^n \phi_i y^i \end{aligned} \quad (3.10)$$

Equations (3.10) give the partial solution because it still contains some assumed constants that are yet to be determined. In order to obtain these constants, we impose the left over boundary conditions in equation (3.6), that is

$$u(1) = 0, \quad \theta(1) = m, \quad \phi(1) = n$$

into equation (3.10) and these give three equations in c , e and f which are solved simultaneously when $m = n = 2$ and $M = Re = \alpha = 1$ $Pr = 0.71$, $Sc = k = 0.5$, $G_r = G_m = 5$, $S_0 = 2.5$, $E = 0.01$ to obtain

$$c = 6.70890773, \quad e = 0.7829362318, \quad f = 1.375214171$$

substituting the calculated constants into equation 3.10 gives

$$\begin{aligned} u(y) &= 6.708907773y - 8.354453886y^2 + 3.222661884y^3 - 2.077953980y^4 + 0.6874688490y^5 - \\ &0.234181473y^6 + 0.05806468498y^7 - 0.01207117966y^8 + 0.1544185067 \times 10^{-2}y^9 + 0.1018991820 \times \\ &10^{-3}y^{10} - 0.1587470222 \times 10^{-3}y^{11} + 0.6998911775 \times 10^{-4}y^{12} \end{aligned} \quad (3.11)$$

$$\begin{aligned} \theta(y) &= 1 + 1.014139121y + 0.391693880y^2 - 1.391626123y^3 + 1.706853878y^4 - \\ &1.906168996y^5 + 1.890937632y^6 - 1.607026349y^7 + 1.260956639y^8 - 0.8864051162y^9 + \\ &0.5836058052y^{10} - 0.358766127y^{11} + 0.2115302496y^{12} \end{aligned} \quad (3.12)$$

$$\begin{aligned} \phi(y) &= 1 + 0.9514981261y - 0.2457648245y^2 + 0.6029344325y^3 - 0.4778351335y^4 + \\ &0.3114865901y^5 - 0.2247770974y^6 + 0.1357278591y^7 - 0.07971112440y^8 + 0.04372175835y^9 - \\ &0.02322012250y^{10} + 0.01174245372y^{11} - 0.005802916773y^{12} \end{aligned} \quad (3.13)$$

equations (3.11), (3.12) and (3.13) are the general solutions of coupled equation (3.6).

Table3.3 Showing the results of $u(y)$ in equation (3.6) in comparison with the results of Weighted Residual method(WRM) via collocation

y	WRM-collocation	DTDM	Difference
0.0	0.0000	0.0000	0.0000
0.1	0.5891806455	0.5903677511	0.0011871056
0.2	1.0281041850	1.0302656840	0.0021614990
0.3	1.3296381570	1.3324636950	0.0028255380
0.4	1.5029480360	1.5060733320	0.0031252960
0.5	1.5539657790	1.5570348740	0.0030690950
0.6	1.4857870260	1.4885034480	0.0027164220
0.7	1.2989969730	1.3011508040	0.0021530310
0.8	0.9919248944	0.9933904004	0.0014655060
0.9	0.5608273421	0.5615537402	0.0007263981
1.0	0.0000	0.0000	0.0000

Table3.4 Showing the results of $\theta(y)$ in equation (3.6) in comparison with the results of Weighted Residual method(WRM) via collocation

y	WRM-collocation	DTDM	Difference
0.0	1.0000	1.0000	1.0000
0.1	1.1034339870	1.1130953450	0.0096613580
0.2	1.2049619360	1.2275978610	0.0226359250
0.3	1.3054941450	1.3392338440	0.0337396990
0.4	1.4056113110	1.4452390070	0.0396276960
0.5	1.5056255470	1.5438509790	0.0382254320
0.6	1.6056224990	1.6341569160	0.0285344170
0.7	1.7054846470	1.7164430990	0.0109584520
0.8	1.8048957040	1.7938179300	0.0110777740
0.9	1.9033261690	1.8773416550	0.0259845140
1.0	2.0000	2.0000	0.0000

Table3.5 Showing the results of $\phi(y)$ in equation (3.6) in comparison with the results of Weighted Residual method(WRM) via collocation

y	WRM-collocation	DTDM	Difference
0.0	1.0000	1.0000	1.0000
0.1	1.1064212120	1.0932502180	0.0131709940
0.2	1.2108152490	1.1846148140	0.0262004350
0.3	1.3133135630	1.2763576140	0.0369559490
0.4	1.4141168120	1.3700805990	0.0440362130
0.5	1.5134775770	1.4668478160	0.0466297610
0.6	1.6116881240	1.5672547910	0.0444333330
0.7	1.7090731900	1.6714518400	0.0376213500
0.8	1.8059878310	1.7791099060	0.0268779250
0.9	1.9028203090	1.8892778570	0.0135424520
1.0	2.0000	2.0000	0.0000

4. Results and Conclusion

Tables 3.1 and 3.2 show the results of equation 3.1 subject to the boundary condition 3.2 in comparison with the results obtained by Saeed and Moradi (2012). From the tables it was observed that the results of the proposed method (*DTDM*) is in good agreement with that of Runge-Kutta method of order four obtained by Saeed and Moradi (2012), however, the proposed method presents the results in analytical form which is not in discretized form; this shows the advantage of the proposed method over Runge Kutta method. Also Tables 3.3 to 3.5 show the solution of velocity, temperature and concentration of equation (3.6). For comparison purpose, this results were compared with the solution obtained while method of weighted residual

is used. The difference between the solutions of the two methods were negligible and this affirms the efficiency of the proposed method. In conclusion, the method of Differential Transform Decomposition has been employed to solve fluid dynamics problem in a finite domain for both single and coupled equations. This method solves this class of differential equations without any need of discretization.

5. References

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