



SDI Review Form 1.6

Journal Name:	Journal of Advances in Mathematics and Computer Science
Manuscript Number:	Ms_JAMCS_42137
Title of the Manuscript:	Recursively-Defined Combinatorial Functions: The Case of Binomial and Multinomial Coefficients and Probabilities
Type of the Article	Original Research Article

General guideline for Peer Review process:

This journal's peer review policy states that **NO** manuscript should be rejected only on the basis of '**lack of Novelty**', provided the manuscript is scientifically robust and technically sound. To know the complete guideline for Peer Review process, reviewers are requested to visit this link:

(<http://www.sciencedomain.org/page.php?id=sdi-general-editorial-policy#Peer-Review-Guideline>)



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PART 1: Review Comments

	Reviewer's comment	Author's comment(if agreed with reviewer, correct the manuscript and highlight that part in the manuscript. It is mandatory that authors should write his/her feedback here)
<u>Compulsory</u> REVISION comments		
<u>Minor</u> REVISION comments	Short description This manuscript presents and characterizes a class of recursively-defined combinatorial quantities, such as the binomial and multinomial coefficients and their corresponding probabilities. Helpful visualizations represent these characterizations in terms of regular acyclic signal flow graphs. Moreover, an iterative approach is demonstrated for the calculation of the binomial coefficient, which is more efficient than theconventional or recursive computation. Some reliability applications are also given. General rules about the format of the mathematical expressions: (a) When you use the MS Word template of the journal, it is highly suggested to use a mathematical formula editing tool, like MathType®, and not the MS Word's own equation editor (which is often problematic and weak). (b) All the mathematical notations/quantities that the authors use have to be in the same size as the text, and in italics (or in "Math" style in MathType®); for example, $X, x, f(x), i, j, t$, etc. instead of $X, x, f(x), i, j, t$, respectively. Known mathematical functionsshould be in upright form (non-italics, or "Function" style in MathType®), like $\sin x, \log y, \exp(x)$, etc.instead of $\sin x, \log y, \exp(x)$ etc.Moreover, numbers should be in upright form (bold or italics are usually for emphasizing). (c) When the authors refer to the probability function, you can write, for example, $\Pr(X > x)$ instead of $P(X > x)$, while the expected value and variance of a random variable X should be written as $E(X)$ and $\text{Var}(X)$, not $E(X)$ and $\text{Var}(X)$. (d) When you use indexes in mathematical notations/quantities that are textual (not variables or other mathematical quantities) please use upright form not italics. For example, you should write A_{total} instead of A_{total}. (e) If you want to use bold letters to symbolize vectors or matrices, use the upright form (non-italics, or "Vector-Matrix" style in MathType®); for example, you can write $\mathbf{x} \in \mathbb{R}^{n \times n}$ instead of $\mathbf{x} \in \mathbb{R}^{n \times n}$. (f) When you write in-line mathematical expressions, avoid (when it is possible) large mathematical notations. For example, you can write $f(x) = x/(1-x)$ instead of $f(x) = \frac{x}{1-x}$. Otherwise, use equation lines, where a new line is devoted for those expressions, for example $f(x) = \frac{x}{1-x}.$ (g) Put the corresponding equation numbers at the right end of equation line (not in the following line). (h) Please put a full stop at the end of an equation line if your sentence ends there, or put a comma if your sentence continues after the equation line. Please apply the above general rules throughout your manuscript.	We appreciate the many helpful suggestions made. We tried to abide by the given rules as much as we can.



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	Here are some minor revision comments: Note: The text in red below represents the suggested changes/additions. Page 1. Abstract: Please rewrite as "...multinomial coefficients are shaping a Pascal Pyramid or Tetrahedron, or Hyper-Pyramid. Each of the four functions is computed using both its conventional and recursive definitions. Moreover, the recursive structures of the binomial coefficient and the corresponding probability are utilized in an iterative scheme, which is substantially more efficient than the conventional or recursive evaluation. Analogous iterative evaluations of the multinomial coefficient and the related probability can be constructed similarly. Applications concerning the reliability evaluation for two-valued and multi-valued k-out-of-n systems are also discussed." Keywords: Please rewrite as "Binomial, multinomial, recursion, boundary condition, signal flow graph, iteration, k-out-of-n". Please apply general rule (b) everywhere in the manuscript. Page 2. Last paragraph: Please rewrite as "...Section 3 presents the concept and conventional..." Page 3. Eq. (1): Please put a full stop at the end of the equation line. Then put the corresponding line numbering (1) at the right end of the equation line. Please apply general rules (g) and (h). Page 4. Eq. (2a): Please delete the extra comma in the subscript of k at the beginning of this equation; i.e. you should write $B(k_1, k_1 + k_2) = B(k_1, k_1 + k_2 - 1) + \dots$. After eq. (2a) you should explain the notation used by saying something like: "where the notation $(k_1, k_2 + k_2) k_2 \leftarrow k_2 - 1$ means that we substitute k_2 with $k_2 - 1$". Paragraph above eq. (4): Please rewrite as "...is the Probability Mass Function (PMF) of the..." Eq. (4): Put a comma at the end of the equation. Please apply general rule (h). Paragraph after eq. (4): Please rewrite as "while the corresponding..." Paragraph after eq. (6): Please rewrite as "...the unreliability and reliability of a (two-valued) k-out-of-n: A system, which is a coherent system is said to be "good" if and only if at least k out of its n components are "good" [1,6,10]." Last paragraph: Please rewrite as "...replace $E(k, n, p)$ by $E(k, n, \mathbf{p})$ where $\mathbf{p} = [p_1 \ p_1 \ p_2 \ \dots \ p_n]^T$...". Please apply general rule (e) everywhere in the manuscript. Please also check if $\mathbf{p} = [p_1 \ p_1 \ p_2 \ \dots \ p_n]^T$ is correct or you mean something like $\mathbf{p} = [p_1 \ p_2 \ p_3 \ \dots \ p_n]^T$.	We implemented as many of the suggestions as appropriate. We implemented this request throughout the paper. We followed this piece of advice. Done. We prefer to write pmf (lower case) as opposed to CDF (upper case) OK. OK. OK. Done. Thank you. Comma and full stop were added. Done. Thank you. Done, but " / " is intended and hence retained. Done. Leftover 'k' deleted. Thank you very much.
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	Page 5. Eqs. (5a)-(6a): Put a comma at the end of eq (5a) and a full stop at the end of eq. (6a). Please, delete “a” from the line numberings (5a) and (6a). Please apply general rule (h) everywhere. Paragraph after eq. (7): Please rewrite as “where $\mathbf{p} \mid p_n = [p_1 \ p_1 \ p_2 \ \dots \ p_{n-1}]^T$.”. Eqs. (10)-(10a): Please, use upright bold letters for p and p_n, replace “/” with “ ”, and use upright, non-bold “T” for the transpose symbol at the end of the matrices. Page 6. Last equation: Please, use upright bold letters for k and put a space between k_1 and k_2. Put also a comma at the end of the equation. Apply general rule (h) everuwhere. Page 7. Paragraph after eq. (11b): There is a leftover “k” at the beginning of the sentence. Please revise. Eq. (13a): Write this equation in a new (equation) line; not as text in a paragraph (which is also full justified). Put also a comma at the end of it. Page 8. Paragraph after eq. (17): Please rewrite as “...is replaced in the right-hand side (RHS) of...”. Eqs. (17)-(18): Please change to black color. Page 11. 1stParagraph: Please rewrite as “Figure 5 pinpoints the main problem with recursion...”.	Done Done. We try to highlight the change in (17) – (18) by using red colour. Done. Thank you very much. We really appreciate your useful comments.
<u>Optional/General</u> comments		